Online Social Networks and Media

Signed Graphs
Positive and Negative Ties
Signed networks

Examples
- Sign expresses agreement/disagreement between users
- Trust/distrust relationships
- Edit conflicts in Wikipedia
- Relationships between countries
- Synonyms and antonym relation between words
Structural Balance Theory

- originated in social psychology, by Heider in the 1940s
- graph-theoretic approach by Cartwright and Harary in the 1960s

Considers the possible ways in which triangles on three individuals can be signed

- All possible relationships between 3 people => 4 cases
Structural Balance

- **3 +**: Mutual friends

- **2 +, 1 -**: A is friend with B and C, but B and C do not get well together

- **1 +, 2 -**: A and B are friends with a mutual enemy

- **3 -**: Mutual enemies

Are there all equally possible?
Structural Balance

**3 +**

Mutual friends
“the friend of my friend is my friend”

Stable or balanced

**2 +, 1 -**

A is friend with B and C, but B and C do not get well together
*Implicit force to make B and C friends (- => +) or turn one of the + to -*

Unstable

**1 +, 2 -**

A and B are friends with a mutual enemy
“the enemy of my enemy is my friend”

Stable or balanced

**3 -**

Mutual enemies
*Forces to team up against the third (turn one of the – to +)*

Unstable
Structural Balance

A labeled complete graph is **balanced** if every one of its triangles is balanced

**Structural Balance Property:** For every set of three nodes, if we consider the three edges connecting them, either all three of these are labeled $+$, or else exactly one of them is labeled $+$, aka **odd number of $+$**

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**What does a balanced network look like?** *(global property)*
Balance Theorem: If a labeled complete graph is balanced,
(a) all pairs of nodes are friends, or
(b) the nodes can be divided into two groups $X$ and $Y$, such that every pair of nodes in $X$ like each other, every pair of nodes in $Y$ like each other, and everyone in $X$ is the enemy of everyone in $Y$.

From a local to a global property
Applications of Structural Balance

- How a network evolves over time
- Political science: International relationships
  - Example: the separation of Bangladesh from Pakistan
A Weaker Form of Structural Balance

Weak Structural Balance Property: There is no set of three nodes such that the edges among them consist of exactly two positive edges and one negative edge.

This is allowed

Unstable
Weakly Balance Theorem: If a labeled complete graph is weakly balanced, its nodes can be divided into groups in such a way that every two nodes belonging to the same group are friends, and every two nodes belonging to different groups are enemies.

From a local to a global property
A Weaker Form of Structural Balance

[Diagram showing relationships between sets and mutual friends inside sets, with mutual antagonism between all sets indicated.]
Generalizations

1. Non-complete graphs

2. Instead of all triangles, “most” triangles, approximately divide the graph

*We shall use the original (“non-weak”) definition of structural balance*
Structural Balance in Arbitrary Graphs

Three possible relations
- Positive edge
- Negative edge
- Absence of an edge

What is a good definition of balance in a non-complete graph?
Balance Definition for General Graphs

1. Based on triangles (local view)
2. Division of the network (global view)
Balance for General Graphs: local

A (non-complete) graph is balanced if it can be completed by adding edges to form a signed complete graph that is balanced.
Balance for General Graphs: local
A (non-complete) graph is **balanced** if it possible to divide the nodes into two sets $X$ and $Y$, such that any edge with both ends inside $X$ or both ends inside $Y$ is positive and any edge with one end in $X$ and one end in $Y$ is negative.

The two definitions are **equivalent**:

An arbitrary signed graph is balanced under the **first definition**, if and **only if**, it is balanced under the **second definition**.
Balance Definition for General Graphs

Algorithm for dividing the nodes?
Balance Characterization

What prevents a network from being balanced?

- Start from a node and place nodes in $X$ or $Y$
- Every time we cross a negative edge, change the set

Cycle with odd number of negative edges
Balance Definition for General Graphs

Is there such a cycle with an odd number of -?
Balance Definition for General Graphs

Is there such a cycle with an odd number of -?
Claim: A signed graph is balanced, if and only if, it contains no cycles with an odd number of negative edges.

Proof by construction

Find a balanced division: partition into sets X and Y, all edges inside X and Y positive, crossing edges negative.

Either succeeds or Stops with a cycle containing an odd number of -

Two steps:
1. Convert the graph into a reduced one with only negative edges
2. Solve the problem in the reduced graph
Balance Characterization: Step 1

Find connected components of $G$ by considering only positive edges, let us call then **supernodes**

Check: Do supernodes contain a negative edge between any pair of their nodes
If Yes, $G$ is unbalanced
Proof: say - between A and B, A and B connected by an all-positive path $\rightarrow$ odd cycle
Balance Characterization: Step 1

Else:
Reduce the problem: a node for each supernode, an edge between two supernodes if an edge in the original
Balance Characterization: Step 2

Note: Only negative edges among supernodes

Start labeling each supernode by either $X$ and $Y$
If successful, then label the nodes of the supernode correspondingly
▪ A cycle with an odd number, corresponds to a (possibly larger) odd cycle in the original
Odd cycle
Determining whether the graph is bipartite
there is no edge between nodes in $X$ or $Y$, the only edges are from nodes in $X$ to nodes in $Y$

Use Breadth-First-Search (BFS)
Two type of edges: (1) between nodes in adjacent levels (2) between nodes in the same level

If only type (1), alternate X and Y labels at each level

If type (2), then odd cycle
Balance Characterization: Step 2

An odd cycle is formed from two equal-length paths leading to an edge inside a single layer.
Status theory in practice

- **Epinions**: product review Web site, where users can indicate their *trust* or *distrust* of the reviews
- **Slashdot**: the social network of the blog where a signed link indicates that one user *likes* or *dislikes* the comments
- **Wikipedia**: its voting network where a signed link indicates a positive or negative *vote* by one user *on the promotion* to admin status of another.
Structural balance theory in practice

All-positive triad $T_3$ is heavily overrepresented in all three datasets. $T_3$ tends to be overrepresented by about 40% in all three datasets.

Triad $T_2$ consisting of two enemies with a common friend is heavily underrepresented. $T_2$ is underrepresented by about 75% in Epinions and Slashdot and 50% in Wikipedia.

More consistent with weak structural balance
A theory of status

**Directed networks**

A *positive edge* \((A, B)\) means that \(A\) regards \(B\) as having *higher status* than \(A\)

A *negative edge* \((A, B)\) means that \(A\) regards \(B\) as having *lower status* than \(A\)

Assuming that all participants agree on status ordering, status theory predicts that when the *direction* of an edge is flipped, its *sign* should flip as well.
A theory of status

Structural balance
A theory of status: local property

For any edge \((u, v)\), and any third node \(w\), possible to assign distinct numerical “status values” to \(u\), \(v\), and \(w\) in such a way that the positive edges among them (if any) go from nodes of lower status to nodes of higher status, and the negative edges among them (if any) go from nodes of higher status to nodes of lower status.

Three nodes \(u\), \(v\), and \(w\) are status-consistent if this condition holds.
Let $G$ be a signed, directed graph, and suppose that all sets of three nodes in $G$ are status-consistent. Then it possible to order the nodes of $G$ as $v_1, v_2, \ldots, v_n$ in such a way that each positive edge $(v_i, v_j)$ satisfies $i < j$, and each negative edge $(v_i, v_j)$ satisfies $i > j$. 
Summary

Signed networks
Two interpretations
- Friendship/Foe (undirected)
- Status (directed)

Both at a local and global level
Questions?
References

Networks, Crowds, and Markets (Chapter 5)
Extra material
USA support to Pakistan?

“[T]he United States’s somewhat surprising support of Pakistan ... becomes less surprising when one considers that the USSR was China’s enemy, China was India’s foe, and India had traditionally bad relations with Pakistan. Since the U.S. was at that time improving its relations with China, it supported the enemies of China’s enemies.

Further reverberations of this strange political constellation became inevitable: North Vietnam made friendly gestures toward India, Pakistan severed diplomatic relations with those countries of the Eastern Bloc which recognized Bangladesh, and China vetoed the acceptance of Bangladesh into the U.N.”
Applications of Structural Balance

✓ International relationships (I)

The conflict of Bangladesh’s separation from Pakistan in 1972 (II)

China?
Applications of Structural Balance

International relationships (II)

(a) *Three Emperors’ League 1872–81*
(b) *Triple Alliance 1882*
(c) *German-Russian Lapse 1890*

(d) *French-Russian Alliance 1891–94*
(e) *Entente Cordiale 1904*
(f) *British Russian Alliance 1907*

Figure 5.5: The evolution of alliances in Europe, 1872-1907 (the nations GB, Fr, Ru, It, Ge, and AH are Great Britain, France, Russia, Italy, Germany, and Austria-Hungary respectively). Solid dark edges indicate friendship while dotted red edges indicate enmity. Note how the network slides into a balanced labeling — and into World War I. This figure and example are from Antal, Krapivsky, and Redner [20].