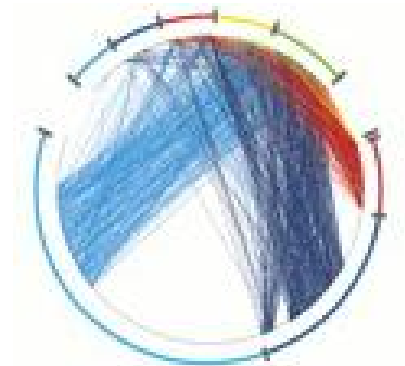
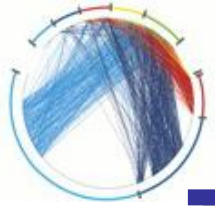


Models and Algorithms for Complex Networks

Networks and Measurements

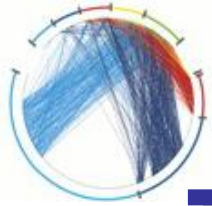
Lecture 3





Types of networks

- § Social networks
- § Knowledge (Information) networks
- § Technology networks
- § Biological networks



Social Networks

§ Links denote a social interaction

§ Networks of acquaintances

§ collaboration networks

- actor networks
- co-authorship networks
- director networks

§ phone-call networks

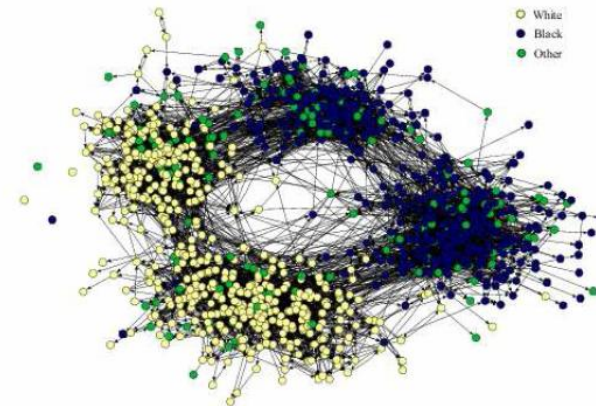
§ e-mail networks

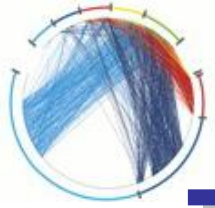
§ IM networks

§ Bluetooth networks

§ sexual networks

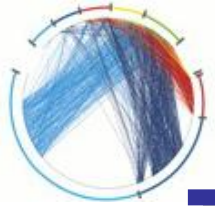
§ home page/blog networks





Knowledge (Information) Networks

- § Nodes store information, links associate information
- § Citation network (directed acyclic)
- § The Web (directed)
- § Peer-to-Peer networks
- § Word networks
- § Networks of Trust
- § Software graphs



Technological networks

§ Networks built for distribution of commodity

§ The Internet

- router level, AS level

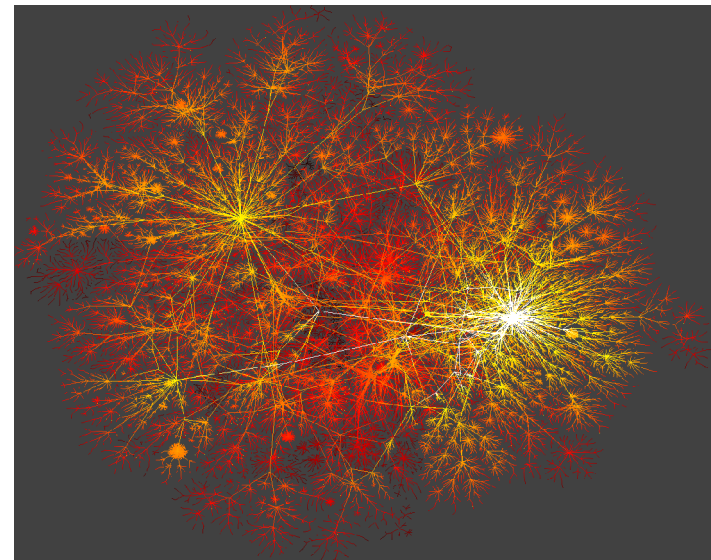
§ Power Grids

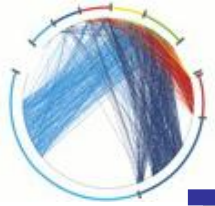
§ Airline networks

§ Telephone networks

§ Transportation Networks

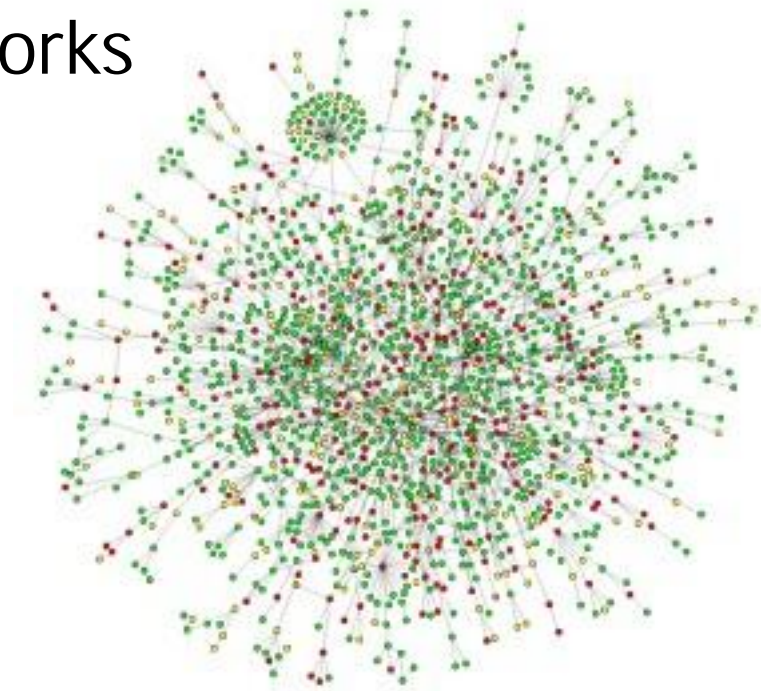
- roads, railways, pedestrian traffic

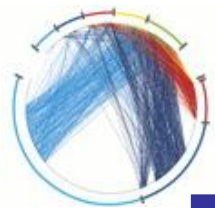




Biological networks

- § Biological systems represented as networks
 - § Protein-Protein Interaction Networks
 - § Gene regulation networks
 - § Gene co-expression networks
 - § Metabolic pathways
 - § The Food Web
 - § Neural Networks





Measuring Networks

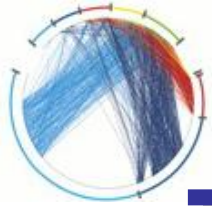
- § Degree distributions
- § Small world phenomena
- § Clustering Coefficient
- § Mixing patterns
- § Degree correlations
- § Communities and clusters



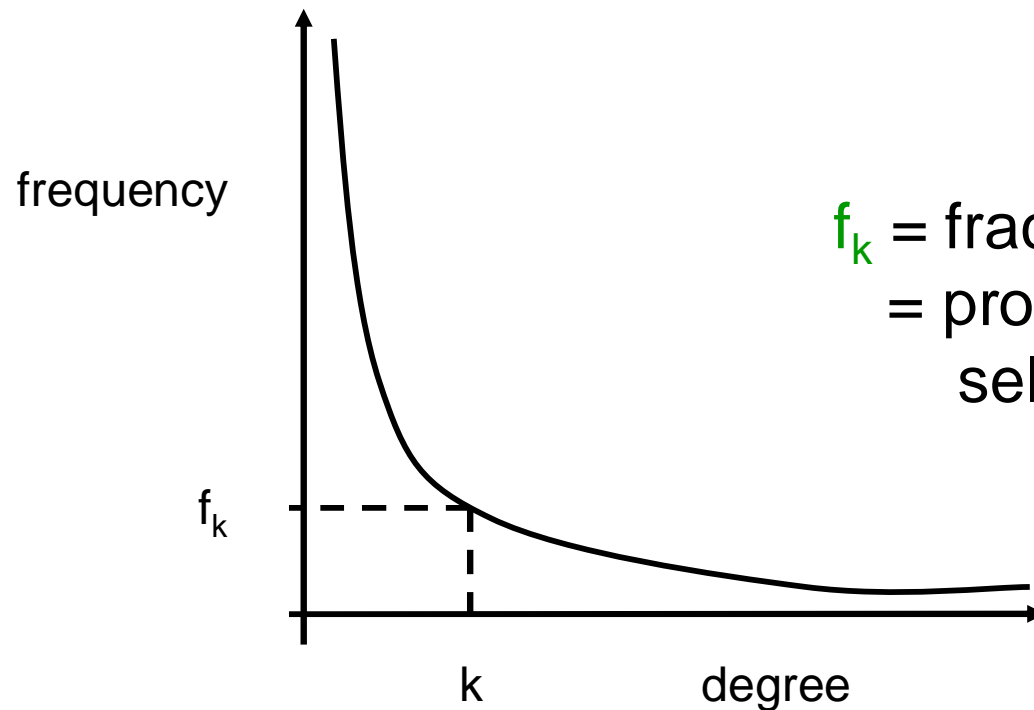
The basic random graph model

- § The measurements on real networks are usually compared against those on “random networks”

- § The basic $G_{n,p}$ (Erdős-Renyi) random graph model:
 - § n : the number of vertices
 - § $0 \leq p \leq 1$
 - § for each pair (i,j) , generate the edge (i,j) **independently** with probability p

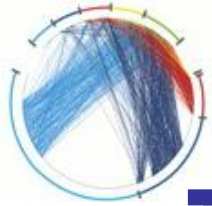


Degree distributions



f_k = fraction of nodes with degree k
= probability of a randomly
selected node to have degree k

§ Problem: find the probability distribution that best fits the observed data



Power-law distributions

- § The degree distributions of most real-life networks follow a **power law**

$$p(k) = Ck^{-\alpha}$$

- § Right-skewed/Heavy-tail distribution

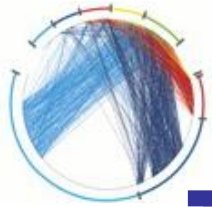
- § there is a non-negligible fraction of nodes that has very high degree (hubs)
- § **scale-free**: no characteristic scale, average is not informative

- § In stark contrast with the random graph model!

- § Poisson degree distribution, $z=np$

$$p(k) = P(k; z) = \frac{z^k}{k!} e^{-z}$$

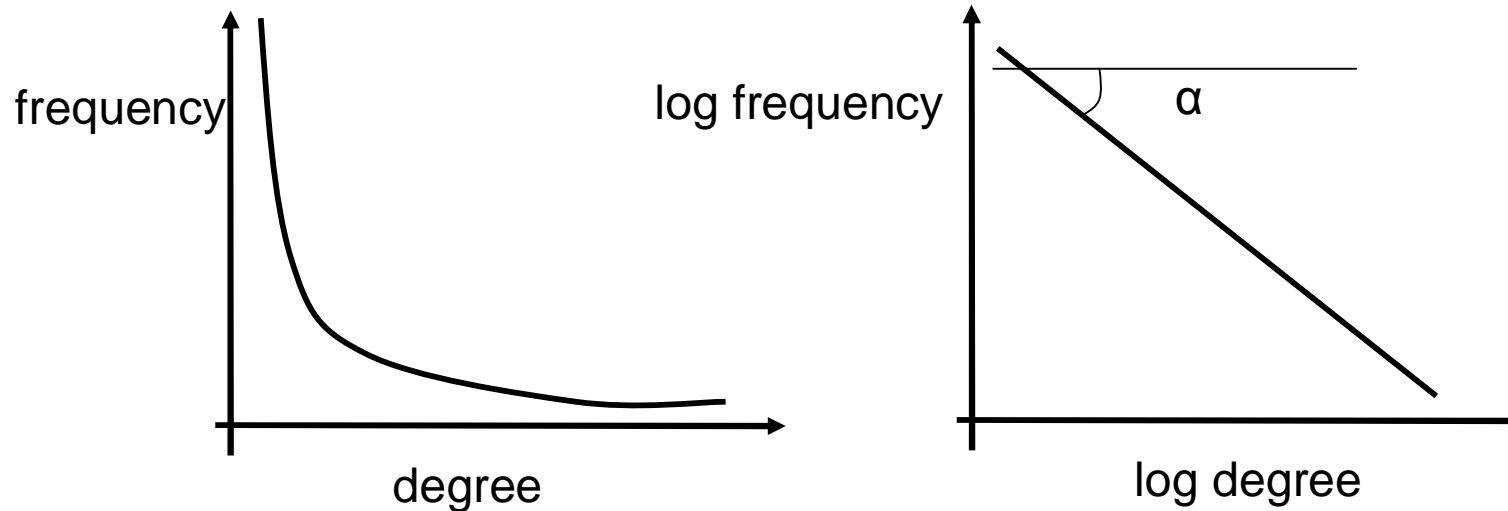
- § highly concentrated around the mean
- § the probability of very high degree nodes is exponentially small



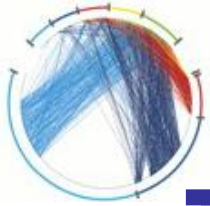
Power-law signature

§ Power-law distribution gives a line in the log-log plot

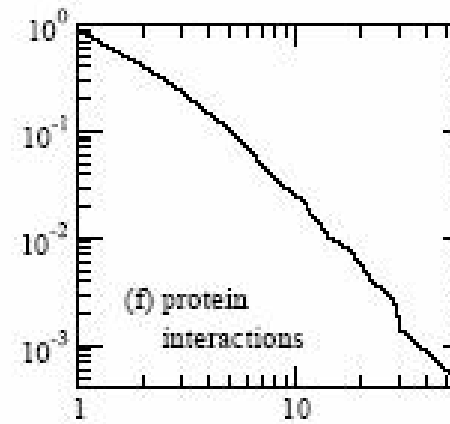
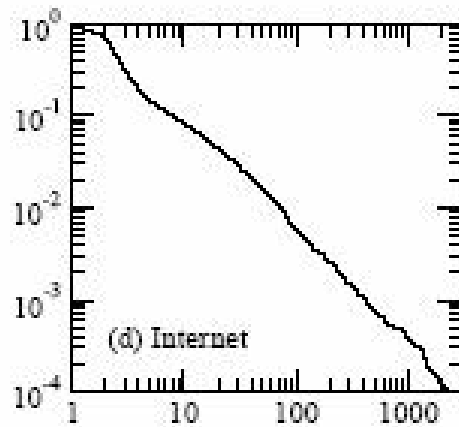
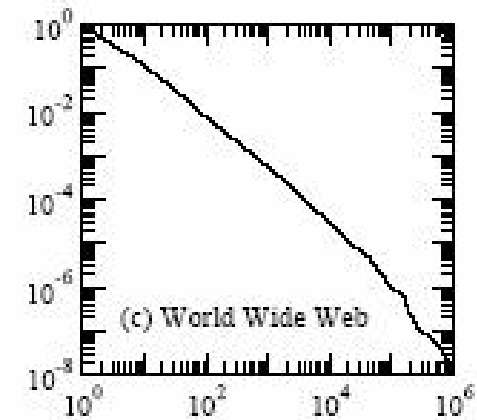
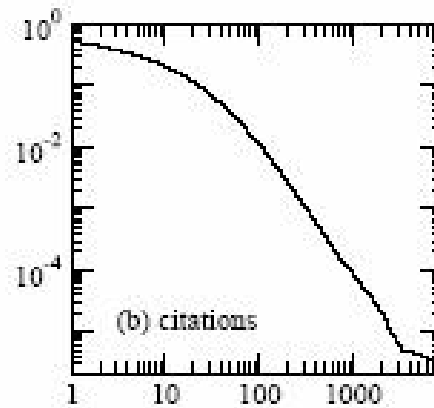
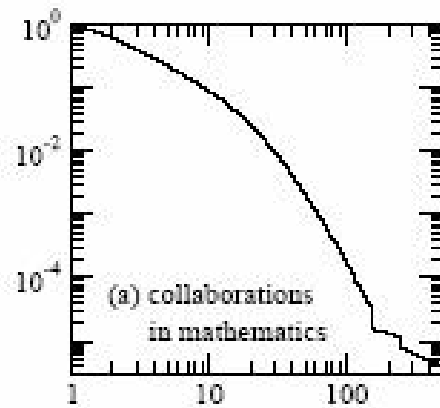
$$\log p(k) = -\alpha \log k + \log C$$



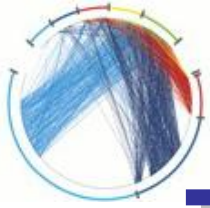
§ α : power-law exponent (typically $2 \leq \alpha \leq 3$)



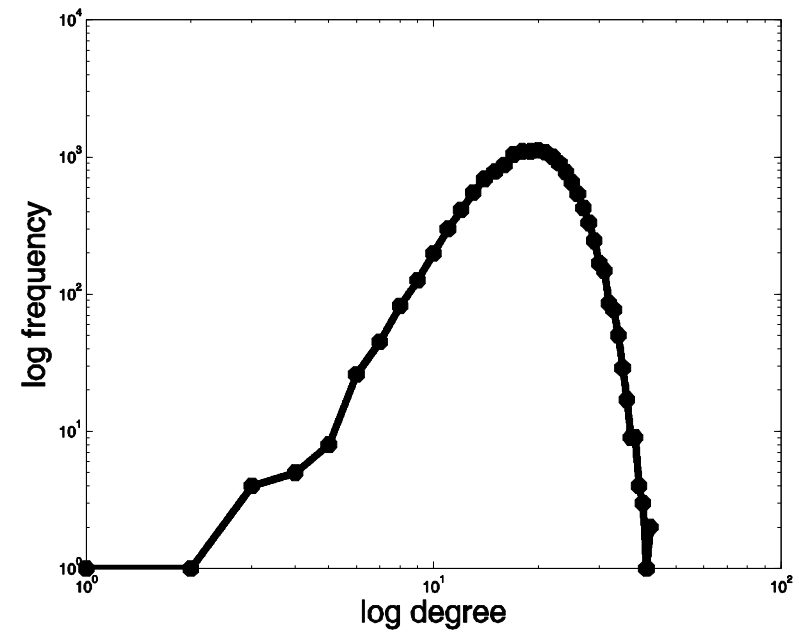
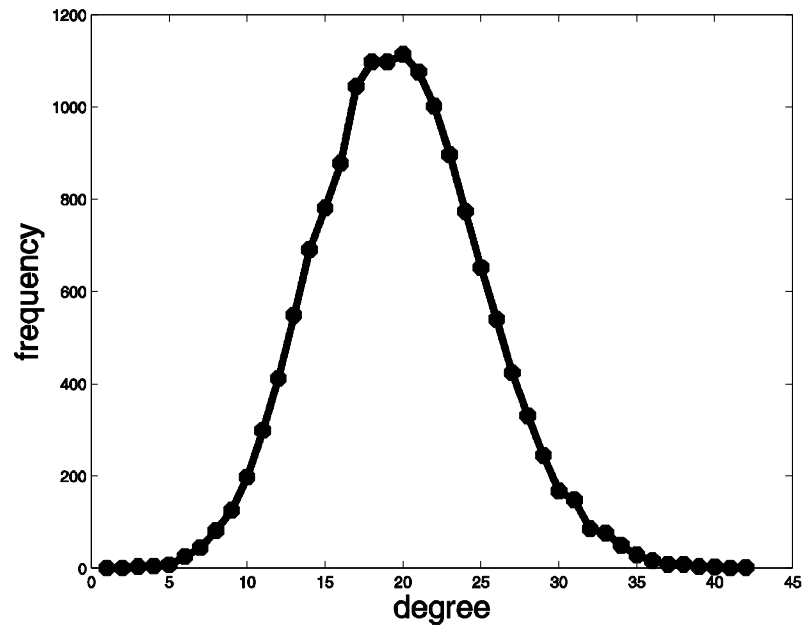
Examples

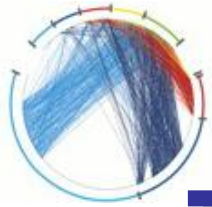


Taken from [Newman 2003]



A random graph example





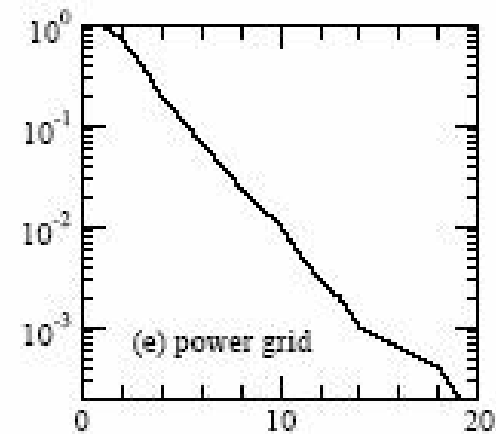
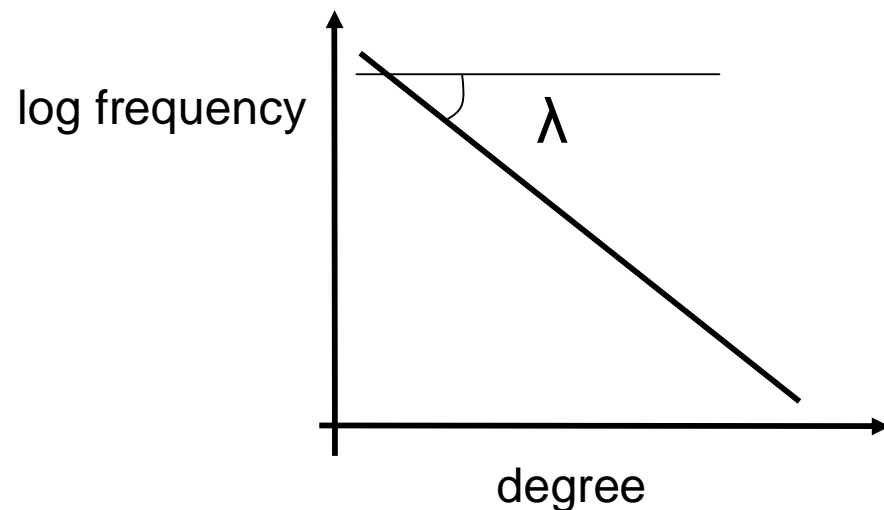
Exponential distribution

§ Observed in some technological or collaboration networks

$$p(k) = \lambda e^{-\lambda k}$$

§ Identified by a line in the log-linear plot

$$\log p(k) = -\lambda k + \log \lambda$$





Average/Expected degree

§ For random graphs $z = np$

§ For power-law distributed degree

§ if $\alpha \geq 2$, it is a constant

§ if $\alpha < 2$, it diverges



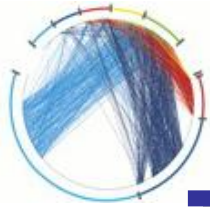
Maximum degree

§ For random graphs, the maximum degree is highly concentrated around the average degree z

§ For power law graphs

$$k_{\max} \approx n^{1/(a-1)}$$

§ Rough argument: solve $nP[X \geq k] = 1$



Collective Statistics (M. Newman 2003)

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	–0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	–0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	–0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	–0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	–0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	–0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	–0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	–0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	–0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	–0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	–0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	–0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	–0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	–0.226	416, 421

TABLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices n ; total number of edges m ; mean degree z ; mean vertex-vertex distance ℓ ; exponent α of degree distribution if the distribution follows a power law (or “–” if not; in/out-degree exponents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r , Sec. III.F. The last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

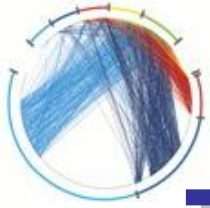


Clustering (Transitivity) coefficient

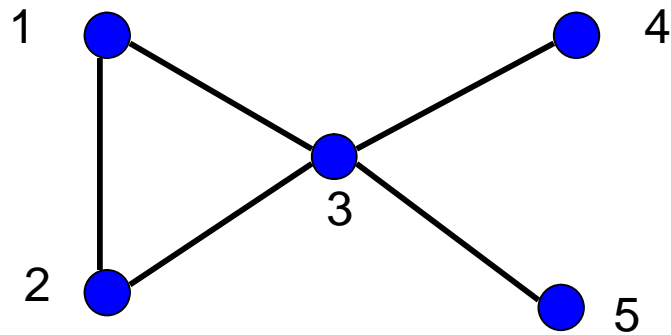
- § Measures the density of triangles (local clusters) in the graph
- § Two different ways to measure it:

$$C^{(1)} = \frac{\sum_i \text{triangles centered at node } i}{\sum_i \text{triples centered at node } i}$$

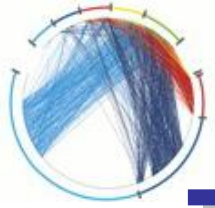
- § The ratio of the means



Example



$$C^{(1)} = \frac{3}{1+1+6} = \frac{3}{8}$$



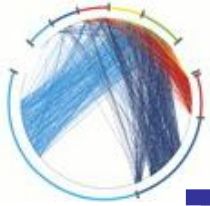
Clustering (Transitivity) coefficient

§ Clustering coefficient for node i

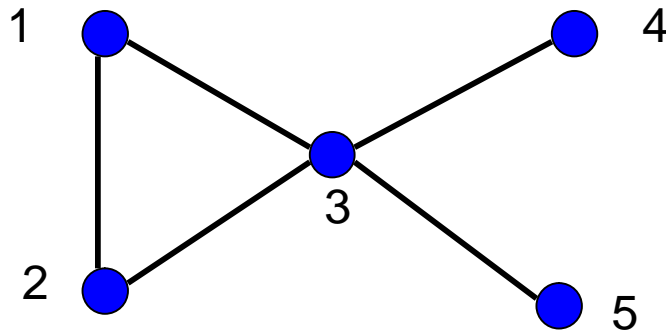
$$C_i = \frac{\text{triangles centered at node } i}{\text{triples centered at node } i}$$

$$C^{(2)} = \frac{1}{n} C_i$$

§ The mean of the ratios



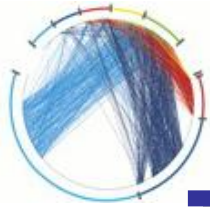
Example



$$C^{(2)} = \frac{1}{5} (1 + 1 + 1/6) = \frac{13}{30}$$

$$C^{(1)} = \frac{3}{8}$$

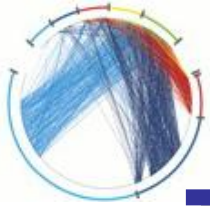
- § The two clustering coefficients give different measures
- § $C^{(2)}$ increases with nodes with low degree



Collective Statistics (M. Newman 2003)

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
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TABLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices n ; total number of edges m ; mean degree z ; mean vertex-vertex distance ℓ ; exponent α of degree distribution if the distribution follows a power law (or “–” if not; in/out-degree exponents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r , Sec. III.F. The last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

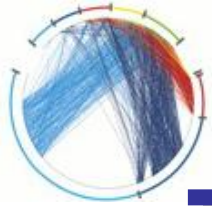


Clustering coefficient for random graphs

- § The probability of two of your neighbors also being neighbors is p , independent of local structure
 - § clustering coefficient $C = p$
 - § when z is fixed $C = z/n = O(1/n)$

Table 1: Clustering coefficients, C , for a number of different networks; n is the number of node, z is the mean degree. Taken from [146].

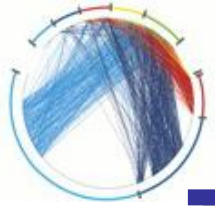
Network	n	z	C measured	C for random graph
Internet [153]	6,374	3.8	0.24	0.00060
World Wide Web (sites) [2]	153,127	35.2	0.11	0.00023
power grid [192]	4,941	2.7	0.080	0.00054
biology collaborations [140]	1,520,251	15.5	0.081	0.000010
mathematics collaborations [141]	253,339	3.9	0.15	0.000015
film actor collaborations [149]	449,913	113.4	0.20	0.00025
company directors [149]	7,673	14.4	0.59	0.0019
word co-occurrence [90]	460,902	70.1	0.44	0.00015
neural network [192]	282	14.0	0.28	0.049
metabolic network [69]	315	28.3	0.59	0.090
food web [138]	134	8.7	0.22	0.065



Millgram's small world experiment

- § Letters were handed out to people in Nebraska to be sent to a target in Boston
- § People were instructed to pass on the letters to someone they knew on first-name basis
- § The letters that reached the destination followed paths of length around 6
- § **Six degrees of separation:** (play of John Guare)

- § Also:
 - § The Kevin Bacon game
 - § The Erdős number
- § Small world project:
<http://smallworld.columbia.edu/index.html>



Measuring the small world phenomenon

§ d_{ij} = shortest path between i and j

§ Diameter:

$$d = \max_{i,j} d_{ij}$$

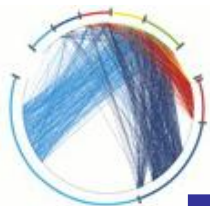
§ Characteristic path length:

$$\} = \frac{1}{n(n-1)/2} \sum_{i>j} d_{ij}$$

§ Harmonic mean

$$\}^{-1} = \frac{1}{n(n-1)/2} \sum_{i>j} d_{ij}^{-1}$$

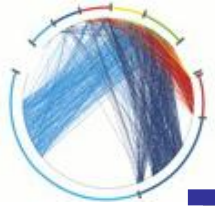
§ Also, distribution of all shortest paths



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	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	–0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	–0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	–0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	–0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	–0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	–0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	–0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	–0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	–0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	–0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	–0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	–0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	–0.226	416, 421

TABLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices n ; total number of edges m ; mean degree z ; mean vertex-vertex distance ℓ ; exponent α of degree distribution if the distribution follows a power law (or “–” if not; in/out-degree exponents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r , Sec. III.F. The last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.



Is the path length enough?

§ Random graphs have diameter

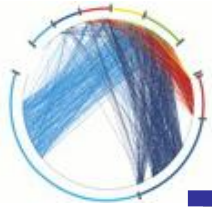
$$d = \frac{\log n}{\log z}$$

§ $d = \log n / \log \log n$ when $z = \omega(\log n)$

§ Short paths should be combined with other properties

§ ease of navigation

§ high clustering coefficient



Degree correlations

- § Do high degree nodes tend to link to high degree nodes?
- § Pastor Satorras et al.
 - § plot the mean degree of the neighbors as a function of the degree

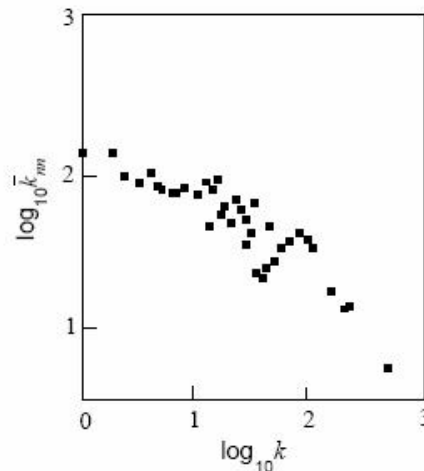
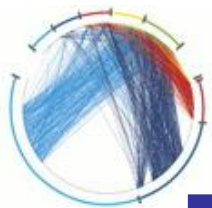


FIG. 3.13. Correlations of the degrees of nearest-neighbour vertices (autonomous systems) in the Internet at the interdomain level (after Pastor-Satorras, Vázquez, and Vespignani 2001). The empirical dependence of the average degree of the nearest neighbours of a vertex on the degree of this vertex is shown in a log-log scale. This empirical dependence was fitted by a power law with exponent approximately 0.5.



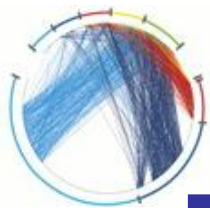
Degree correlations

§ Newman

§ compute the **correlation coefficient** of the degrees of the two endpoints of an edge

§ assortative/disassortative

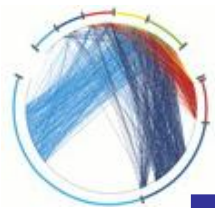
$$r = \frac{M^{-1} \sum_i j_i k_i - [M^{-1} \sum_i \frac{1}{2}(j_i + k_i)]^2}{M^{-1} \sum_i \frac{1}{2}(j_i^2 + k_i^2) - [M^{-1} \sum_i \frac{1}{2}(j_i + k_i)]^2},$$



Collective Statistics (M. Newman 2003)

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	–0.029	45
sexual contacts	undirected	2 810				3.2				265, 266	
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	–0.067	14, 34
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biological	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	–0.156	212
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TABLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices n ; total number of edges m ; mean degree z ; mean vertex-vertex distance ℓ ; exponent α of degree distribution if the distribution follows a power law (or “–” if not; in/out-degree exponents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r , Sec. III.F. The last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.



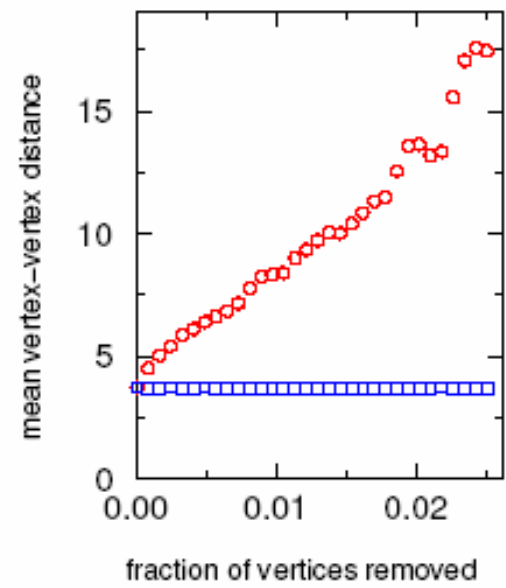
Connected components

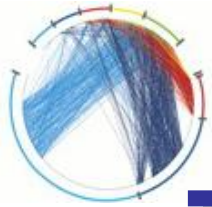
- § For undirected graphs, the size and distribution of the connected components
 - § is there a **giant component**?
- § For directed graphs, the size and distribution of strongly and weakly connected components



Network Resilience

§ Study how the graph properties change when performing random or targeted node deletions





Graph eigenvalues

§ For random graphs

§ semi-circle law

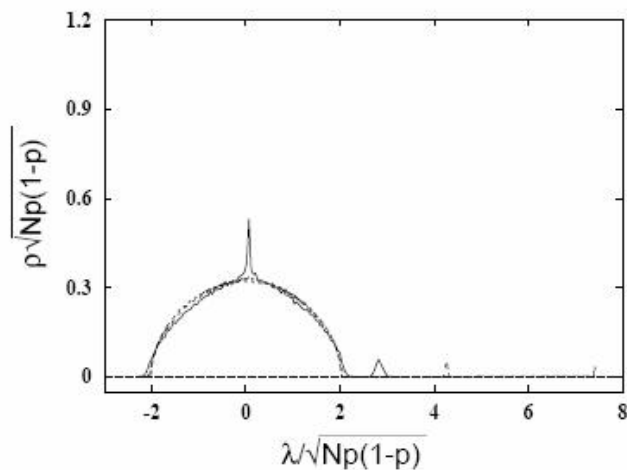
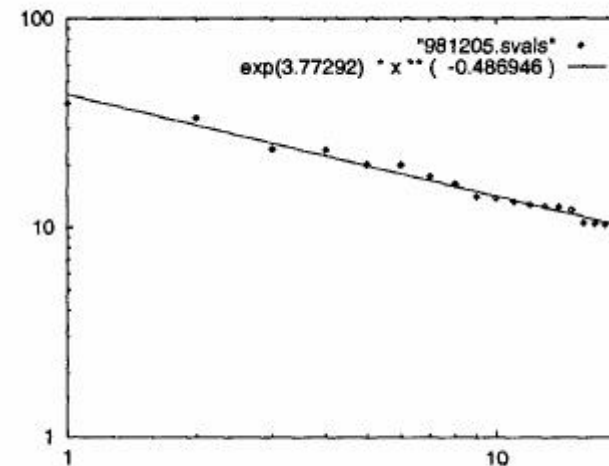
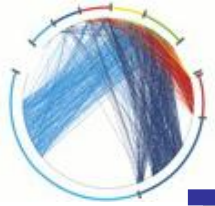


FIG. 10. Rescaled spectral density of three random graphs having $p = 0.05$ and size $N = 100$ (continuous line), $N = 300$ (dashed line) and $N = 1000$ (short-dashed line). The isolated peak corresponds to the principal eigenvalue. After Farkas *et al.* 2001.

§ For the Internet
(Faloutsos³)



(a) Int-12-98



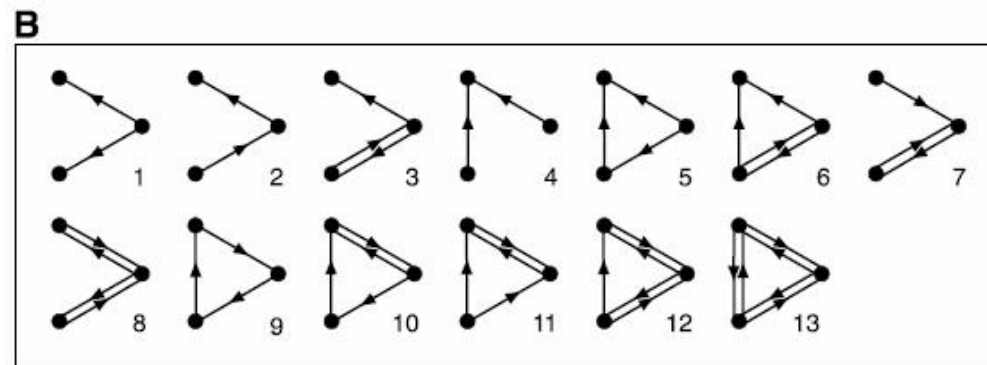
Motifs

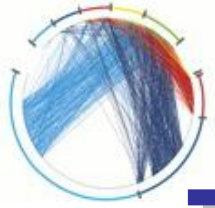
- § Most networks have the same characteristics with respect to global measurements
 - § can we say something about the local structure of the networks?
- § Motifs: Find small subgraphs that over-represented in the network



Example

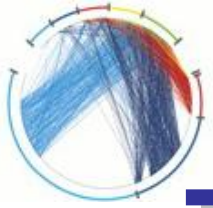
§ Motifs of size 3 in a directed graph





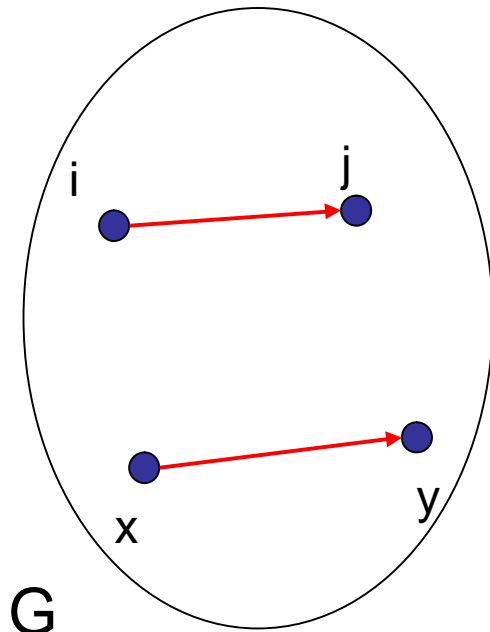
Finding interesting motifs

- § Sample a part of the graph of size S
- § Count the frequency of the motifs of interest
- § Compare against the frequency of the motif in a random graph with the same number of nodes **and** the same degree distribution

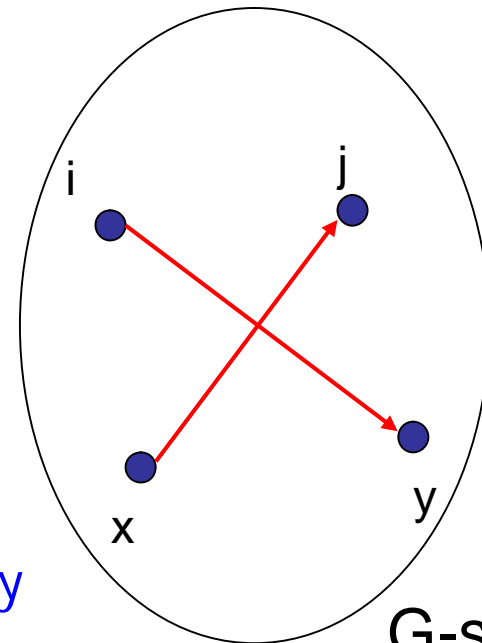


Generating a random graph

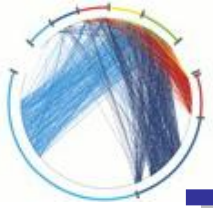
- § Find edges (i,j) and (x,y) such that edges (i,y) and (x,j) do not exist, and swap them
- § repeat for a large enough number of times



degrees of i,j,x,y
are preserved



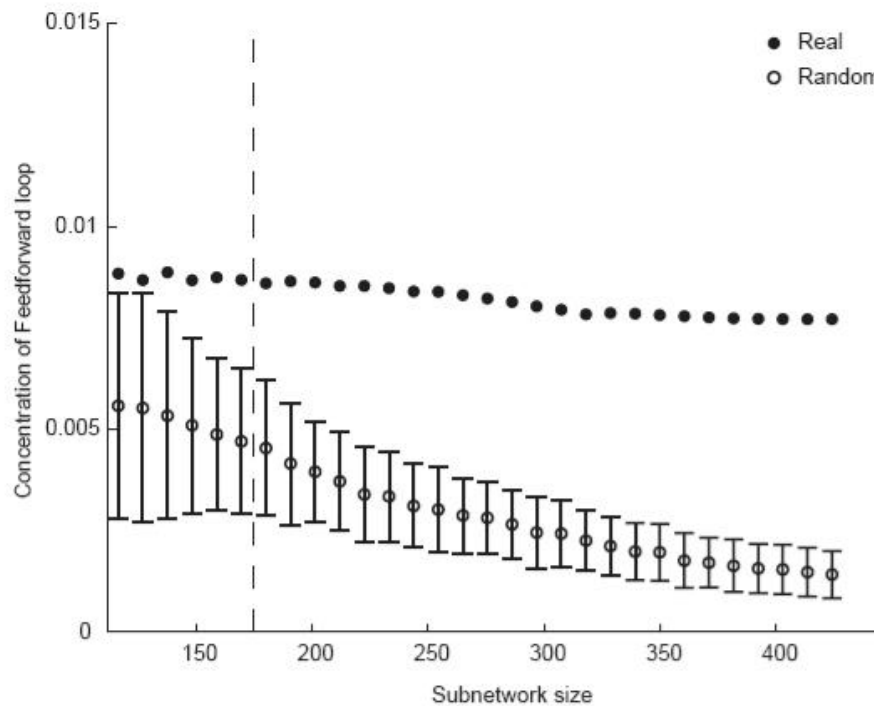
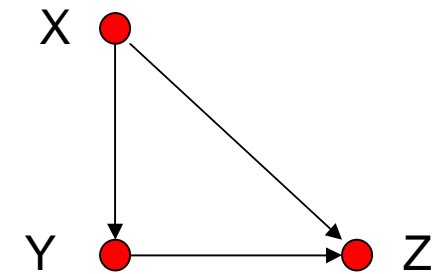
G-swapped



The feed-forward loop

§ Over-represented in gene-regulation networks

§ a signal delay mechanism

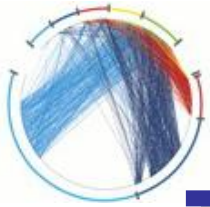




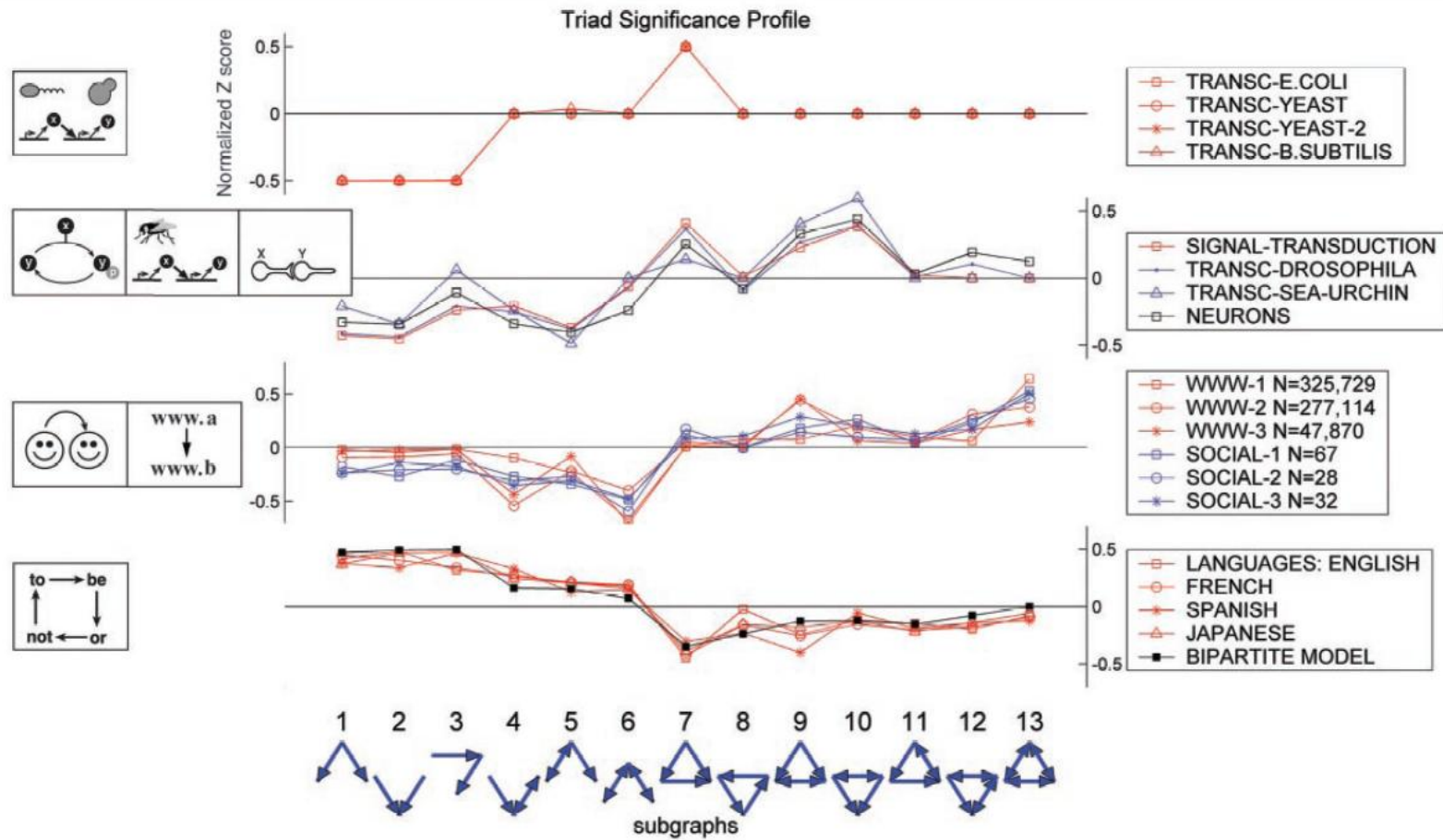
Families of networks

§ Compute the relative frequency of different motifs, and group the networks if they exhibit similar frequencies

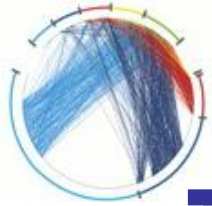
$$Z_i = (N_{\text{real}_i} - \langle N_{\text{rand}_i} \rangle) / \text{std}(N_{\text{rand}_i})$$



Experiments



Milo et al. 2004



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- § R. Albert and A.-L. Barabási, [Statistical mechanics of complex networks](#), *Reviews of Modern Physics* 74, 47-97 (2002).
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