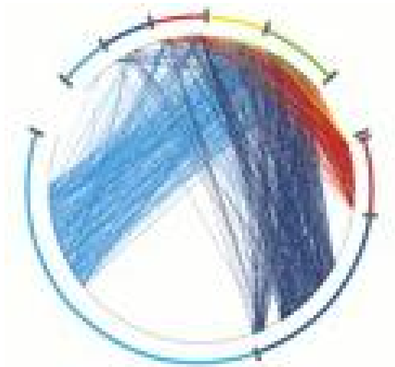
Information Networks

Failures and Epidemics in Networks



Spread in Networks

- Understanding the spread of viruses (or rumors, information, failures etc) is one of the driving forces behind network analysis
  - predict and prevent epidemic outbreaks (e.g. the Bird-flu outbreak)
  - protect computer networks (e.g. against worms)
  - predict and prevent cascading failures (U.S. power grid)
  - understanding of fads, rumors, trends
    - viral marketing
  - anti-terrorism?

Percolation in Networks

- Site Percolation: Each node of the network is randomly set as occupied or not-occupied. We are interested in measuring the size of the largest connected component of occupied vertices
- Bond Percolation: Each edge of the network is randomly set as occupied or not-occupied. We are interested in measuring the size of the largest component of nodes connected by occupied edges

- Good model for failures or attacks

Percolation Threshold

- How many nodes should be occupied in order for the network to not have a giant component? (the network does not percolate)

Percolation Threshold for the configuration model

- If p_k is the fraction of nodes with degree k, then if a fraction q of the nodes is occupied, the probability of a node to have degree m is

$$p_m' = \sum_{k=m}^{\infty} p_k \binom{k}{m} q^m (1-q)^{k-m}$$

- This defines a new configuration model
  - apply the known threshold
- For scale free graphs we have $q_c \leq 0$ for power law exponent less than 3!
  - there is always a giant component (the network always percolates)

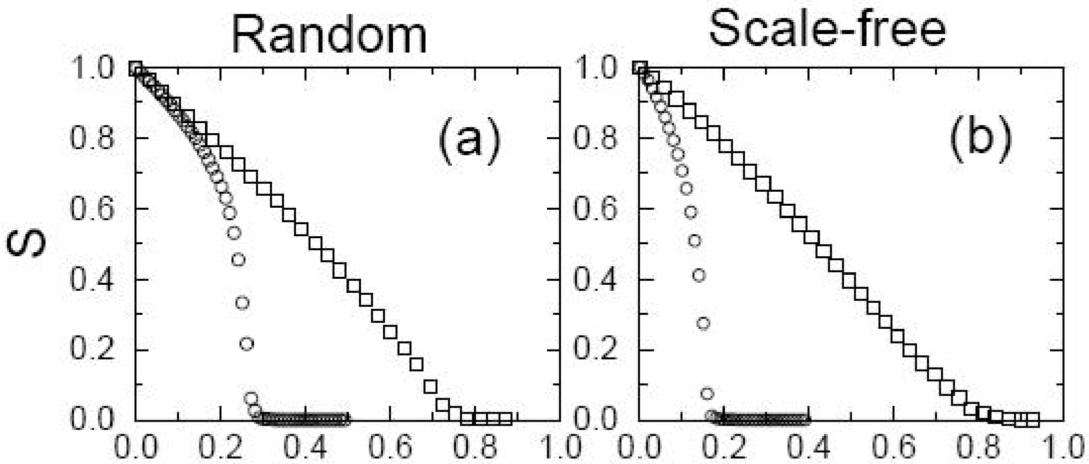
Percolation threshold

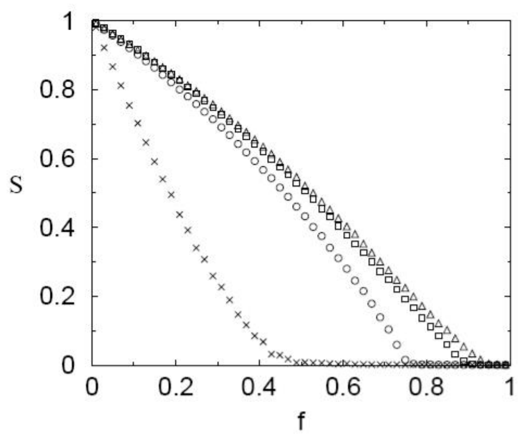
- An analysis for general graphs is and general occupation probabilities is possible
  - for scale free graphs it yields the same results

- But ... if the nodes are removed preferentially (according to degree), then it is easy to disconnect a scale free graph by removing a small fraction of the edges

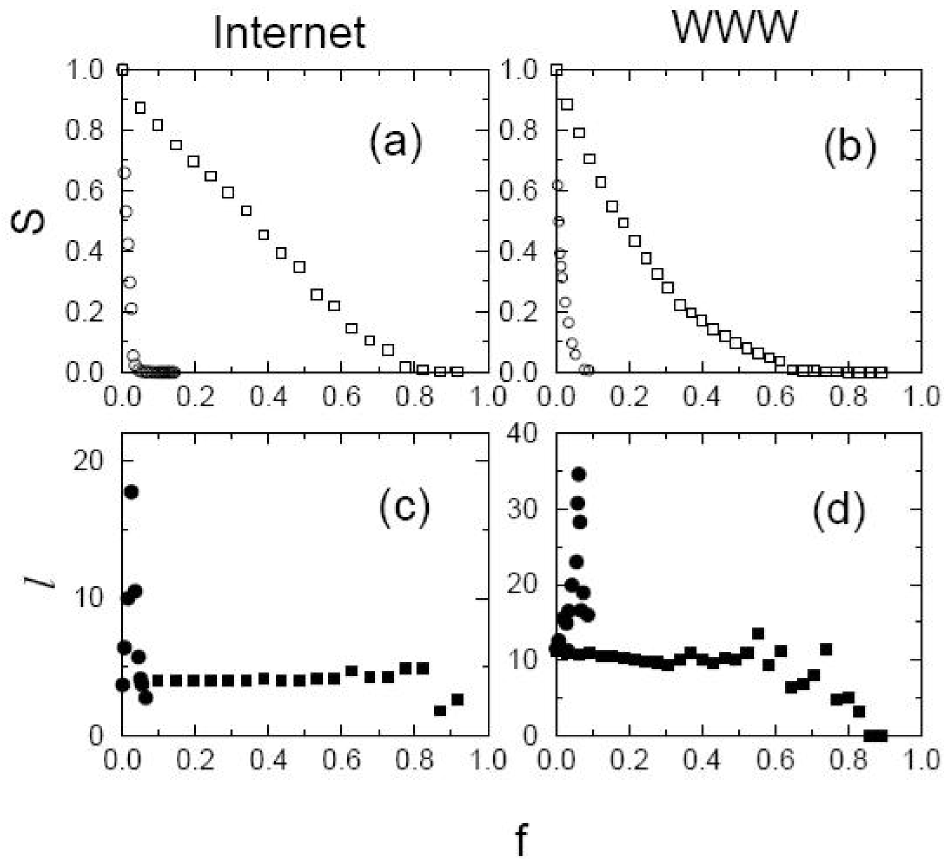
Network resilience

§ Scale-free graphs are resilient to random attacks, but sensitive to targeted attacks. For random networks there is smaller difference between the two





Real networks



Cascading failures

- Each node has a load and a capacity that says how much load it can tolerate.
- When a node is removed from the network its load is redistributed to the remaining nodes.
- If the load of a node exceeds its capacity, then the node fails

Cascading failures: example

- The load of a node is the betweeness centrality of the node
- The capacity of the node is $C = (1+b)L$
  - the parameter b captures the additional load a node can handle

Cascading failures in SF graphs

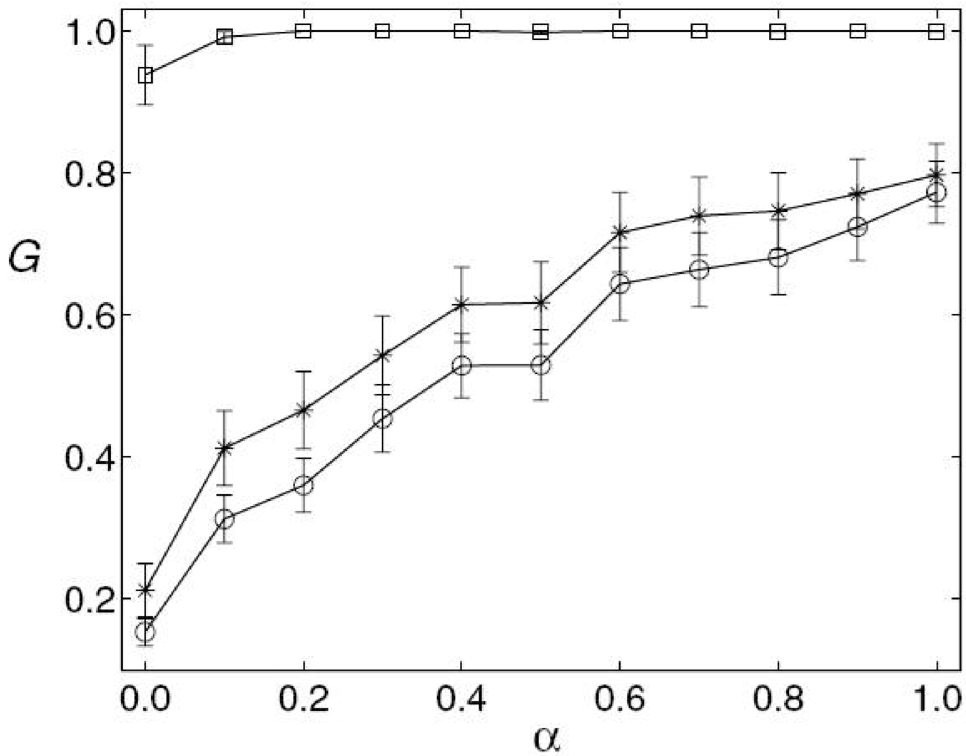


Fig. 2. Cascading failure in scale-free networks with scaling exponent $\gamma = 3$, as triggered by the removal of one node chosen at random (squares), or among those with largest connectivities (stars) or highest loads (circles). Each curve corresponds to the average over 5 triggers and 10 realizations of the network. The error bars represent the standard deviation. The number of nodes in the largest component is $5000 \leq N \leq 5100$.

The SIR model

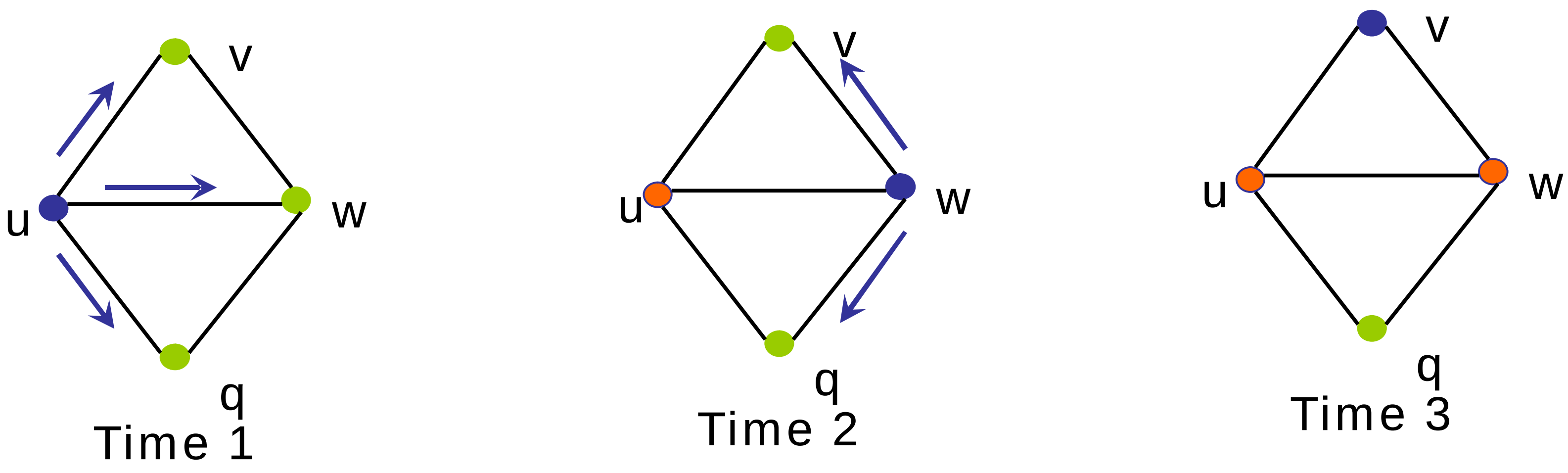
- Each node may be in the following states
  - Susceptible: healthy but not immune
  - Infected: has the virus and can actively propagate it
  - Recovered: (or Removed/Immune/Dead) had the virus but it is no longer active
- Infection rate p: probability of getting infected by a neighbor per unit time
- Immunization rate q: probability of a node getting recovered per unit time

The SIR model

- It can be shown that virus propagation can be reduced to the bond-percolation problem for appropriately chosen probabilities
  - again, there is no percolation threshold for scale-free graphs

A simple SIR model

- Time proceeds in discrete time-steps
- If a node is infected at time t it infects all its neighbors with probability p
- Then the node becomes recovered (q = 1)



The caveman small-world graphs

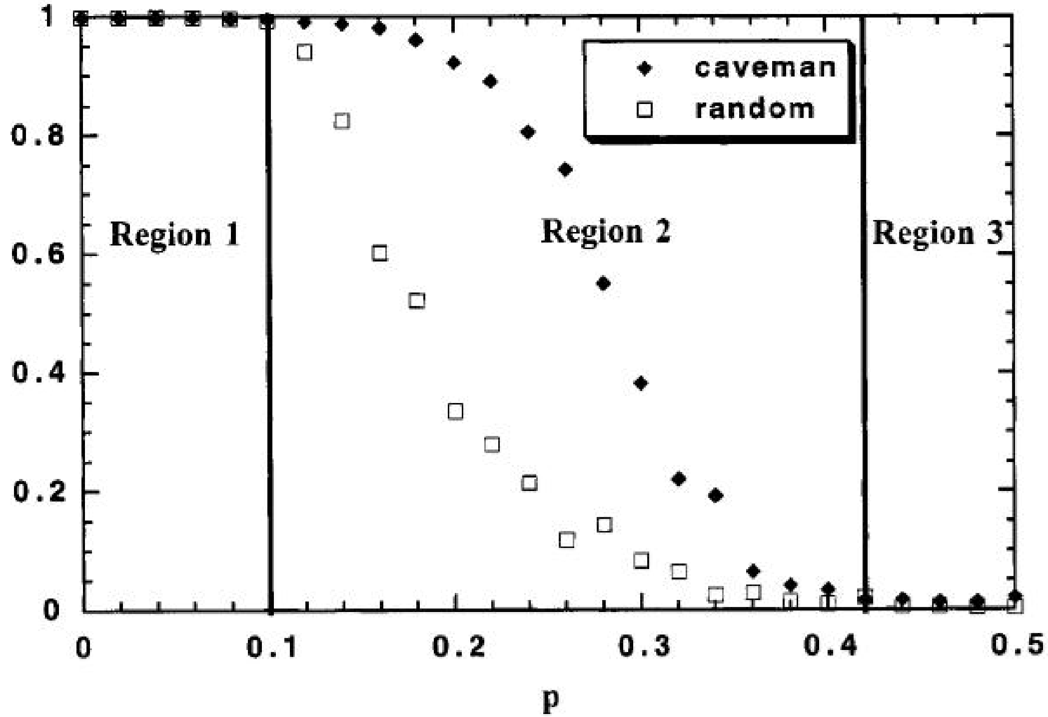


FIG. 11.—Fraction of uninfected survivors (F_s) versus infectiousness (p) for disease spreading dynamics on a network generated by the α-model at clustered and random extremes.

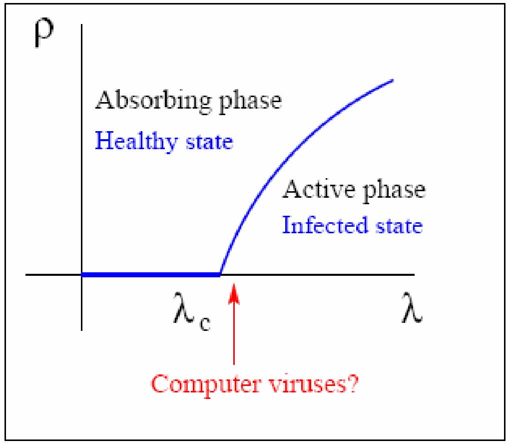
The SIS model

- Susceptible-Infected-Susceptible:
  - each node may be healthy (susceptible) or infected
  - a healthy node that has an infected neighbor becomes infected with probability p
  - an infected node becomes healthy with probability q
  - spreading rate r=p/q

Epidemic Threshold

- The epidemic threshold for the SIS model is a value r_c such that for $r < r_c$ the virus dies out, while for $r > r_c$ the virus spreads.
- For homogeneous graphs,

$$r_c = \frac{1}{\langle k \rangle}$$



- For scale free graphs

$$r_c = \frac{\langle k \rangle}{\langle k^2 \rangle}$$

- For exponent less than 3, the variance is infinite, and the epidemic threshold is zero

An eigenvalue point of view

- Consider the SIS model, where every neighbor may infect a node with probability p. The probability of getting cured is q
- If A is the adjacency matrix of the network, then the virus dies out if

$$\lambda_1(A) \leq \frac{q}{p}$$

- That is, the epidemic threshold is $r_c = 1/\lambda_1(A)$

The SIS model

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  - a healthy node that has an infected neighbor becomes infected with probability p
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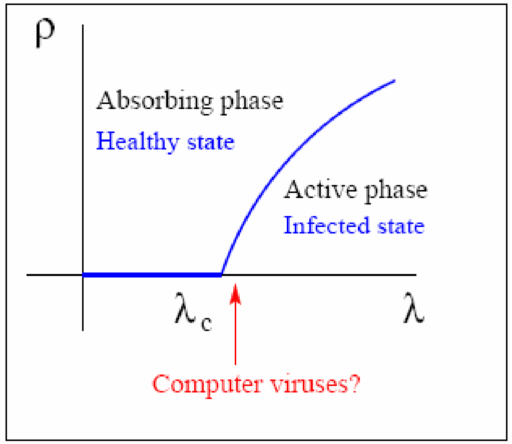

- For scale free graphs

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An eigenvalue point of view

- Time proceeds in discrete timesteps. At time t,
  - an infected node u infects a healthy neighbor v with probability p.
  - node u becomes healthy with probability q
- If A is the adjacency matrix of the network, then the virus dies out if

$$\lambda_1(A) \leq \frac{q}{p}$$

- That is, the epidemic threshold is $r_c = 1/\lambda_1(A)$

Multiple copies model

- Each node may have multiple copies of the same virus
  - v: state vector
    - v_i : number of virus copies at node i

- At time $t = 0$, the state vector is initialized to v^0
- At time t,

  For each node i

    For each of the v_i^t virus copies at node i

      the copy is propagated to a neighbor j with prob p

      the copy dies with probability q

Analysis

- The expected state of the system at time t is given by

$$\overline{\mathbf{v}^{t}} = \left(p\mathbf{A} + (1-q)\mathbf{I}\right)\overline{\mathbf{v}^{t-1}}$$

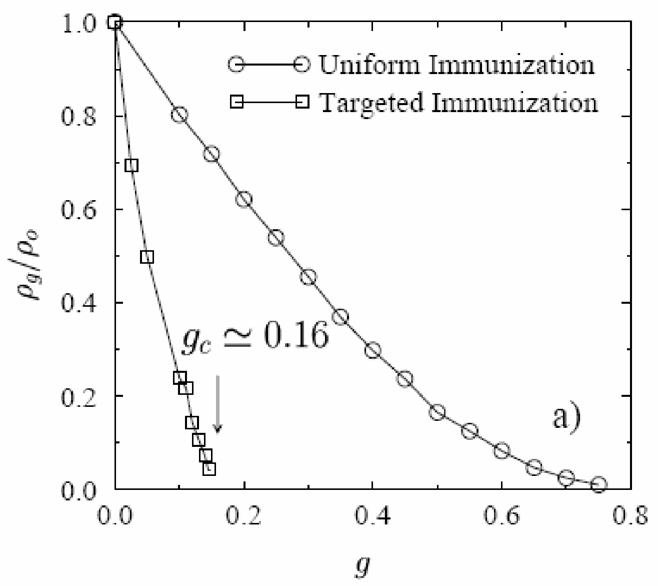
- As t à ∞
  - if $\lambda_1\left(p\mathbf{A} + (1-q)\mathbf{I}\right) < 1 \Leftrightarrow \lambda_1(\mathbf{A}) < q/p$ then $\overline{\mathbf{v}^{t}} \rightarrow 0$
    - the probability that all copies die converges to 1
  - if $\lambda_1\left(p\mathbf{A} + (1-q)\mathbf{I}\right) = 1 \Leftrightarrow \lambda_1(\mathbf{A}) = q/p$ then $\overline{\mathbf{v}^{t}} \rightarrow \mathbf{c}$
    - the probability that all copies die converges to 1
  - if $\lambda_1\left(p\mathbf{A} + (1-q)\mathbf{I}\right) > 1 \Leftrightarrow \lambda_1(\mathbf{A}) = q/p$ then $\overline{\mathbf{v}^{t}} \rightarrow \infty$
    - the probability that all copies die converges to a constant < 1

Immunization

- Given a network that contains viruses, which nodes should we immunize in order to contain the spread of the virus?
- The flip side of the percolation theory

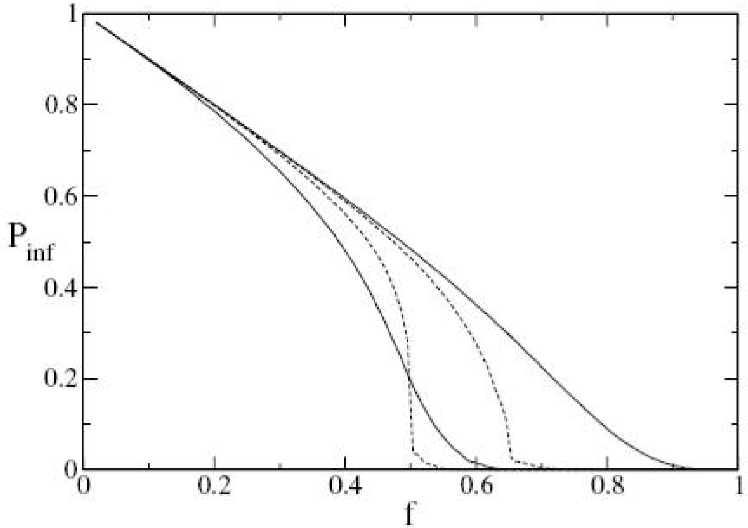
Immunization of SF graphs

- Uniform immunization vs Targeted immunization



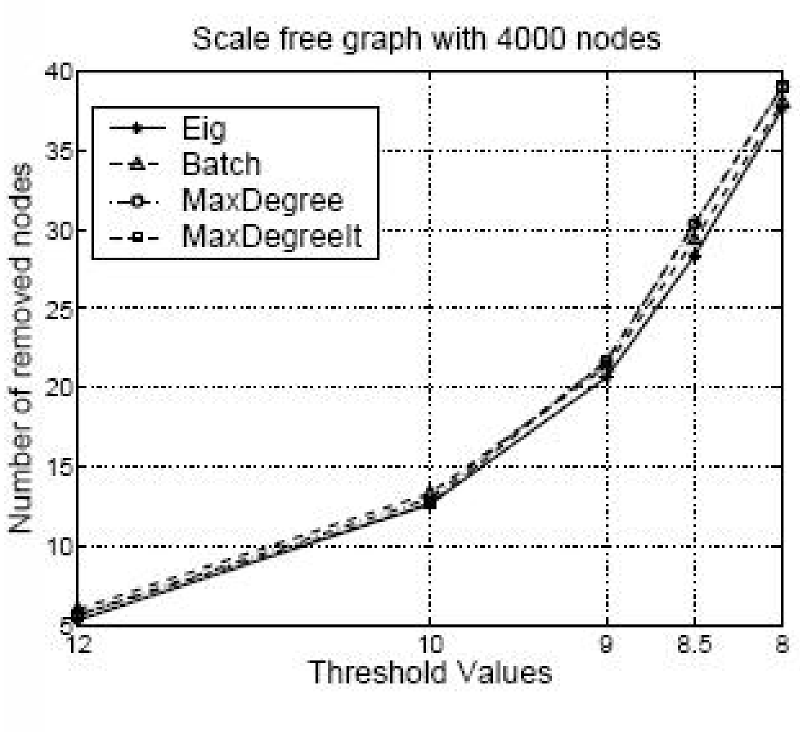
Immunizing aquaintances

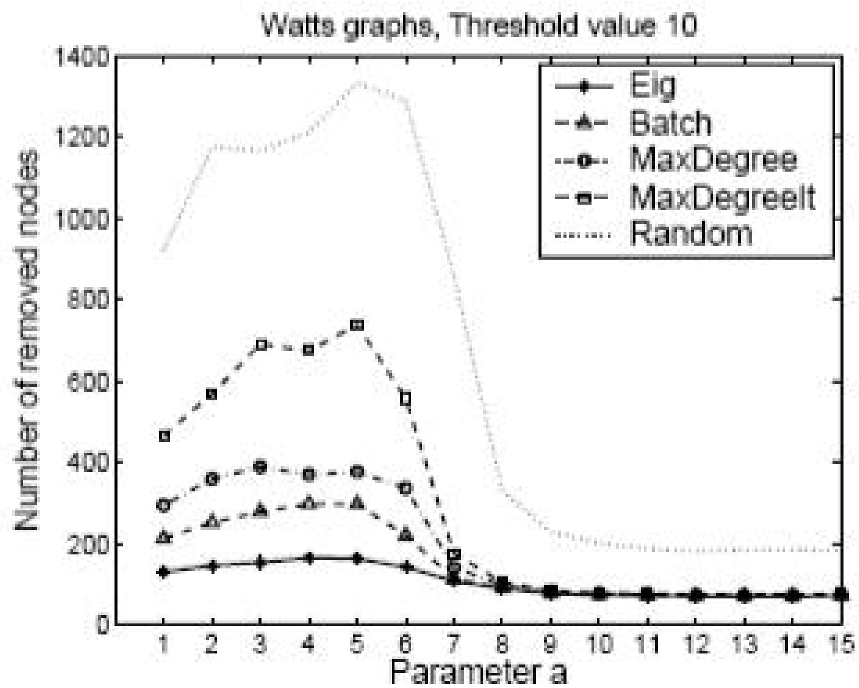
- Pick a fraction f of nodes in the graph, and immunize one of their acquaintances
  - you should gravitate towards nodes with high degree



Reducing the eigenvalue

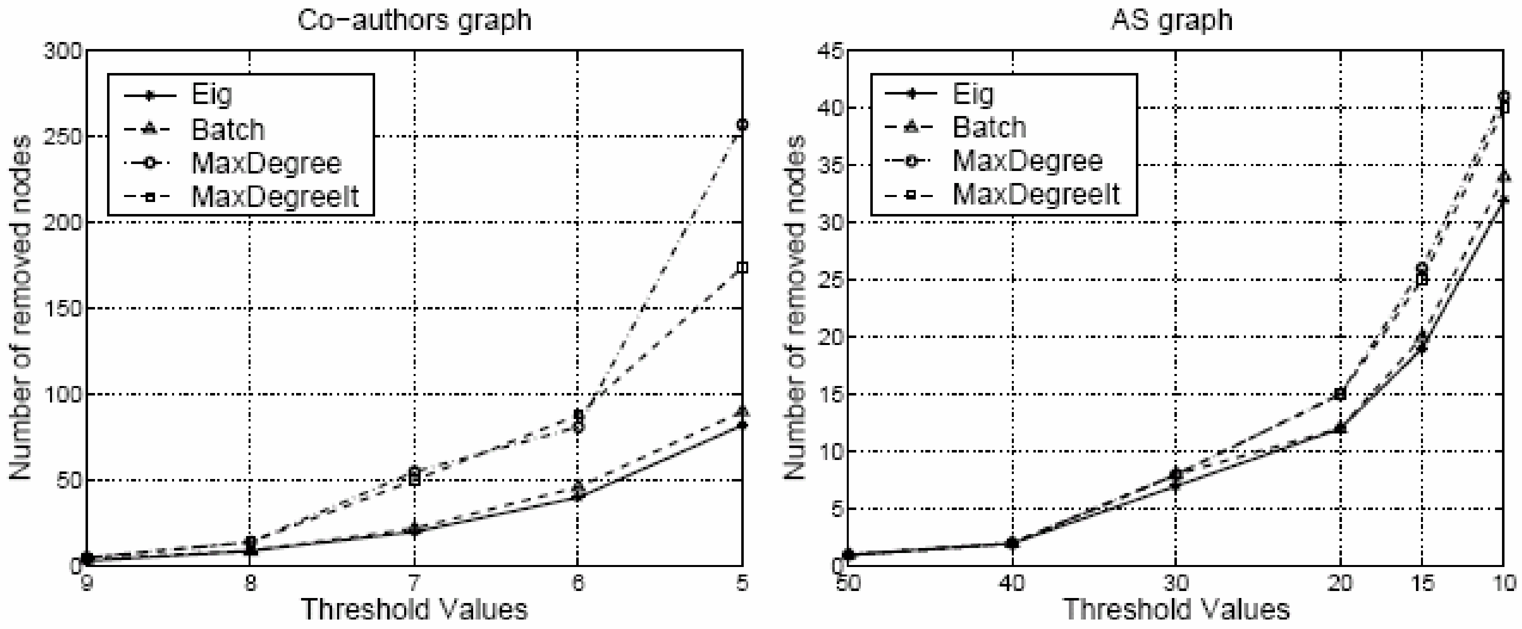
§ Repeatedly remove the node with the highest value in the principal eigenvector





Reducing the eigenvalue

- Real graphs



Gossip

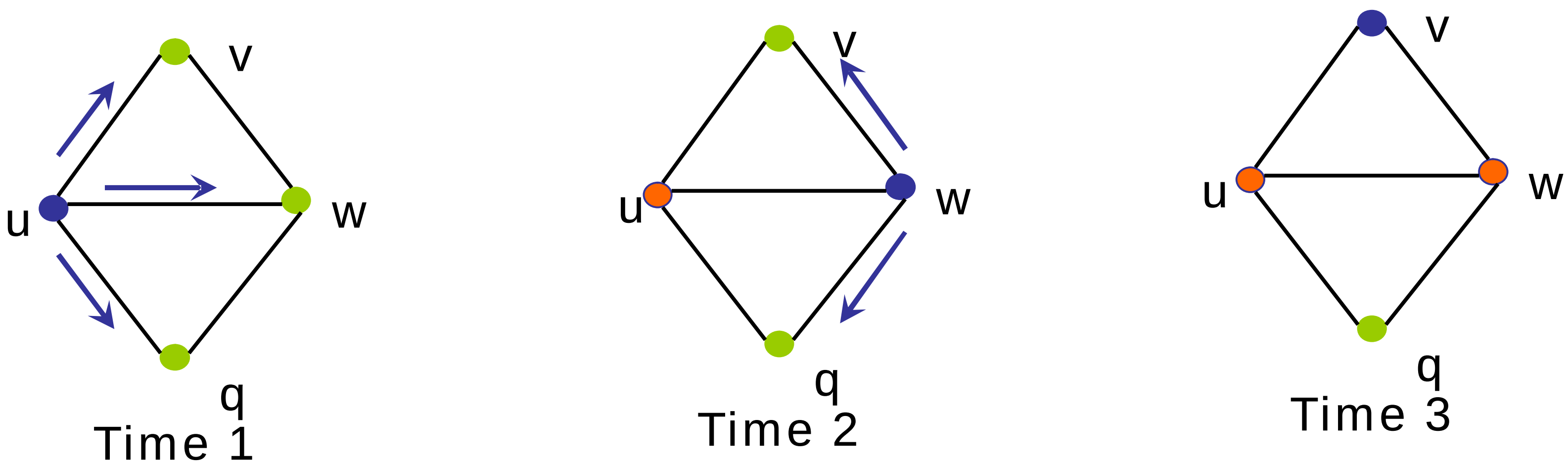
- Gossip can also be thought of as a virus that propagates in a social network.
- Understanding gossip propagation is important for understanding social networks, but also for marketing purposes
- Provides also a diffusion mechanism for the network

Independent cascade model

- Each node may be active (has the gossip) or inactive (does not have the gossip)
- Time proceeds at discrete time-steps. At time t, every node v that became active in time $t-1$ actives a non-active neighbor w with probability p_{uw}. If it fails, it does not try again
  - the same as the simple SIR model

A simple SIR model

- Time proceeds in discrete time-steps
- If a node u is infected at time t it infects neighbor v with probability p_{uv}
- Then the node becomes recovered (q = 1)



Linear threshold model

- Each node may be active (has the gossip) or inactive (does not have the gossip)
- Every directed edge (u,v) in the graph has a weight b_{uv}, such that

$$\sum_{\text{v is a neighbor of u}} b_{uv} \leq 1$$

- Each node u has a threshold value T_u (set uniformly at random)
- Time proceeds in discrete time-steps. At time t an inactive node u becomes active if

$$\sum_{\text{v is an active neighbor of u}} b_{vu} > T_u$$

Influence maximization

- Influence function: for a set of nodes A (target set) the influence s(A) is the expected number of active nodes at the end of the diffusion process if the gossip is originally placed in the nodes in A.

- Influence maximization problem [KKT03]: Given an network, a diffusion model, and a value k, identify a set A of k nodes in the network that maximizes s(A).

- The problem is NP-hard

Submodular functions

- Let f:2^U à R be a function that maps the subsets of universe U to the real numbers
- The function f is submodular if

$$f(S \cup \{v\}) - f(S) \geq f(T \cup \{v\}) - f(T)$$

when $S \subseteq T$

  - the principle of diminishing returns

Approximation algorithms for maximization of submodular functions

- The problem: given a universe U, a function f, and a value k compute the subset S of U of size k that maximizes the value $f(S)$

- The Greedy algorithm
  - at each round of the algorithm add to the solution set S the element that causes the maximum increase in function f

- Theorem: For any submodular function f, the Greedy algorithm computes a solution S that is a $(1-1/e)$-approximation of the optimal solution S^*
  - $f(S) \geq (1-1/e) f(S^*)$
  - f(S) is no worse than 63% of the optimal

Submodularity of influence

- How do we deal with the fact that influence is defined as an expectation?

- Express s(A) as an expectation over the input rather than the choices of the algorithm

Independent cascade model

- Each edge (u,v) is considered only once, and it is "activated" with probability p_{uv}.
- We can assume that all random choices have been made in advance
  - generate a subgraph of the input graph where edge (u,v) is included with probability p_{uv}
  - propagate the gossip deterministically on the input graph
  - the active nodes at the end of the process are the nodes reachable from the target set A
- The influence function is obviously submodular when propagation is deterministic
- The weighted combination of submodular functions is also a submodular function

Linear Threshold model

- Setting the thresholds in advance does not work
- For every node u, sample one of the edges pointing to node u, with probability b_{vu} and make it “live”, or select no edge with probability $1-\sum_v b_{vu}$
- Propagate deterministically on the resulting graph

Model equivalence

- For a target set A, the following two distributions are equivalent
  - The distribution over active sets obtained by running the Linear Threshold model starting from A
  - The distribution over sets of nodes reachable from A, when live edges are selected as previously described.

Simple case: DAG

- Compute the topological sort of the nodes in the graph and consider them in this order.
- If S_i neighbors of node i are active then the probability that it becomes active is $\sum_{j \in S_i} b_{ji}$
- This is also the probability that one of the nodes in S_i is sampled
- Proceed inductively

General graphs

- Let A_t be the set of active nodes at the end of the t-th iteration of the algorithm
- Prob that inactive node v becomes active at time t, given that it has not become active so far, is

$$\frac{\sum_{u \in A_t - A_{t-1}} b_{uv}}{1 - \sum_{u \in A_{t-1}} b_{uv}}$$

General graphs

- Starting from the target set, at each step we reveal the live edges from reachable nodes
- Each live edge is revealed only when the source of the link becomes reachable
- The probability that node v becomes reachable at time t, given that it was not reachable at time $t-1$ is the probability that there is an live edge from the set $A_t - A_{t-1}$

$$\frac{\sum_{u \in A_t - A_{t-1}} b_{uv}}{1 - \sum_{u \in A_{t-1}} b_{uv}}$$

Experiments

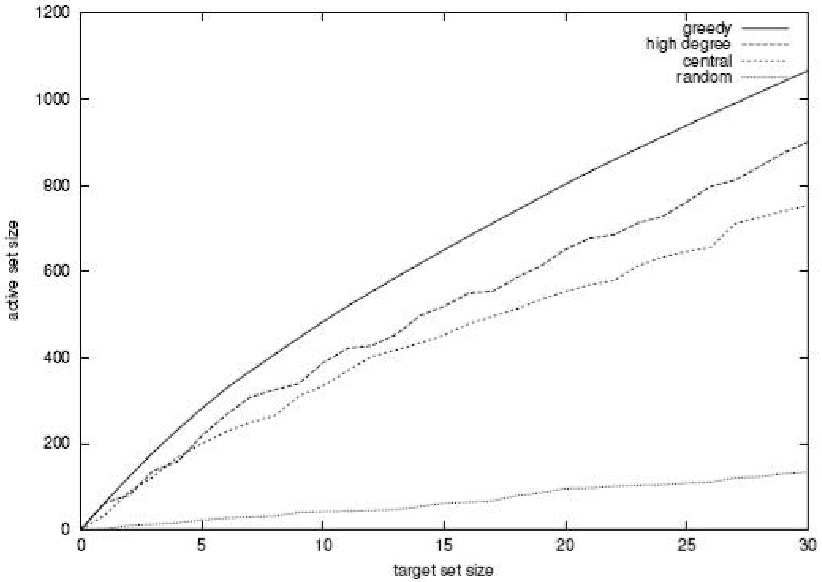


Figure 1: Results for the linear threshold model

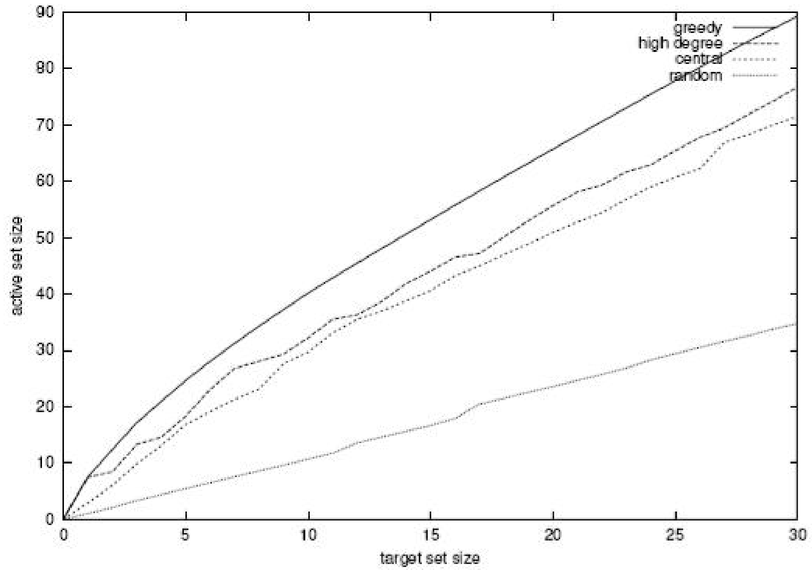


Figure 3: Independent cascade model with probability 1%

Gossip as a method for diffusion of information

- In a sensor network a node acquires some new information. How does it propagate the information to the rest of the sensors with a small number of messages?

- We want
  - all nodes to receive the message fast (in logn time)
  - the neighbors that are (spatially) closer to the node to receive the information faster (in time independent of n)

Information diffusion algorithms

- Consider points on a lattice

- Randomized rumor spreading: at each round each node sends the message to a node chosen uniformly at random
  - time to inform all nodes O(logn)
  - same time for a close neighbor to receive the message

- Neighborhood flooding: a node sends the message to all of its neighbors, one at the time, in a round robin fashion
  - a node at distance d receives the message in time O(d)
  - time to inform all nodes is $O(\sqrt{n})$

Spatial gossip algorithm

- At each round, each node u sends the message to the node v with probability proportional to d_{uv}^{-Dr}, where D is the dimension of the lattice and $1 < r < 2$

- The message goes from node u to node v in time logarithmic in d_{uv}. On the way it stays within a small region containing both u and v

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