Social Media Mining

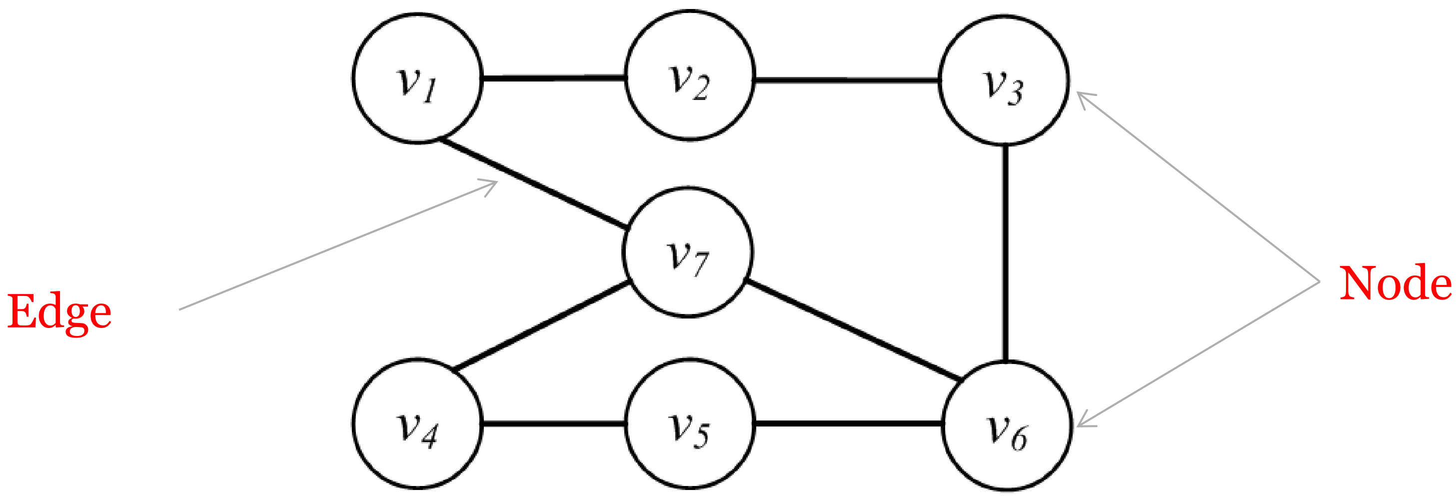
Graph Essentials

Graph Basics

Nodes and Edges

A network is a graph

- **nodes, actors**, or **vertices** (plural of **vertex**)
- Connections, **edges** or **ties**



Nodes and Edges

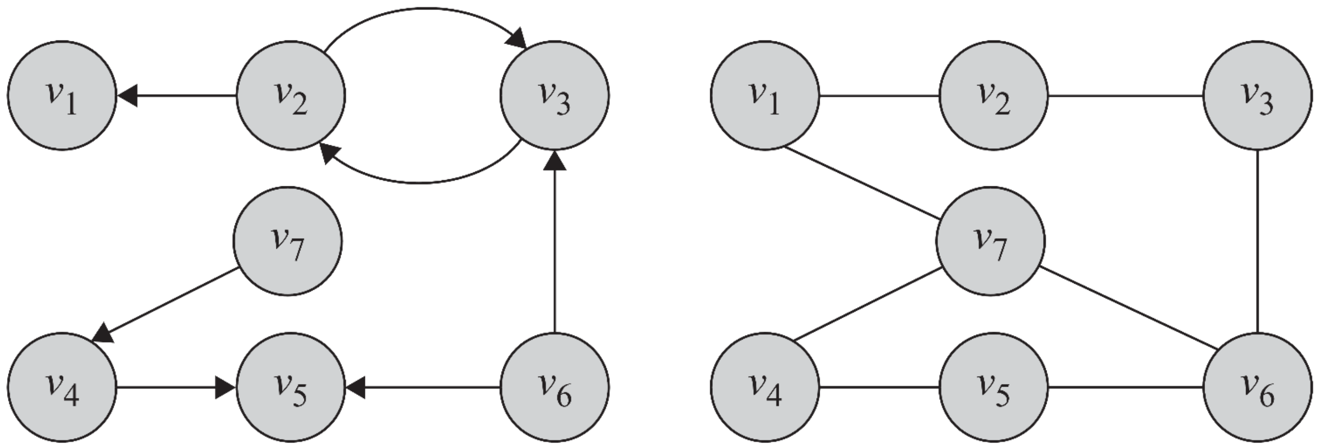
- In a social graph, nodes are people and any pair of people connected denotes the friendship, relationships, social ties between them
- In a web graph, "nodes" represent sites and the connection between nodes indicates web-links between them

  – The size of the graph is $|V| = \mathbf{n}$
  – Number of edges (size of the edge-set $|E| = \mathbf{m}$

$$V = \{v_1, v_2, \ldots, v_n\}$$
$$E = \{e_1, e_2, \ldots, e_m\}$$

Directed Edges and Directed Graphs

- Edges can have directions. A directed edge is sometimes called an arc



(a) Directed Graph (b) Undirected Graph

- Edges are represented using their end-points e(v2,v1). In undirected graphs both representations are the same

Neighborhood and Degree (In-degree, out-degree)

- For any node *v*, the set of nodes it is connected to via an edge is called its neighborhood and is represented as *N(v)*

- The number of edges connected to one node is the degree of that node (the size of its neighborhood)
  - Degree of a node *i* is usually presented using notation d_i
  - In case of directed graphs
    - d_i^{in} In-degrees is the number of edges pointing towards a node
    - d_i^{out} Out-degree is the number of edges pointing away from a node

Degree and Degree Distribution

- **Theorem 1.** The summation of degrees in an undirected graph is twice the number of edges

$$\sum_i d_i = 2|E|$$

- **Lemma 1.** The number of nodes with odd degree is even
- **Lemma 2.** In any directed graph, the summation of in-degrees is equal to the summation of out-degrees,

$$\sum_i d_i^{out} = \sum_j d_j^{in}$$

Degree Distribution

When dealing with very large graphs, how nodes' degrees are distributed is an important concept to analyze and is called ***Degree Distribution*** p_d

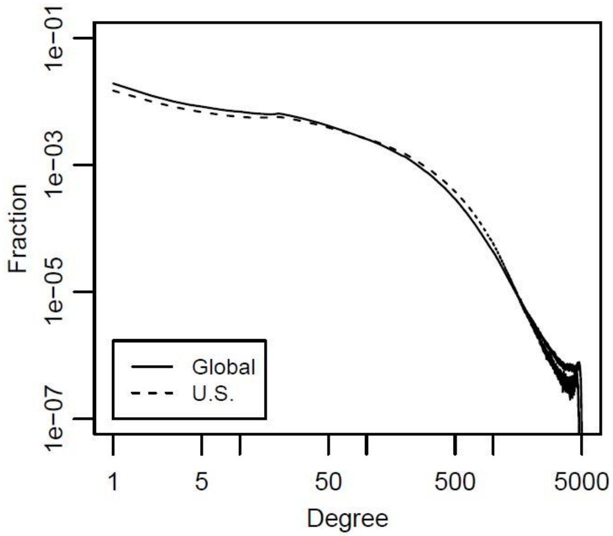
$$p_d = \frac{n_d}{n},$$

- Where n_d is the number of nodes with degree d
- Degree distribution can be computed from **degree sequence**:

$$\pi(d) = \{d_1, d_2, \ldots, d_n\}$$

Degree distribution histogram

– The x-axis represents the degree and the y-axis represents the number of nodes (frequency) having that degree



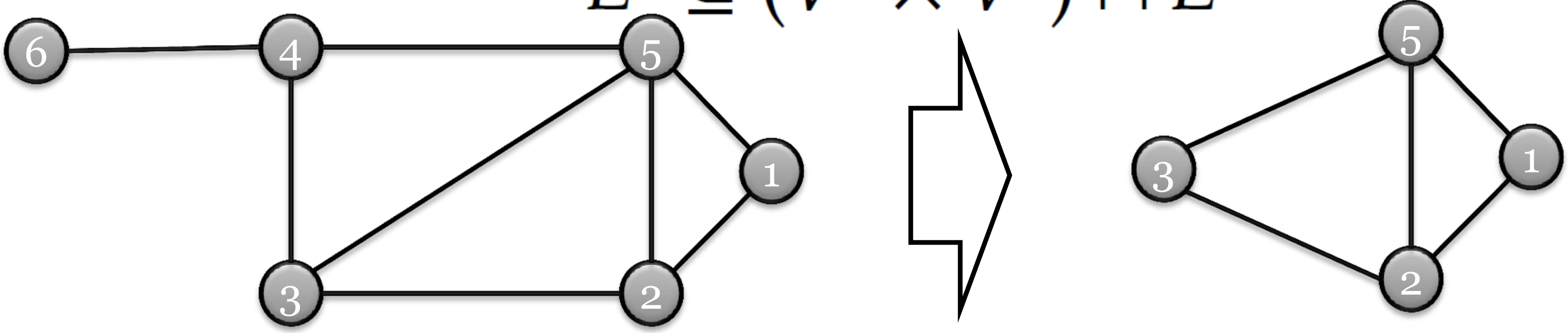
Subgraph

- Graph G can be represented as a pair G(V, E), where V is the node set and E is the edge set

- G'(V', E') is a subgraph of G(V, E) (induced subgraph)

$$V' \subseteq V,$$

$$E' \subseteq (V' \times V') \cap E$$

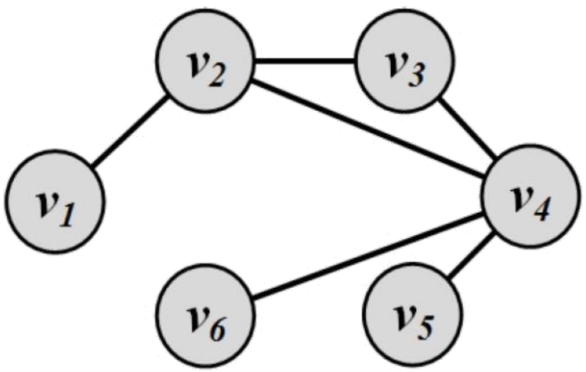


Graph Representation

- **Adjacency Matrix**
- **Adjacency List**
- **Edge List**

Adjacency Matrix

$$A_{ij} = \begin{cases} 1, \text{ if there is an edge between nodes } vi \text{ and } vj \\ 0, \text{ otherwise} \end{cases}$$



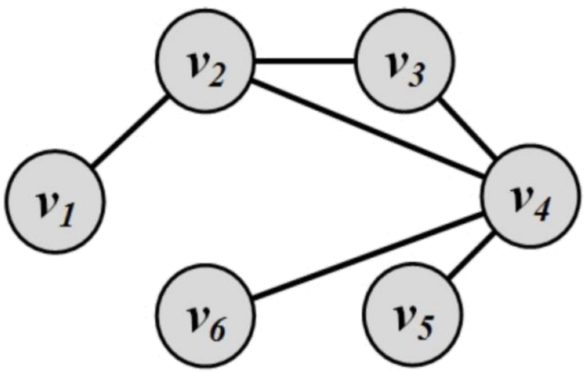
	v_1	v_2	v_3	v_4	v_5	v_6
v_1	0	1	0	0	0	0
v_2	1	0	1	1	0	0
v_3	0	1	0	1	0	0
v_4	0	1	1	0	1	1
v_5	0	0	0	1	0	0
v_6	0	0	0	1	0	0

Diagonal Entries are self-links or loops

Social media networks have very sparse adjacency matrices

Adjacency List

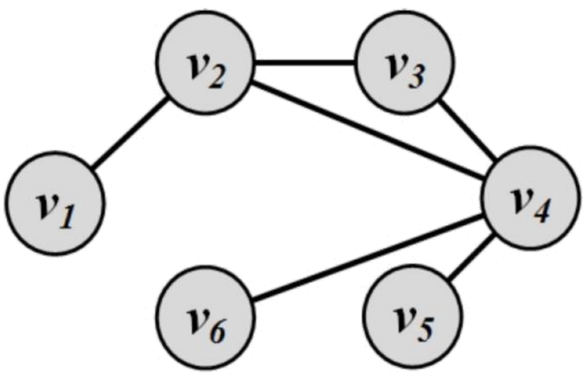
- In an adjacency list for every node, we maintain a list of all the nodes that it is connected to
- The list is usually sorted based on the node order or other preferences



Node	Connected To
v_1	v_2
v_2	v_1, v_3, v_4
v_3	v_2, v_4
v_4	v_2, v_3, v_5, v_6
v_5	v_4
v_6	v_4

Edge List

- In this representation, each element is an edge and is usually represented as *(u, v)*, denoting that node *u* is connected to node *v* via an edge



(v_1, v_2)
(v_2, v_3)
(v_2, v_4)
(v_3, v_4)
(v_4, v_5)
(v_4, v_6)

Types of Graphs

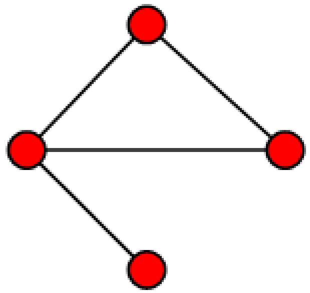
- **Null, Empty, Directed/Undirected/Mixed, Simple/Multigraph, Weighted, Signed Graph**

Directed-Undirected

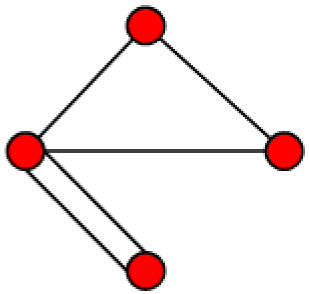
- The adjacency matrix for directed graphs is not symmetric ($A \neq A^T$)
  - ($A_{ij} \neq A_{ji}$)
- The adjacency matrix for undirected graphs is symmetric ($A = A^T$)

Simple Graphs and Multigraphs

- Simple graphs are graphs where only a single edge can be between any pair of nodes
- Multigraphs are graphs where you can have *multiple edges* between two nodes and loops



Simple graph



Multigraph

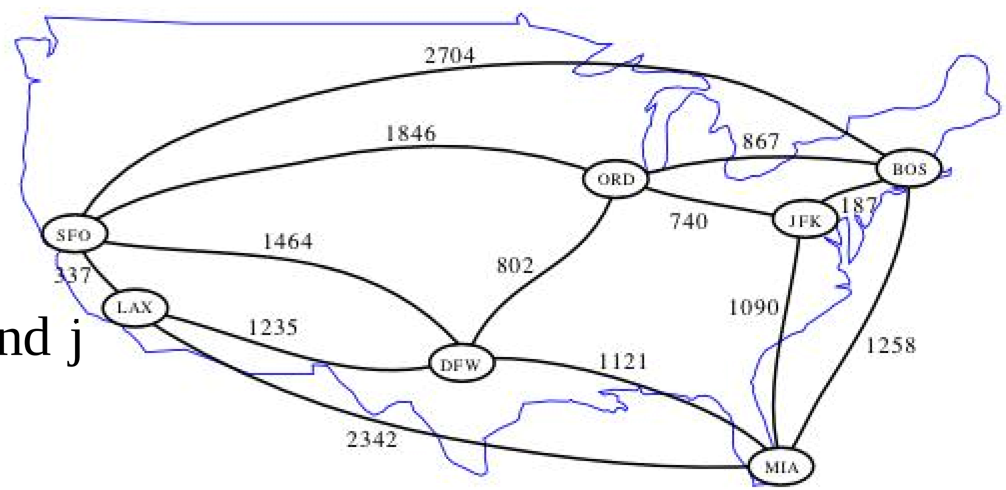
- The adjacency matrix for multigraphs can include numbers larger than one, indicating multiple edges between nodes

Weighted Graph

- A weighted graph is one where edges are associated with weights
  - For example, a graph could represent a map where nodes are cities and edges are routes between them
    - The weight associated with each edge could represent the distance between these cities

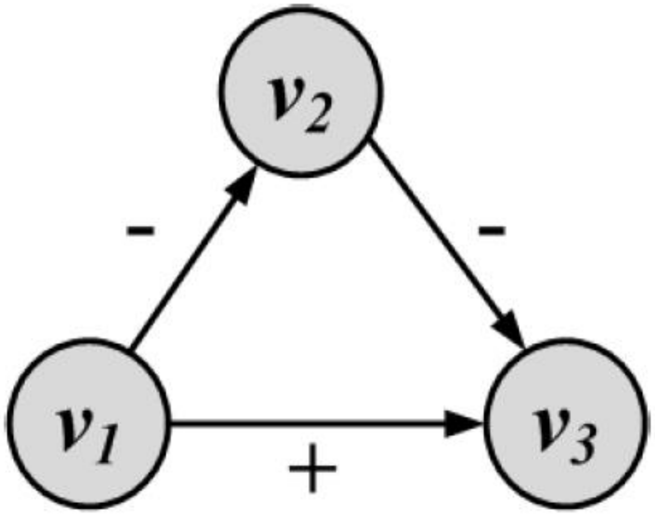
G(V, E, W)

$$A_{ij} = \begin{cases} w, w \in R \\ 0, \text{There is no edge between i and j} \end{cases}$$



Signed Graph

- When weights are binary (0/1, -1/1, +/-) we have a signed graph



- It is used to represent friends or foes
- It is also used to represent social status

Connectivity in Graphs

- **Adjacent nodes/Edges, Walk/Path/Trail/Tour/Cycle,**

Adjacent nodes and Incident Edges

Two nodes are adjacent if they are connected via an edge.

Two edges are incident, if they share on end-point

When the graph is directed, edge directions must match for edges to be incident

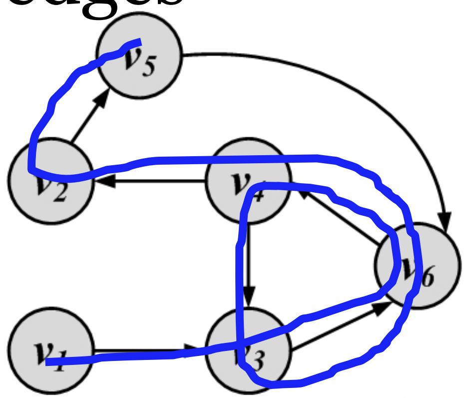
Walk, Path, Trail, Tour, and Cycle

Walk: A walk is a sequence of incident edges visited one after another

- **Open walk**: A walk does not end where it starts
- **Close walk**: A walk returns to where it starts

- Representing a walk:
  - A sequence of edges: $e_1, e_2, \ldots, e_n$
  - A sequence of nodes: $v_1, v_2, \ldots, v_n$
- Length of walk: the number of visited edges

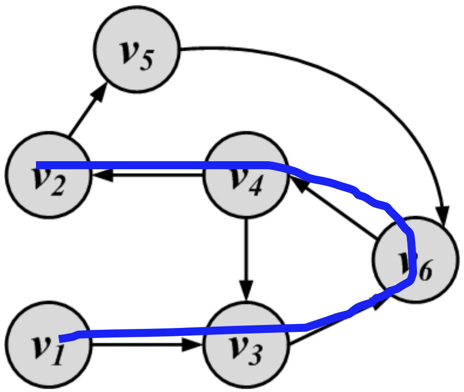
Length of walk= 8



Path

- A walk where **nodes and edges are distinct** is called a **path** and a closed path is called a **cycle**
- The length of a path or cycle is the number of edges visited in the path or cycle

Length of path= 4



Random walk

- A walk that in each step the next node is selected *randomly among the neighbors*
  - The weight of an edge can be used to define the probability of visiting it
  - For all edges that start at v_i the following equation holds

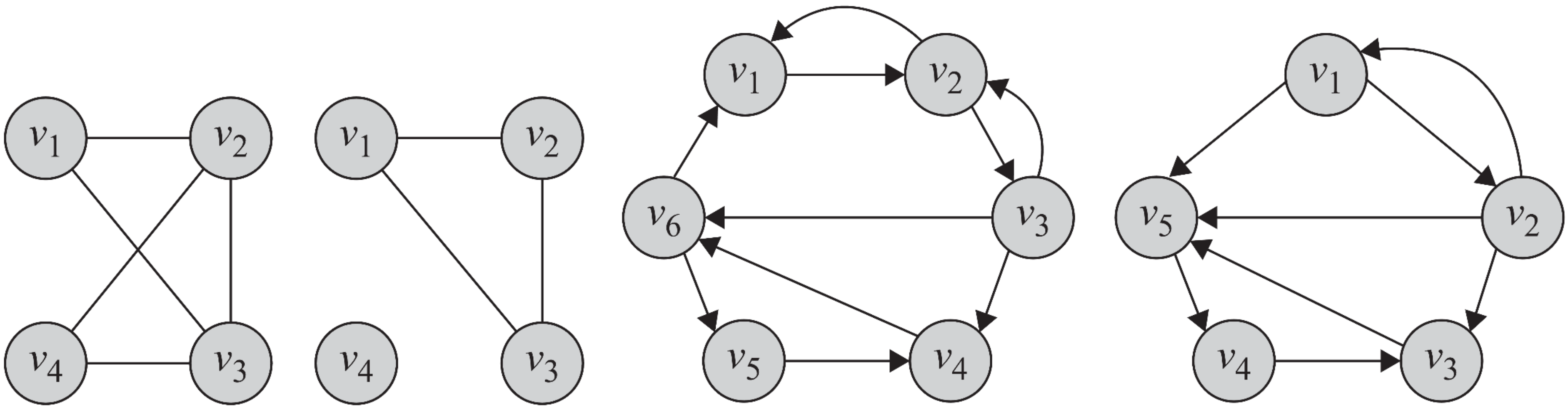
$$\sum_x w_{i,x} = 1, \forall i, j \; w_{i,j} \geq 0.$$

Connectivity

- A node v_i is **connected** to node v_j (or reachable from v_j) if it is adjacent to it or there exists a path from v_i to v_j.

- A graph is **connected**, if there exists a path between any pair of nodes in it
  - In a directed graph, **a graph is strongly connected** if there exists a directed path between any pair of nodes
  - In a directed graph, **a graph is weakly connected** if there exists a path between any pair of nodes, without following the edge directions
- A graph is **disconnected**, if it not connected.

Connectivity: Example



(a) Connected (b) Disconnected (c) Strongly connected (d) Weakly connected

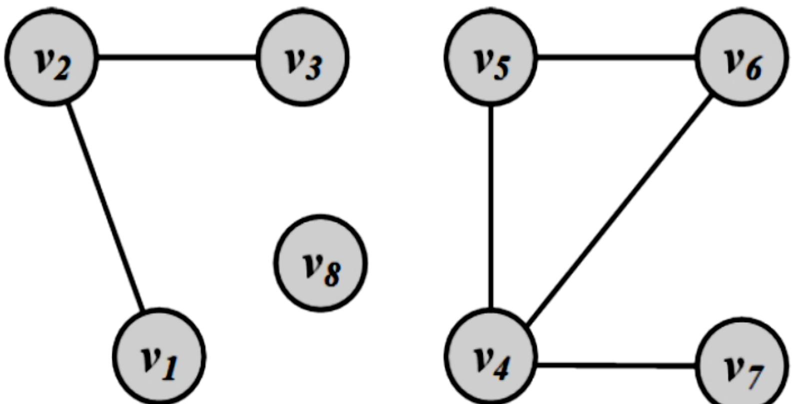
Component

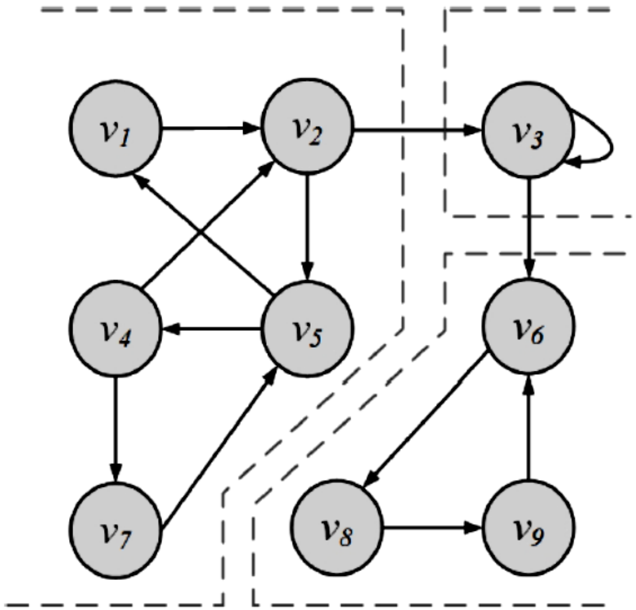
- A **component** in an undirected graph is a *connected* ***subgraph***, i.e., there is a path between every pair of nodes inside the component

- In directed graphs, we have a **strongly connected** components when there is a path from u to v and one from v to u for every pair (u,v).

- The component is **weakly connected** if replacing directed edges with undirected edges results in a connected component

Component Examples:





3 components

3 Strongly-connected components

Shortest Path

- Shortest Path is the path between two nodes that has the shortest length.

- The concept of the neighborhood of a node can be generalized using shortest paths. An n-hop neighborhood of a node is the set of nodes that are within n hops distance from the node.

Diameter

- The **diameter** of a graph is the length of the longest shortest path between any pair of nodes between any pairs of nodes in the graph

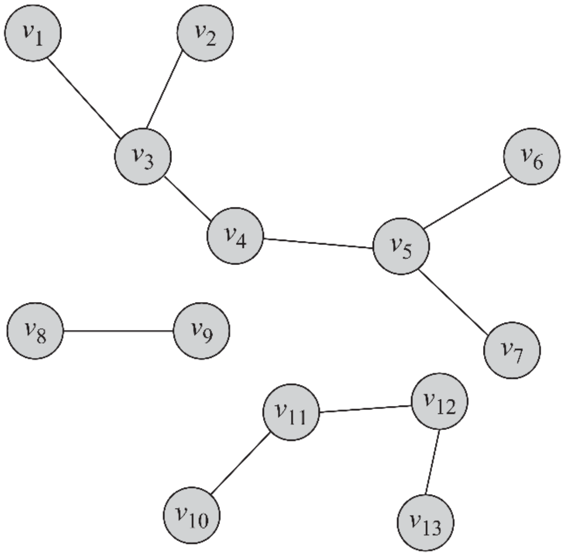
$$diameter_G = \max_{(v_i,v_j)\in V\times V} l_{i,j}.$$

- How big is the diameter of the web?

Special Graphs

Trees and Forests

- **Trees** are special cases of undirected graphs
- A tree is a graph structure that has no cycle in it
- In a tree, there is exactly one path between any pair of nodes
- In a tree: $|V| = |E| + 1$
- A set of disconnected trees is called a **forest**



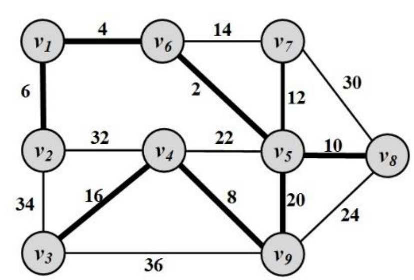
A forest containing 3 trees

Special Subgraphs

Spanning Trees

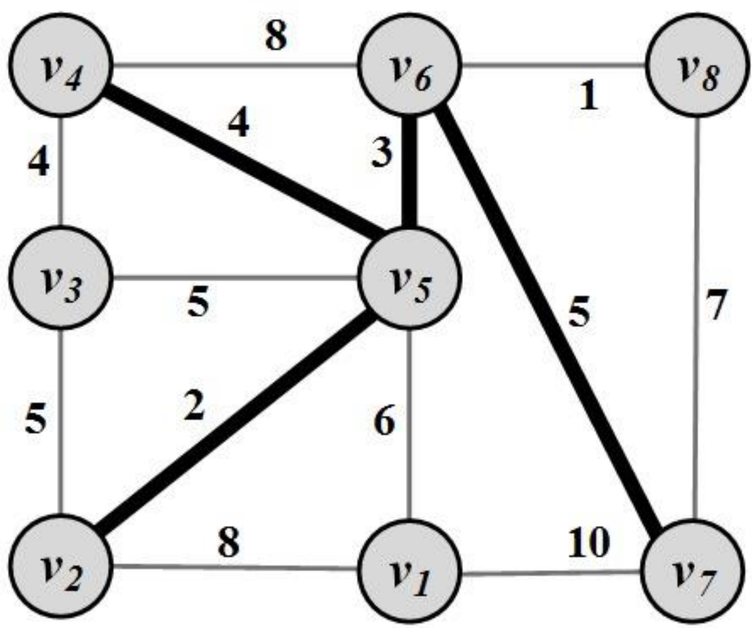
- For any connected graph, the spanning tree is a subgraph and a tree that includes all the nodes of the graph
- There may exist multiple spanning trees for a graph.
- For a weighted graph and one of its spanning tree, the weight of that spanning tree is the summation of the edge weights in the tree.
- Among the many spanning trees found for a weighted graph, the one with the minimum weight is called the

minimum spanning tree (MST)



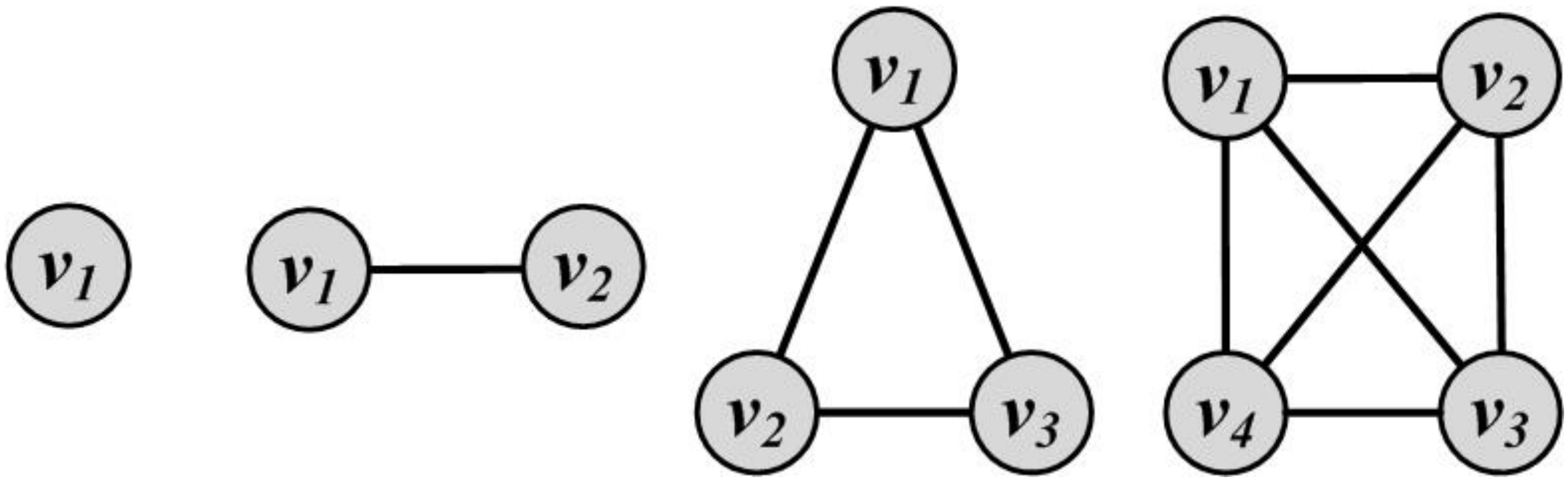
Steiner Trees

- Given a weighted graph G : (V, E, W) and a *subset* of nodes V' ⊆ V (terminal nodes), the Steiner tree problem aims to find a tree such that it spans all the V' nodes and the weight of this tree is minimized



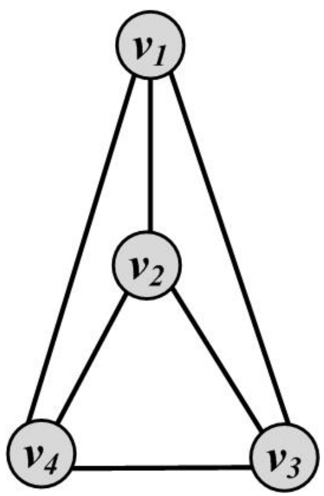
Complete Graphs

- A complete graph is a graph where for a set of nodes V, *all possible edges* exist in the graph
- In a complete graph, any pair of nodes are connected via an edge

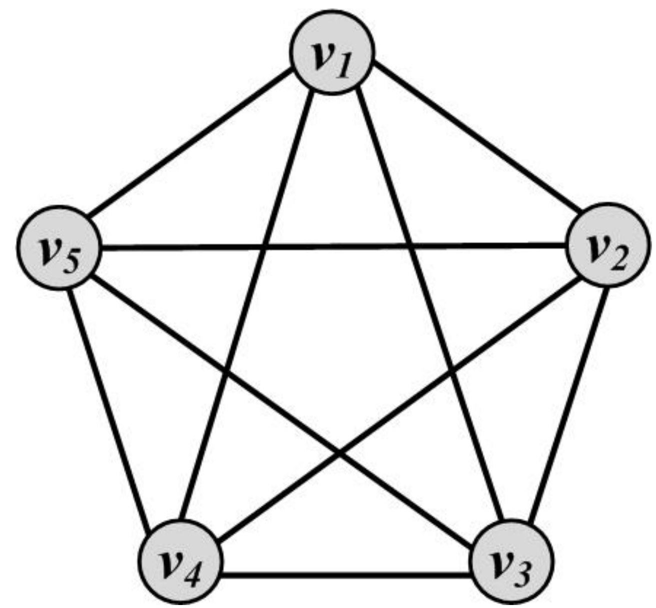


Planar Graphs

- A graph that can be drawn in such a way that no two edges cross each other (other than the endpoints) is called planar



Planar Graph

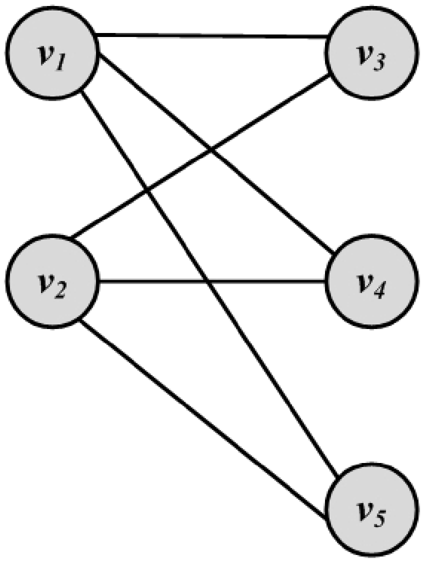


Non-planar Graph

Bipartite Graphs

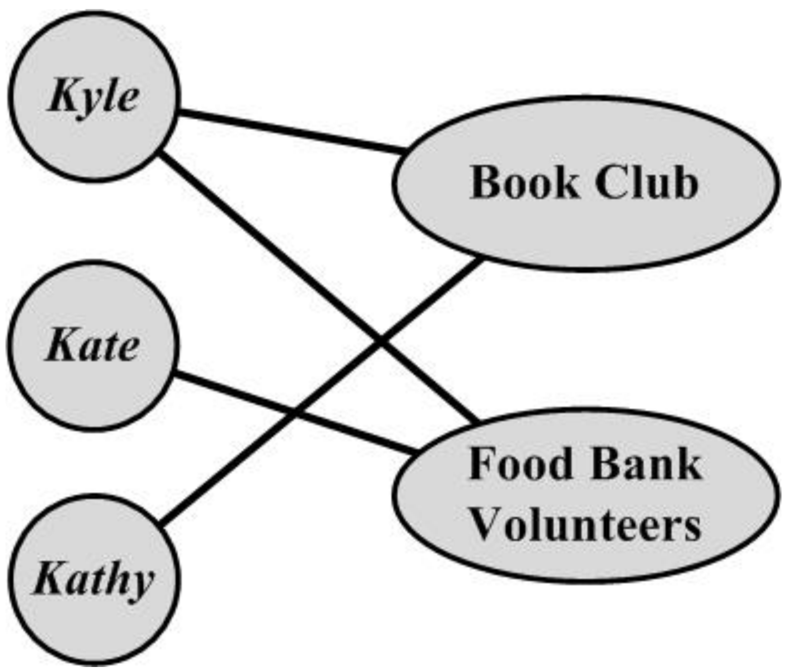
- A bipartite graph G(V; E) is a graph where the node set can be partitioned into two sets such that, for all edges, one end-point is in one set and the other end-point is in the other set.

$$
\begin{cases}
V = V_L \cup V_R, \\
V_L \cap V_R = \emptyset, \\
E \subset V_L \times V_R.
\end{cases}
$$



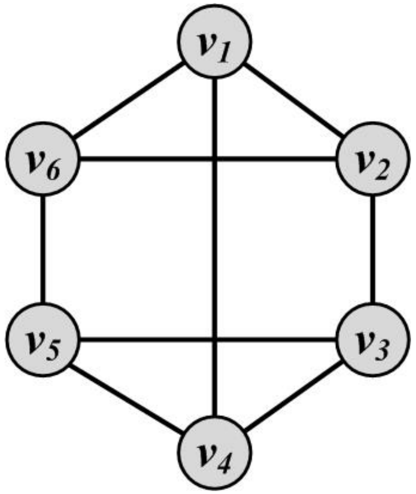
Affiliation Networks

- An affiliation network is a bipartite graph. If an individual is associated with an affiliation, an edge connects the corresponding nodes.



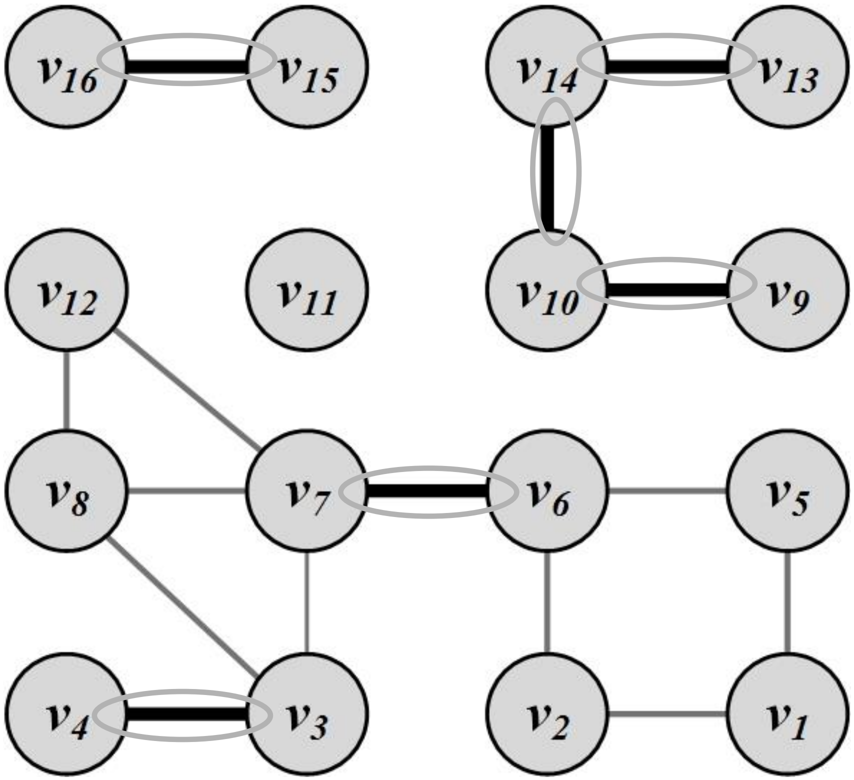
Regular Graphs

- A regular graph is one in which all nodes have the same degree
- Regular graphs can be connected or disconnected
- In a k-regular graph, all nodes have degree k
- Complete graphs are examples of regular graphs



Bridges (cut-edges)

- Bridges are edges whose removal will increase the number of connected components



Graph Algorithms

Graph/Network Traversal Algorithms

Graph/Tree Traversal

Traversal

1. All users are visited; and
2. No user is visited more than once.

- There are two main techniques:
  - **Depth-First Search (DFS)**
  - **Breadth-First Search (BFS)**

Depth-First Search (DFS)

- Depth-First Search (DFS) starts from a node i, selects one of its neighbors j from N(i) and performs Depth-First Search on j before visiting other neighbors in N(i).

- The algorithm can be used both for trees and graphs
  - The algorithm can be implemented using a *stack structure*

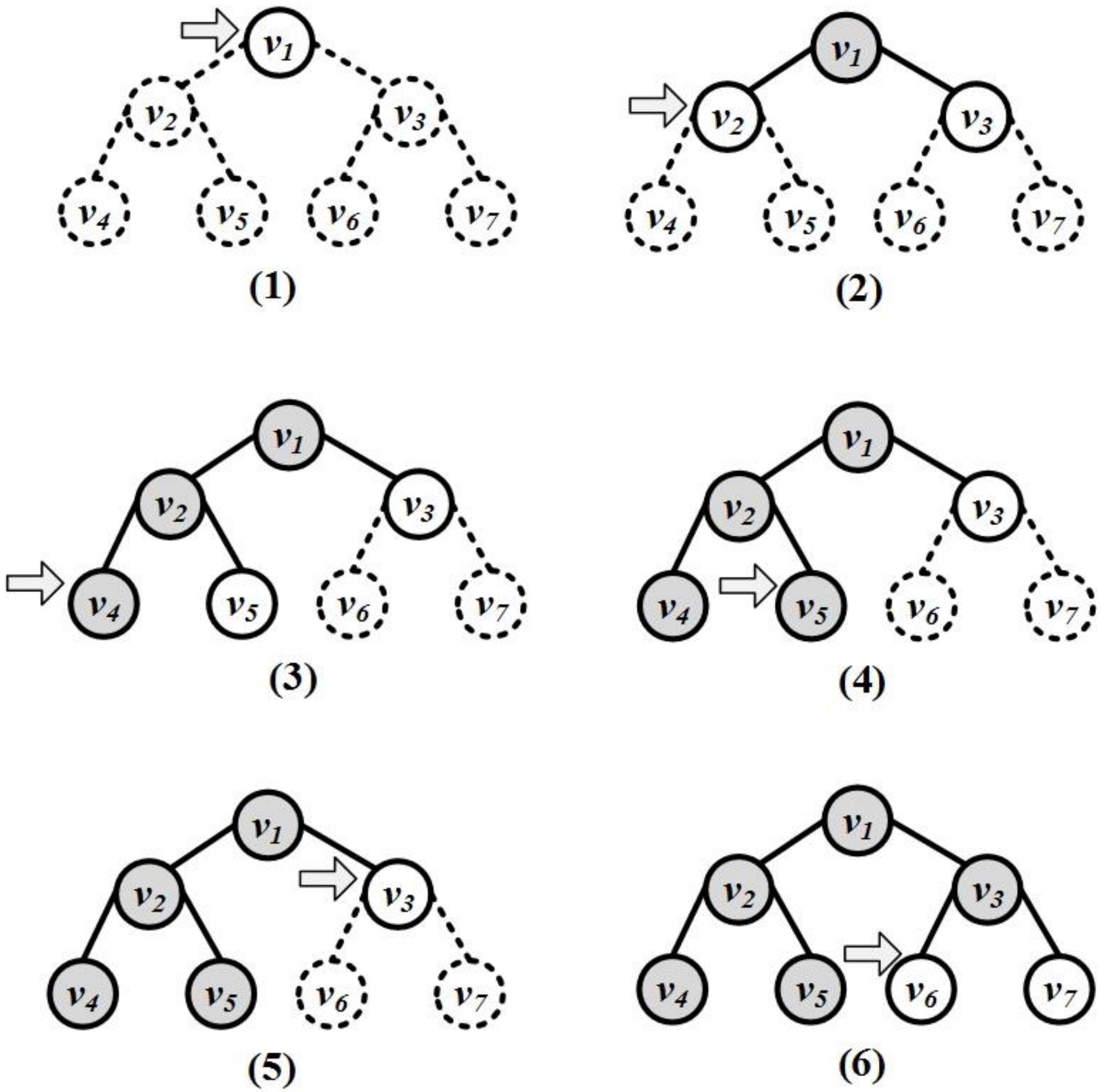
DFS Algorithm

Algorithm 2.2 Depth-First Search (DFS)

Require: Initial node v, graph/tree G:(V, E), stack S

```
return An ordering on how nodes in G are visited
Push v into S;
visitOrder = 0;
while S not empty do
    node = pop from S;
    if node not visited then
        visitOrder = visitOrder +1;
        Mark node as visited with order visitOrder; //or print node
        Push all neighbors/children of node into S;
    end if
end while
Return all nodes with their visit order.
```

Depth-First Search (DFS): An Example



Breadth-First Search (BFS)

- BFS starts from a node, visits all its immediate neighbors first, and then moves to the second level by traversing their neighbors.
- The algorithm can be used both for trees and graphs
  - The algorithm can be implemented using a *queue structure*

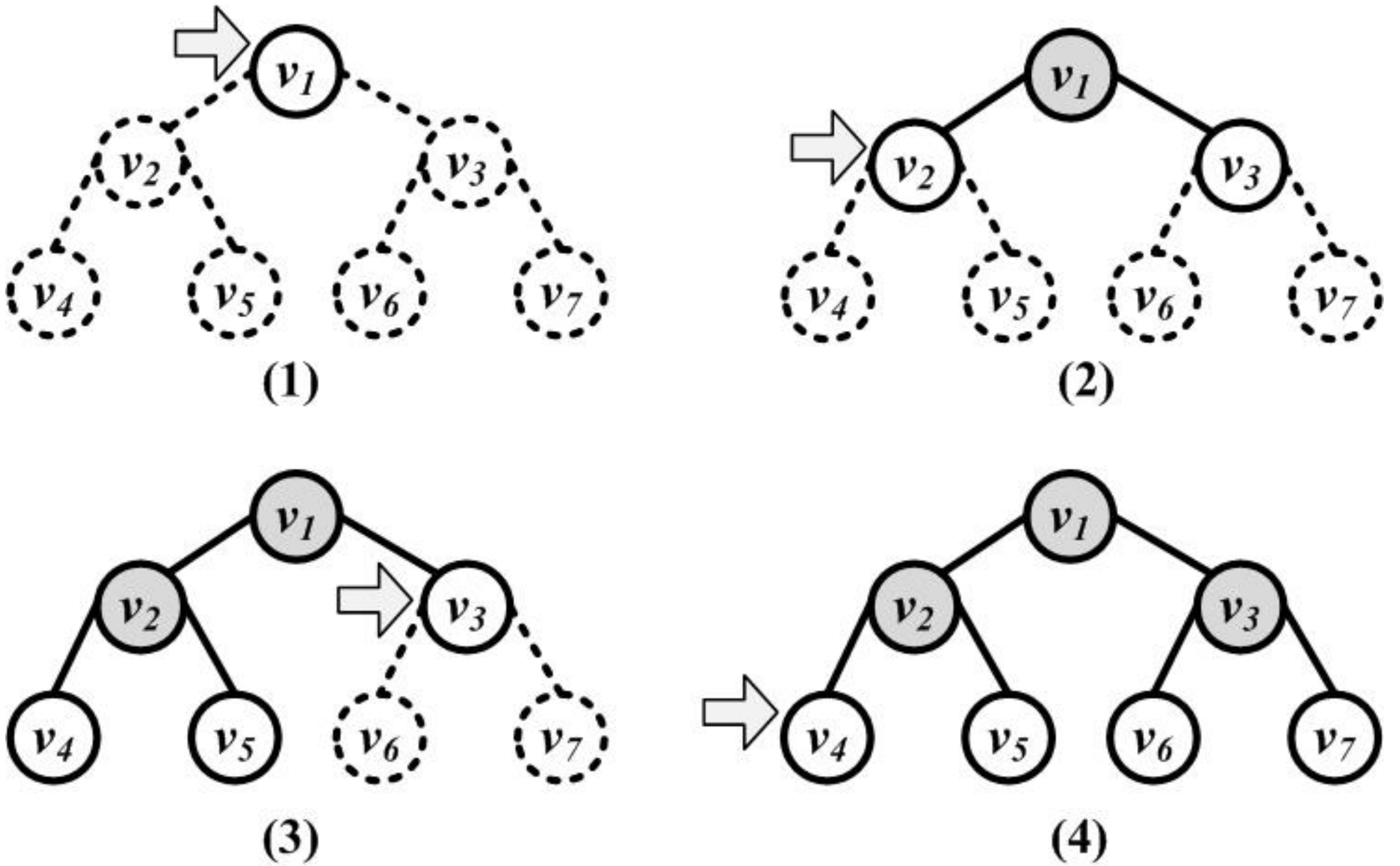
BFS Algorithm

Algorithm 2.3 Breadth-First Search (BFS)

Require: Initial node v, graph/tree $G(V, E)$, queue Q

```
return An ordering on how nodes are visited
Enqueue v into queue Q;
visitOrder = 0;
while Q not empty do
    node = dequeue from Q;
    if node not visited then
        visitOrder = visitOrder +1;
        Mark node as visited with order visitOrder; //or print node
        Enqueue all neighbors/children of node into Q;
    end if
end while
```

Breadth-First Search (BFS)



Shortest Path

When a graph is connected, there is a chance that multiple paths exist between any pair of nodes

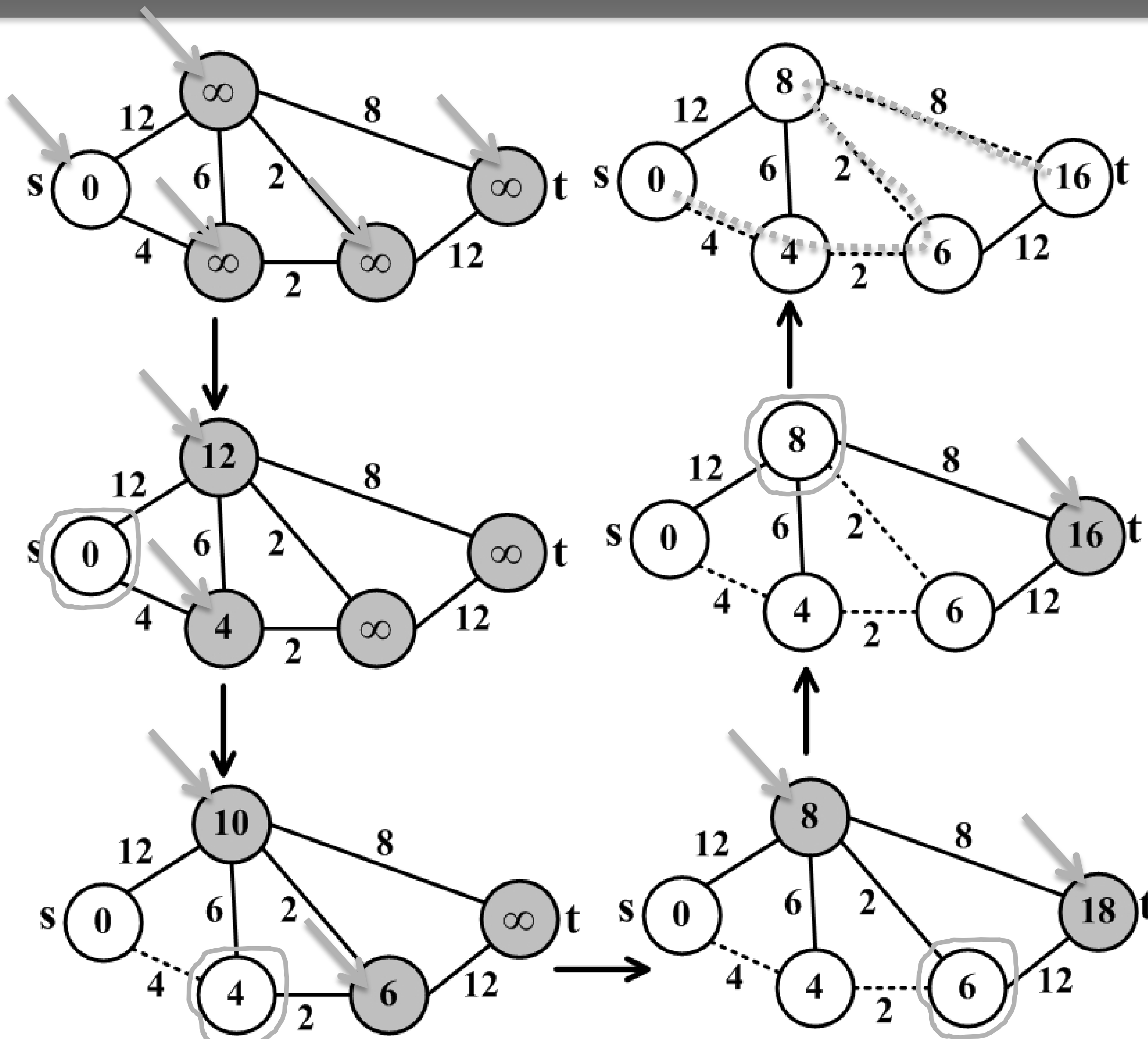
- In many scenarios, we want the shortest path between two nodes in a graph

- **Dijkstra's Algorithm**
  - It is designed for weighted graphs with non-negative edges
  - It finds shortest paths that start from a provided node *s* to all other nodes
  - It finds both shortest paths and their respective lengths

Dijkstra's Algorithm: Finding the shortest path

1. Initiation:
   - Assign zero to the source node and infinity to all other nodes
   - Mark all nodes unvisited
   - Set the source node as current
2. For the current node, consider all of its unvisited neighbors and calculate their *tentative* distances
   - If tentative distance (current node's distance + edge weight) is smaller than neighbor's distance, then Neighbor's distance = tentative distance
3. After considering all of the neighbors of the current node, mark the current node as visited and remove it from the *unvisited set*
   - A visited node will never be checked again and its distance recorded now is final and minimal
4. If the destination node has been marked visited or if the smallest tentative distance among the nodes in the *unvisited set* is infinity, then stop
5. Set the unvisited node marked with the smallest tentative distance as the next "current node" and go to step 2

Dijkstra’s Algorithm Execution Example



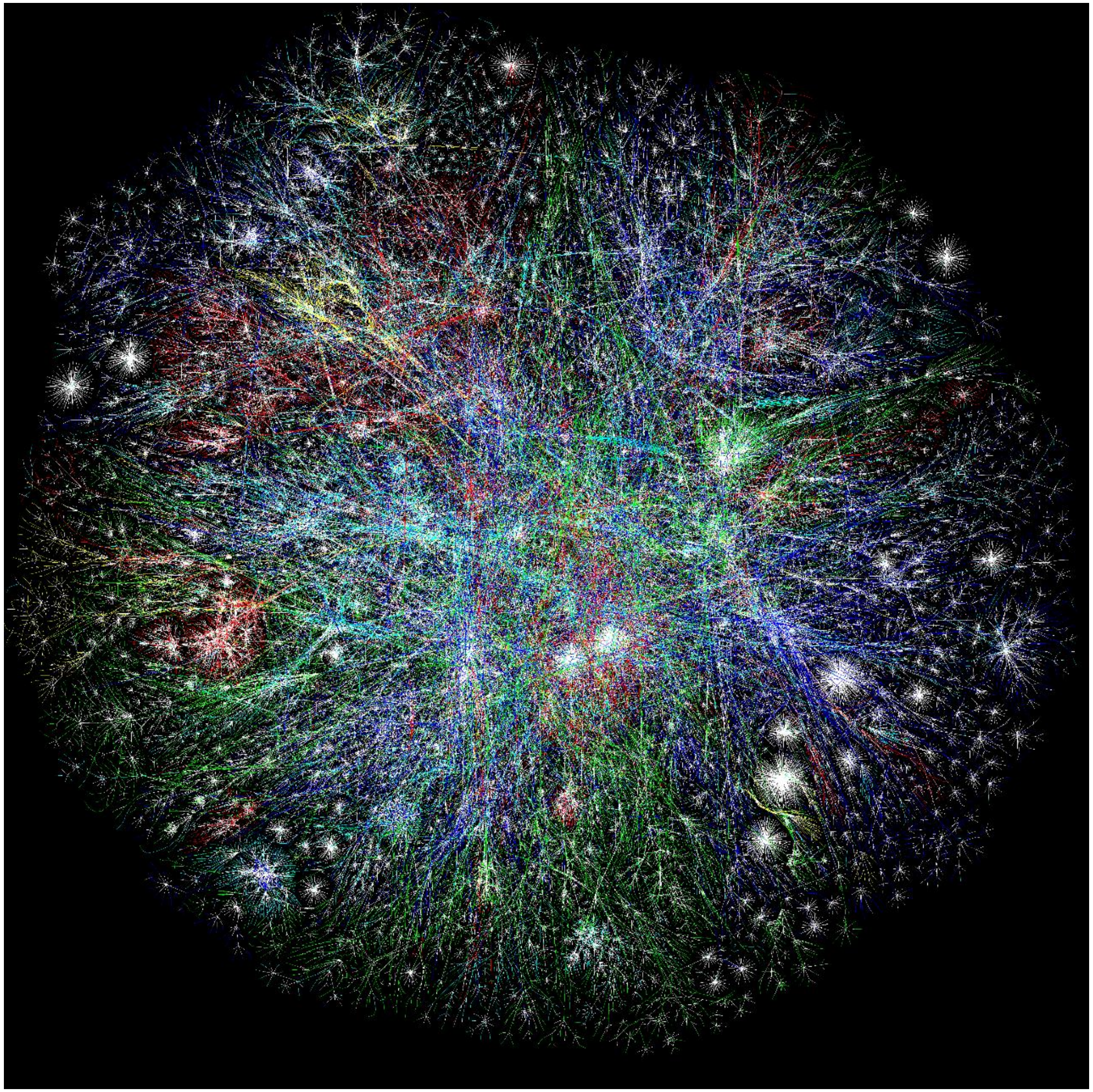
Dijkstra's Algorithm

- Dijkstra's algorithm is source-dependent and finds the shortest paths between the source node and all other nodes. To generate all-pair shortest paths, one can run dijsktra's algorithm *n times* or use other algorithms such as Floyd-Warshall algorithm.

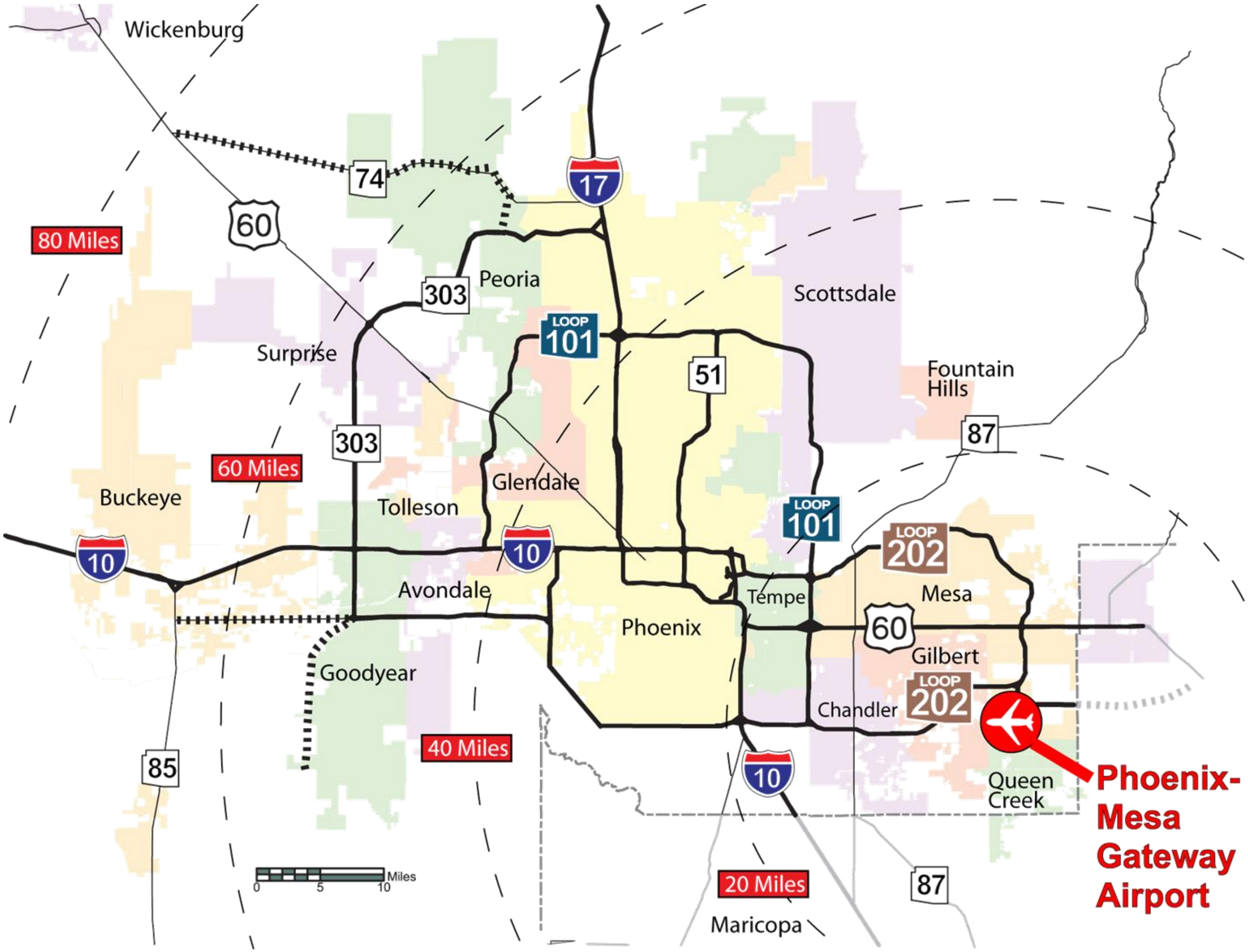
- If we want to compute the shortest path from source v to destination d, we can stop the algorithm once the shortest path to the destination node has been determined

Other slides

Internet



Phoenix Road Network



Social Networks and Social Network Analysis

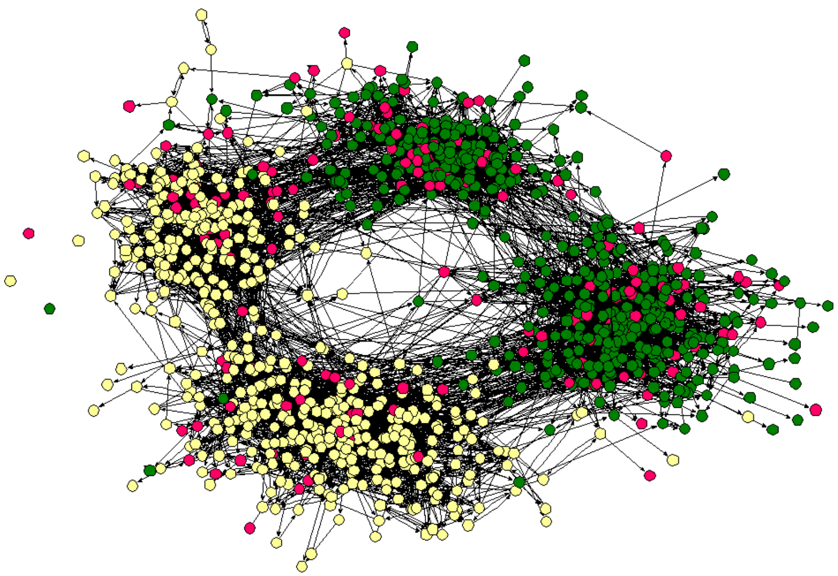
- A social network
  - A network where elements have a social structure
    - A set of actors (such as individuals or organizations)
    - A set of ties (connections between individuals)

- Social networks examples:
  - your family network, your friend network, your colleagues ,etc.

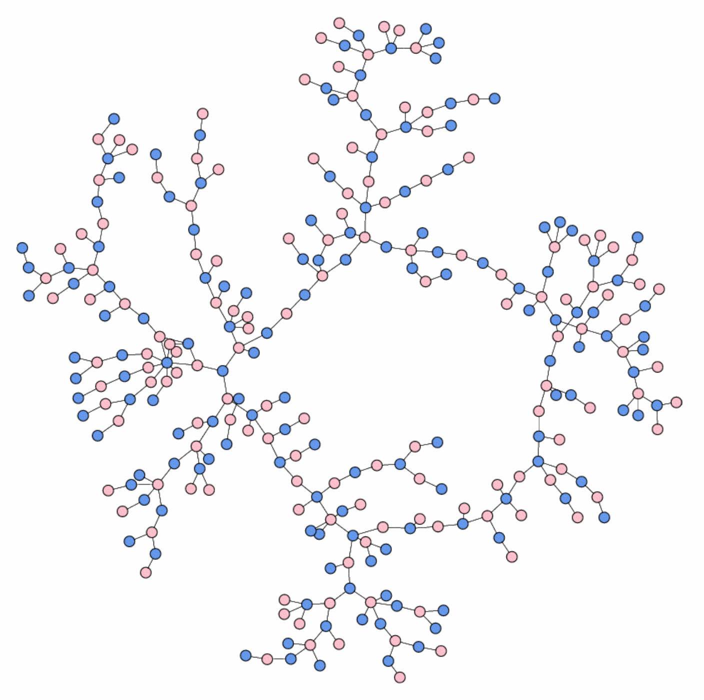
- To analyze these networks we can use Social Network Analysis (SNA)

- Social Network Analysis is an interdisciplinary field from social sciences, statistics, graph theory, complex networks, and now computer science

Social Networks: Examples



High school friendship

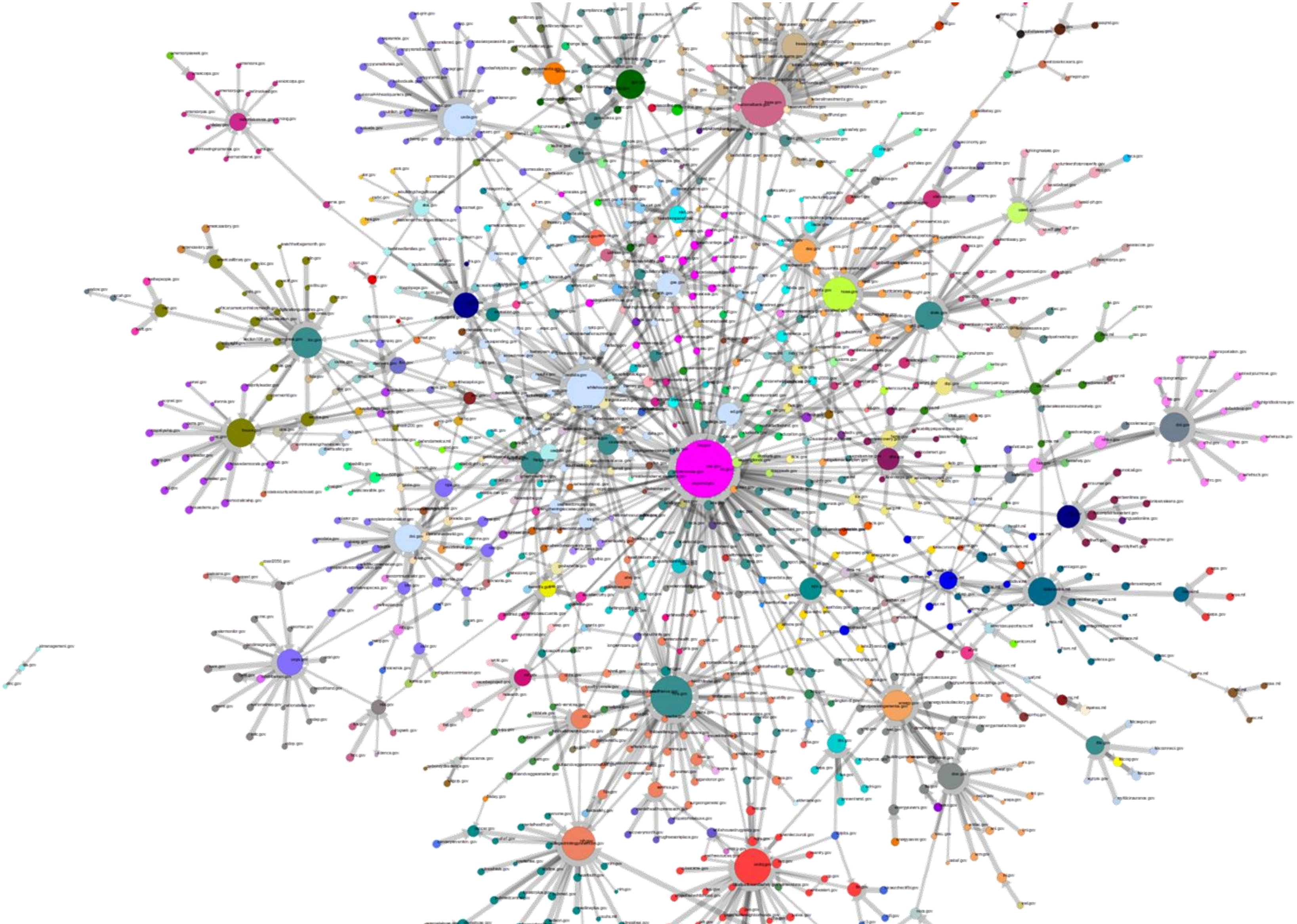


High school dating

Webgraph

- A webgraph is a way of representing how internet sites are connected on the web
- In general, a web graph is a directed multigraph
- Nodes represent sites and edges represent links between sites.
- Two sites can have multiple links pointing to each other and can have loops (links pointing to themselves)

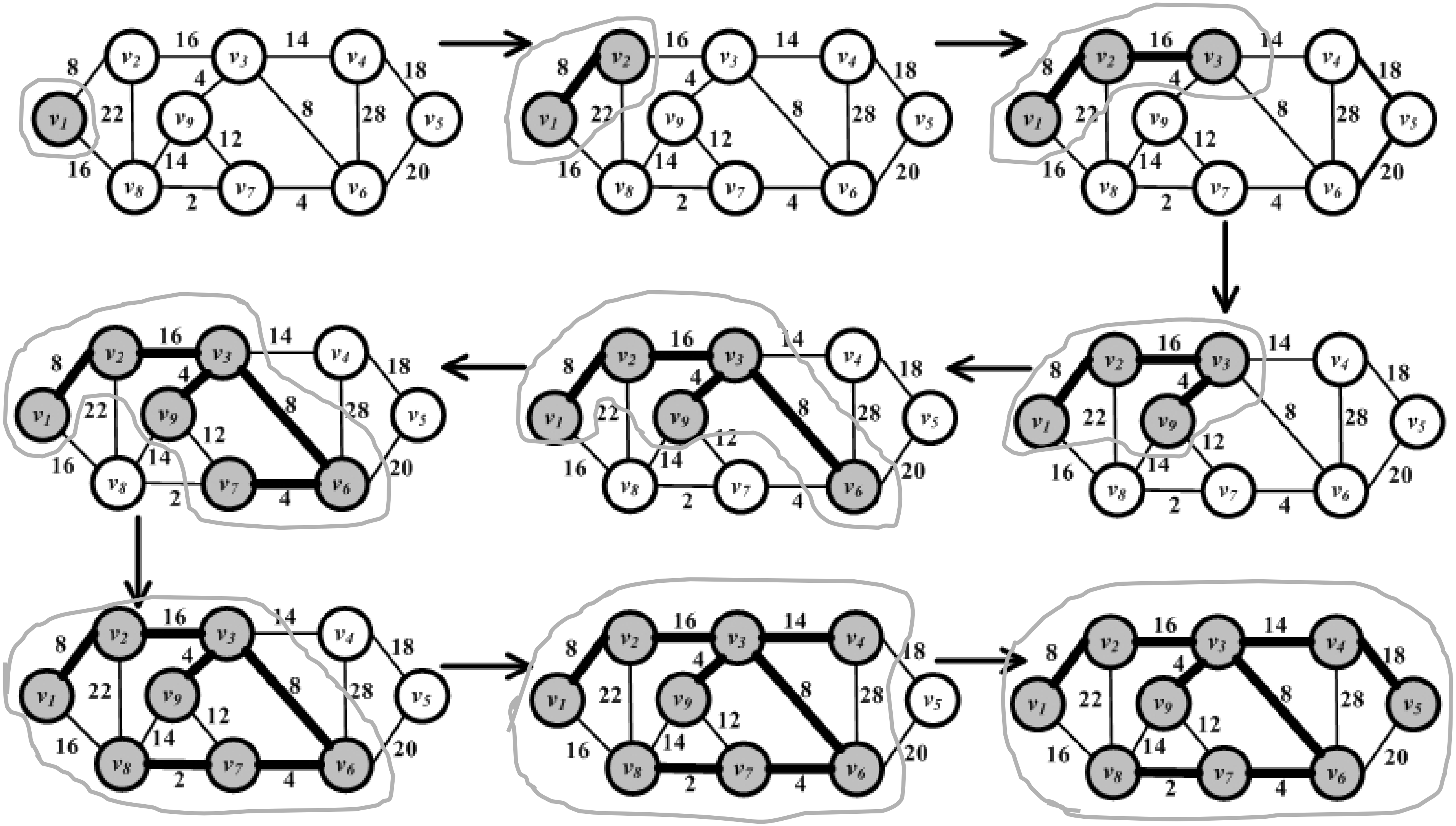
Webgraph: Government Agencies



Prim's Algorithm: Finding Minimum Spanning Tree

- It finds minimal spanning trees in a weighted graph
  - It starts by selecting a random node and adding it to the spanning tree.
  - It then grows the spanning tree by selecting edges which have one endpoint in the existing spanning tree and one endpoint among the nodes that are not selected yet. Among the possible edges, the one with the minimum weight is added to the set (along with its end-point).
  - This process is iterated until the graph is fully spanned

Prim's Algorithm Execution Example



Bridge Detection

Algorithm 2.7 Bridge Detection Algorithm

Require: Connected graph $G(V, E)$

return Bridge Edges
$bridgeSet = \{\}$
for $e(u, v) \in E$ **do**
    G' = Remove e from G
    Disconnected = False;
    if BFS in G' starting at u does not visit v **then**
        Disconnected = True;
    end if
    if Disconnected **then**
        $bridgeSet = bridgeSet \cup \{e\}$
    end if
end for
Return $bridgeSet$