Λ14 Διαδικτυακά Κοινωνικά Δίκτυα και Μέσα

Link Prediction

Motivation

- Recommending *new friends* in online social networks.
- Predicting the participation of *actors* in events
- Suggesting *interactions* between the members of a company/organization that are external to the hierarchical structure of the organization itself.
- Predicting *connections* between members of terrorist organizations who have not been directly observed to work together.
- Suggesting *collaborations* between researchers based on co-authorship.

Problem Definition

Link prediction problem: Given the links in a social network at time *t*, ***predict*** the edges that will be added to the network during the time interval from time *t* to a given future time *t'*

- Based solely on the *topology* of the network (social proximity) (the more general problem also considers attributes of the nodes and links)

- Different from the problem of *inferring missing* (hidden) links (there is a temporal aspect)

  To save experimental effort in the laboratory or in the field

Problem Formulation

Consider a social network $G = (V, E)$ where each edge $e = <u, v> \in E$ represents an interaction between u and v that took place at a particular time $t(e)$

(multiple interactions between two nodes as parallel edges with different timestamps)

For two times, $t < t'$, let $G[t, t']$ *denote* subgraph of G consisting of all edges with a timestamp between t and t'

- For four times, $t_0 < t'_0 < t_1 < t'_1$, given $G[t_0, t'_0]$, we wish to output a list of edges not in $G[t_0, t'_0]$ that are predicted to appear in $G[t_1, t'_1]$

✓ $[t_0, t'_0]$ training interval
✓ $[t_1, t'_1]$ test interval

What about new nodes?

Two parameters: $\kappa_{training}$ and κ_{test}

Core: all nodes that are incident to at least $\kappa_{training}$ edges in $G[t_0, t'_0]$, and at least κ_{test} edges in $G[t_1, t'_1]$

❖ *Predict new edges between the nodes in Core*

Example Dataset: co-authorship

	training period			Core		
	authors	papers	collaborations[1]	authors	$\lvert E_{old} \rvert$	$\lvert E_{new} \rvert$
astro-ph	5343	5816	41852	1561	6178	5751
cond-mat	5469	6700	19881	1253	1899	1150
gr-qc	2122	3287	5724	486	519	400
hep-ph	5414	10254	47806	1790	6654	3294
hep-th	5241	9498	15842	1438	2311	1576

t_0 = 1994, t'_0 = 1996: **training interval -> [1994, 1996]**
t_1 = 1997, t'_1 = 1999: **test interval -> [1997, 1999]**

- G_{collab} = <V, E_{old}> = G[1994, 1996]
- E_{new}: authors in V that co-author a paper during the test interval but not during the training interval

$\kappa_{training} = 3$, κ_{test} = 3: **Core** consists of all authors who have written at least 3 papers during the training period and at least 3 papers during the test period

Predict E_{new}

Methods for Link Prediction

Assign a connection weight score(x, y) to pairs of nodes <x, y> based on the input graph (G_{collab}) and produce a ranked list of decreasing order of score

How to assign the score between two nodes x and y?

✓ Some form of **similarity** or **node proximity**

How to Evaluate the Prediction

Each link predictor p outputs a ranked list L_p of pairs in $V \times V - E_{old}$: predicted new collaborations in decreasing order of confidence

In this paper, focus on Core, thus

$$E*_{new} = E_{new} \cap (\text{Core} \times \text{Core}),\ n = |E*_{new}|$$

Evaluation method: *Size of the intersection* of

- the first n edge predictions from L_p that are in Core × Core, and
- the set $E*_{new}$

❖ *How many of the (relevant) top-n predictions are correct (precision?)*

Methods for Link Prediction: Shortest Path

For $x, y \in V \times V - E_{old}$,

score(x, y) = (negated) length of *shortest path* between x and y

✓ If there are more than *n* pairs of nodes tied for the shortest path length, order them at random.

Methods for Link Prediction: Neighborhood-based

The "larger" the overlap of the neighbors of two nodes, the more likely the nodes to be linked in the future

Let $\Gamma(x)$ denote the set of neighbors of x in G_{collab}

Common neighbors:

$$\mathrm{score}(x, y) = |\Gamma(x) \cap \Gamma(y)|$$

A adjacency matrix -> $A_{x,y}{}^2$
Number of different paths of length 2

Jaccard coefficient:

$$\mathrm{score}(x, y) = \frac{|\Gamma(x) \cap \Gamma(y)|}{|\Gamma(x) \cup \Gamma(y)|}$$

The probability that both x and y have a feature f, for a randomly selected feature that either x or y has

Methods for Link Prediction: Neighborhood-based

Adamic/Adar:

$$\mathrm{score}(x, y) = \sum_{z \in \Gamma(x) \cap \Gamma(y)} \frac{1}{\log |\Gamma(z)|}$$

✓ Assigns large weights to common neighbors *z* of *x* and *y* which themselves have few neighbors (weight rare features more heavily)

- Neighbors who are linked with **2** nodes are assigned weight = 1/log(2) = **1.4**
- Neighbors who are linked with **5** nodes are assigned weight = 1/log(5) = **0.62**

Methods for Link Prediction: Neighborhood-based

Preferential attachment:

Based on the premise that the probability that a new edge has node x as its endpoint is proportional to $|\Gamma(x)|$, i.e., nodes like to form ties with 'popular' nodes

$$\mathrm{score}(x, y) = |\Gamma(x)||\Gamma(y)|$$

✓ Researchers found empirical evidence to suggest that co-authorship is correlated with the product of the neighborhood sizes

❖This depends ***on the degrees*** of the nodes not on their neighbors per se

Methods for Link Prediction: based on the ensemble of all paths

Not just the shortest, but *all* paths between two nodes

Methods for Link Prediction: based on the ensemble of all paths

Katz$_\beta$ measure:

$$\text{score}(x, y) := \sum_{\ell=1}^{\infty} \beta^{\ell} \cdot |\text{paths}_{x,y}^{\langle \ell \rangle}|$$

$$\sum_{l=1}^{\infty} \beta^{l} \cdot |\text{paths}_{xy}^{(l)}| = \beta A_{xy} + \beta^2 (A^2)_{xy} + \beta^3 (A^3)_{xy} + \cdots$$

Sum over all paths of length *l*, β > 0 is a parameter of the predictor, exponentially damped to count short paths more heavily

✓*Small β predictions much like common neighbors*

Closed form: $(I - \beta A)^{-1} - I$

1. *Unweighted* version, in which $\text{path}_{x,y}^{(1)}$ = **1**, if x and y have collaborated, **0** otherwise
2. *Weighted* version, in which $\text{path}_{x,y}^{(1)}$ = **#times** x and y have collaborated

Methods for Link Prediction: based on the ensemble of all paths

Consider a *random walk* on G_{collab} that starts at *x* and iteratively moves to a neighbor of *x* chosen uniformly at random from $\Gamma(x)$.

The Hitting Time $H_{x,y}$ from x to y is the expected number of steps it takes for the random walk starting at x to reach y.

$$score(x, y) = -H_{x,y}$$

The Commute Time $C_{x,y}$ from x to y is the expected number of steps to travel from x to y and from y to x

$$score(x, y) = -(H_{x,y} + H_{y,x})$$

Can also consider stationary-normed versions:

$score(x, y) = -H_{x,y}\,\pi_y$

$score(x, y) = -(H_{x,y}\,\pi_y + H_{y,x}\,\pi_x)$

Methods for Link Prediction: based on the ensemble of all paths

*The hitting time and commute time measures are sensitive to parts of the graph far away from x and y -> periodically **reset the walk***

Random walk on G_{collab} that starts at *x* and has a probability of α of returning to x at each step

Rooted Page Rank: Starts from x, with probability ($1 - a$) moves to a random neighbor and with probability *a* returns to x

score(x, y) = stationary probability of y in a rooted PageRank

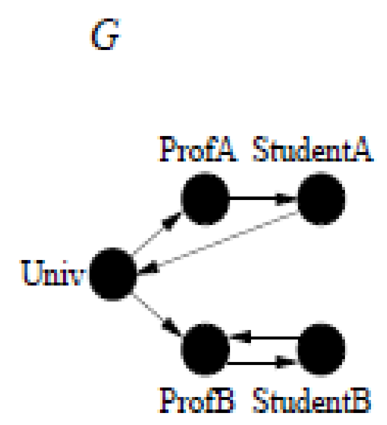
Methods for Link Prediction: based on the ensemble of all paths

SimRank

Two objects are *similar*, if they are *related to similar objects*

Two objects x and y are similar, if respectively they are related to objects a and b, and a and b are themselves similar

$$\text{similarity}(x, y) := \gamma \cdot \frac{\sum_{a \in \Gamma(x)} \sum_{b \in \Gamma(y)} \text{similarity}(a, b)}{|\Gamma(x)| \cdot |\Gamma(y)|}$$



similarity(x, y) = 1, if x = y

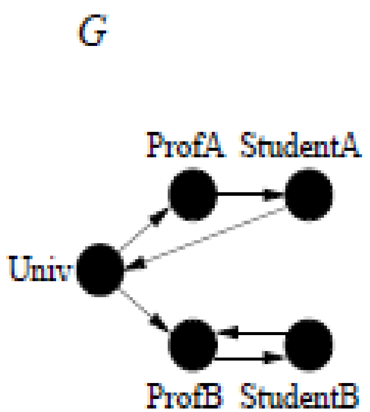
Average similarity between in- neighbors of x and in-neighbors of y

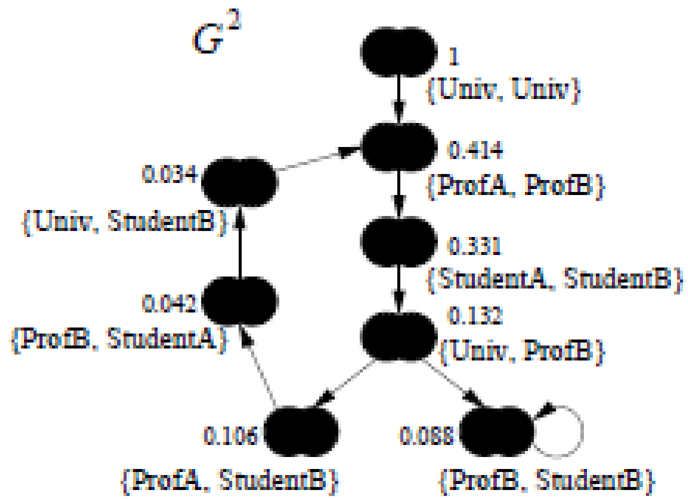
A set of n^2 equations for a graph of size n

score(x, y) = similarity(x, y)

SimRank

Graph G^2:
A node for each pair of nodes
(x, y) → (a, b), if x → a and y → b
Scores *flow* from a node to its neighbors
γ gives the rate of *decay* as similarity flows across edges (γ = 0.8 in the example)



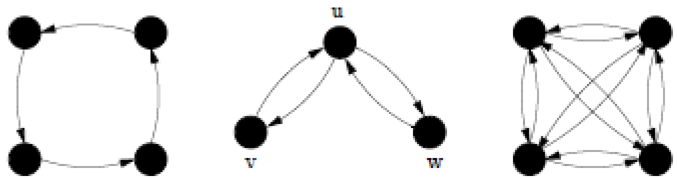


Iterative computation

$s_0(x, y) = 1$ if $x = y$ and 0 otherwise
s_{k+1} based on the s_k values of its (in-neighbors) computed at iteration k

SimRank: Random surfer model

How soon two random surfers are expected to meet at the same node if they started at nodes x and y and randomly walked the graph backwards



Let us consider G^2

A node (a, b) as a state of the tour in G, a moves to c, b moves to d in G, then (a, b) moves to (c, d) in G^2

A tour in G^2 of length n represents a pair of tours in G where each has length n

What are the states in G^2 that correspond to "meeting" points?

Singleton nodes (common neighbors)

Hitting time (expected distance over all tours) d(u, v) : the expected number of steps that it would take a random surfer who at each step follows a random out-edge before it reaches v starting from u

The sum is taken over all walks that start from (x, y) which end at a singleton node

Methods for Link Prediction: High-level approaches

Low rank approximations

M adjacency matrix

Apply SVD (singular value decomposition)

The rank-k matrix that best approximates M

Methods for Link Prediction: High-level approaches

Unseen Bigrams

Unseen bigrams: pairs of word that co-occur in a test corpus, but not in the corresponding training corpus

Not just score(x, y) but score(z, y) for nodes z that are similar to x

$S_x^{(\delta)}$ the δ nodes *most related to x*

$$\text{score}^*_{unweighted}(x,y) \quad := \quad \left|\{z : z \in \Gamma(y) \cap S_x^{\langle\delta\rangle}\}\right|$$

$$\text{score}^*_{weighted}(x,y) \quad := \quad \sum\nolimits_{z \in \Gamma(y) \cap S_x^{\langle\delta\rangle}} \text{score}(x,z)$$

Methods for Link Prediction: High-level approaches

Clustering

- Compute score(x, y) for al edges in E_{old}
- Delete the (1-p) fraction of the edges whose score is the lowest, for some parameter p
- Recompute score(x, y) for all pairs in the subgraph

Evaluation: baseline

Baseline: random predictor

Randomly select pairs of authors who did not collaborate in the training interval

Probability that a random prediction is correct:

$$\frac{|E_{new}|}{\binom{|\text{Core}|}{2} - |E_{old}|}$$

In the datasets, from 0.15% (cond-mat) to 0.48% (astro-ph)

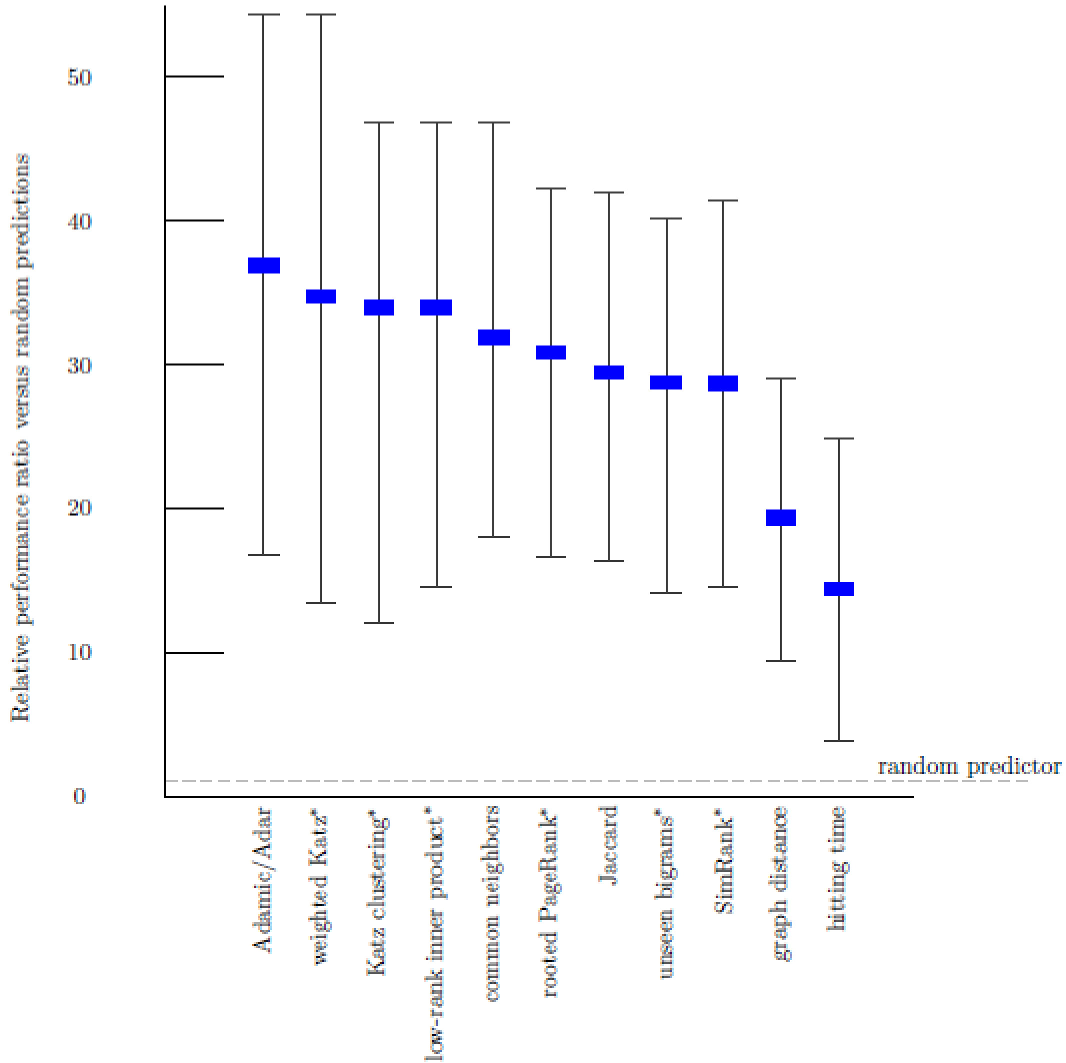
Evaluation: Factor improvement over random

predictor		astro-ph	cond-mat	gr-qc	hep-ph	hep-th
probability that a random prediction is correct		0.475%	0.147%	0.341%	0.207%	0.153%
graph distance (all distance-two pairs)		*9.4*	*25.1*	*21.3*	*12.0*	*29.0*
common neighbors		**18.0**	**40.8**	**27.1**	**26.9**	**46.9**
preferential attachment		4.7	6.0	7.5	*15.2*	7.4
Adamic/Adar		*16.8*	**54.4**	**30.1**	**33.2**	**50.2**
Jaccard		*16.4*	**42.0**	19.8	**27.6**	*41.5*
SimRank	$\gamma = 0.8$	*14.5*	*39.0*	*22.7*	*26.0*	*41.5*
hitting time		6.4	23.7	*24.9*	3.8	13.3
hitting time—normed by stationary distribution		5.3	23.7	11.0	11.3	21.2
commute time		5.2	15.4	**33.0**	*17.0*	23.2
commute time—normed by stationary distribution		5.3	16.0	11.0	11.3	16.2
rooted PageRank	$\alpha = 0.01$	*10.8*	*27.8*	**33.0**	*18.7*	*29.1*
	$\alpha = 0.05$	*13.8*	*39.6*	**35.2**	*24.5*	*41.1*
	$\alpha = 0.15$	*16.6*	**40.8**	**27.1**	**27.5**	*42.3*
	$\alpha = 0.30$	*17.1*	**42.0**	*24.9*	**29.8**	*46.5*
	$\alpha = 0.50$	*16.8*	**40.8**	*24.2*	**30.6**	*46.5*
Katz (weighted)	$\beta = 0.05$	3.0	21.3	19.8	2.4	12.9
	$\beta = 0.005$	*13.4*	**54.4**	**30.1**	*24.0*	**51.9**
	$\beta = 0.0005$	*14.5*	**53.8**	**30.1**	**32.5**	**51.5**
Katz (unweighted)	$\beta = 0.05$	*10.9*	**41.4**	**37.4**	*18.7*	**47.7**
	$\beta = 0.005$	*16.8*	**41.4**	**37.4**	*24.1*	**49.4**
	$\beta = 0.0005$	*16.7*	**41.4**	**37.4**	*24.8*	**49.4**

Evaluation: Factor improvement over random

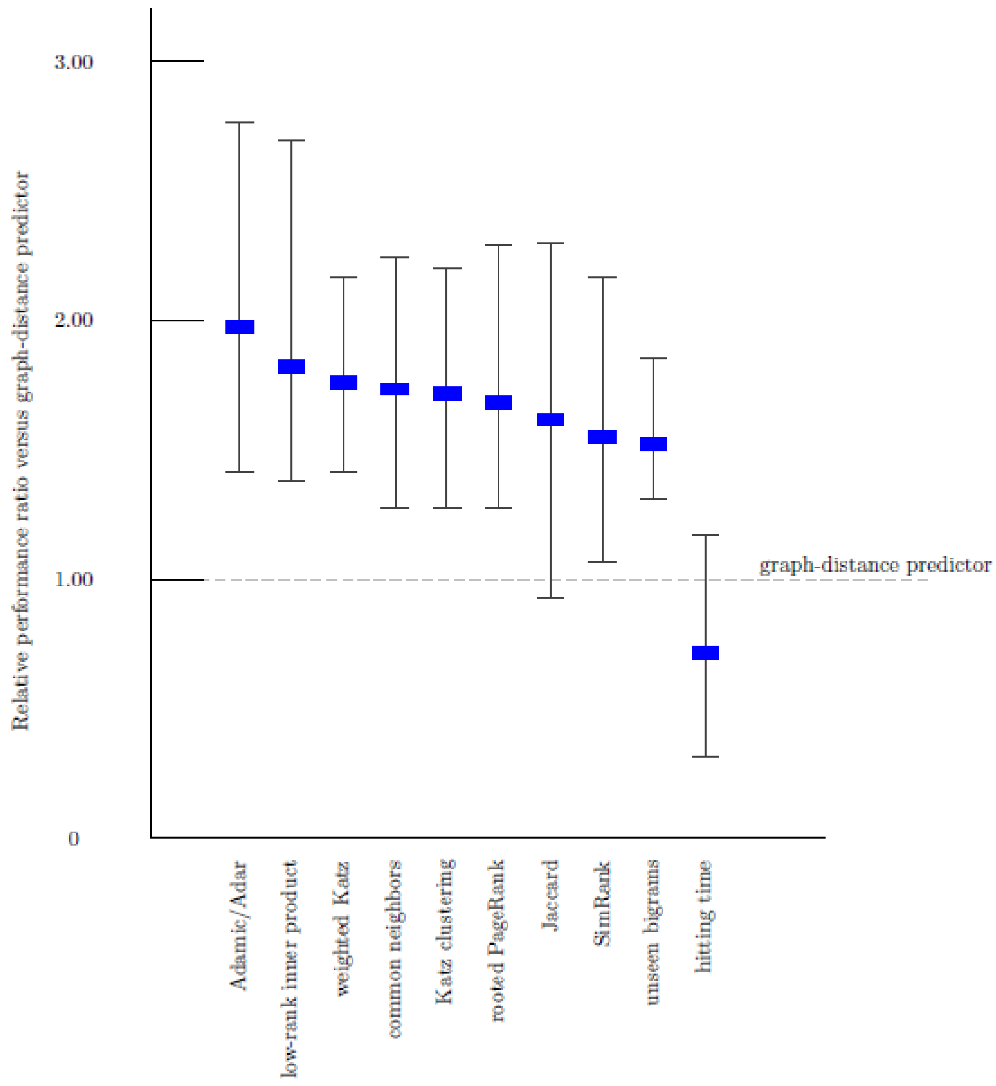
predictor		astro-ph	cond-mat	gr-qc	hep-ph	hep-th
probability that a random prediction is correct		0.475%	0.147%	0.341%	0.207%	0.153%
graph distance (all distance-two pairs)		*9.4*	*25.1*	*21.3*	*12.0*	*29.0*
common neighbors		**18.0**	**40.8**	**27.1**	**26.9**	**46.9**
Low-rank approximation: Inner product	rank = 1024	*15.2*	**53.8**	**29.3**	**34.8**	**49.8**
	rank = 256	*14.6*	**46.7**	**29.3**	**32.3**	**46.9**
	rank = 64	*13.0*	**44.4**	**27.1**	**30.7**	**47.3**
	rank = 16	*10.0*	21.3	**31.5**	**27.8**	*35.3*
	rank = 4	8.8	15.4	**42.5**	*19.5*	22.8
	rank = 1	6.9	5.9	**44.7**	*17.6*	14.5
Low-rank approximation: Matrix entry	rank = 1024	8.2	16.6	6.6	*18.5*	21.6
	rank = 256	*15.4*	*36.1*	8.1	*26.2*	*37.4*
	rank = 64	*13.7*	**46.1**	16.9	**28.1**	*40.7*
	rank = 16	9.1	21.3	*26.4*	*23.1*	*34.0*
	rank = 4	8.8	15.4	**39.6**	*20.0*	22.4
	rank = 1	6.9	5.9	**44.7**	*17.6*	14.5
Low-rank approximation: Katz ($\beta = 0.005$)	rank = 1024	*11.4*	*27.2*	**30.1**	**27.0**	*32.0*
	rank = 256	*15.4*	**42.0**	11.0	**34.2**	*38.6*
	rank = 64	*13.1*	**45.0**	19.1	**32.2**	*41.1*
	rank = 16	9.2	21.3	**27.1**	*24.8*	*34.9*
	rank = 4	7.0	15.4	**41.1**	*19.7*	22.8
	rank = 1	0.4	5.9	**44.7**	*17.6*	14.5
unseen bigrams (weighted)	common neighbors, $\delta = 8$	*13.5*	*36.7*	**30.1**	*15.6*	**46.9**
	common neighbors, $\delta = 16$	*13.4*	*39.6*	**38.9**	*18.5*	**48.6**
	Katz ($\beta = 0.005$), $\delta = 8$	*16.8*	*37.9*	*24.9*	*24.1*	**51.1**
	Katz ($\beta = 0.005$), $\delta = 16$	*16.5*	*39.6*	**35.2**	*24.7*	**50.6**
unseen bigrams (unweighted)	common neighbors, $\delta = 8$	*14.1*	*40.2*	**27.9**	*22.2*	*39.4*
	common neighbors, $\delta = 16$	*15.3*	*39.0*	**42.5**	*22.0*	*42.3*
	Katz ($\beta = 0.005$), $\delta = 8$	*13.1*	*36.7*	**32.3**	*21.6*	*37.8*
	Katz ($\beta = 0.005$), $\delta = 16$	*10.3*	*29.6*	**41.8**	*12.2*	*37.8*
clustering: Katz ($\beta_1 = 0.001, \beta_2 = 0.1$)	$\rho = 0.10$	7.4	*37.3*	**46.9**	**32.9**	*37.8*
	$\rho = 0.15$	*12.0*	**46.1**	**46.9**	*21.0*	*44.0*
	$\rho = 0.20$	4.6	*34.3*	19.8	*21.2*	*35.7*
	$\rho = 0.25$	3.3	*27.2*	20.5	*19.4*	17.4

Evaluation: Average relevance performance (random)

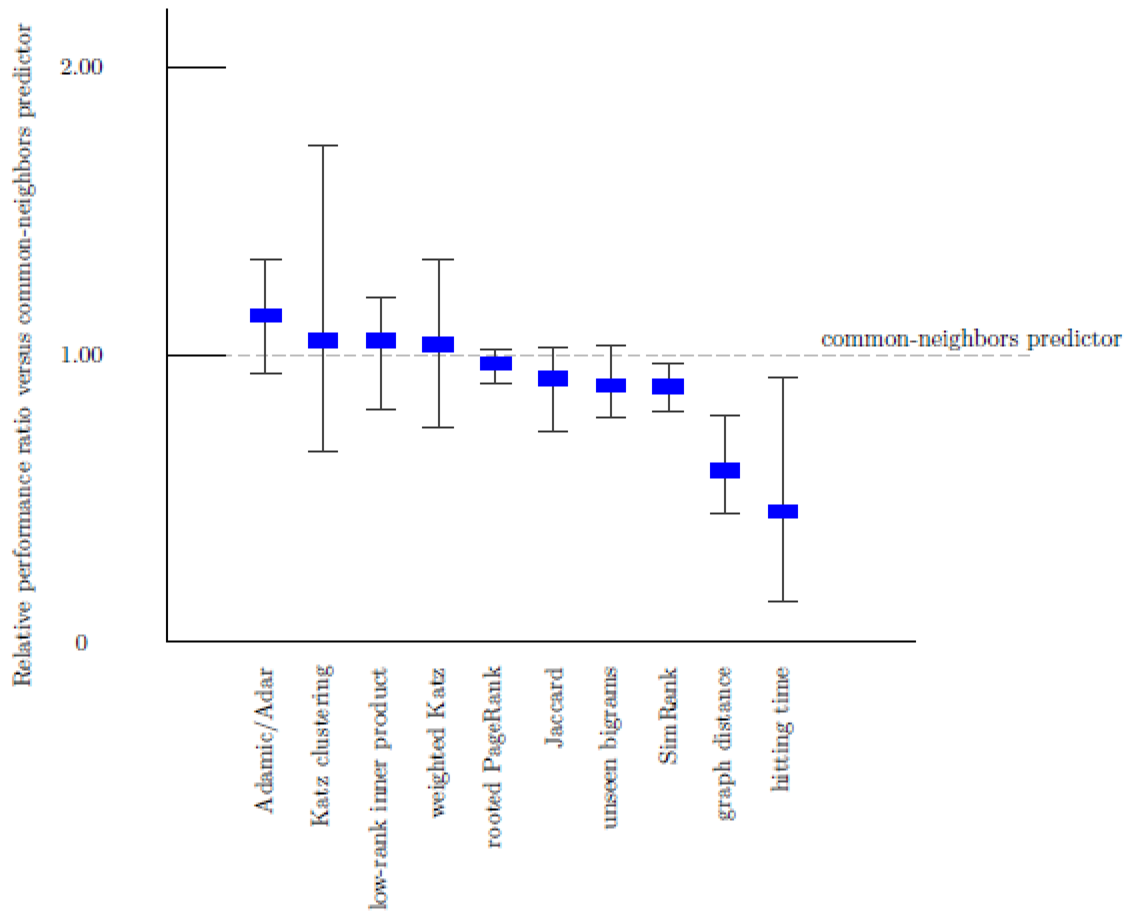


- average ratio over the five datasets of the given predictor's performance *versus a baseline* predictor's performance.
- the error bars indicate the minimum and maximum of this ratio over the five datasets.
- the parameters for the starred predictors are as follows: (1) for weighted Katz, $\beta = 0.005$; (2) for Katz clustering, $\beta 1 = 0.001$; $\rho = 0.15$; $\beta 2 = 0.1$; (3) for low-rank inner product, rank = 256; (4) for rooted Pagerank, $\alpha = 0.15$; (5) for unseen bigrams, unweighted
- common neighbors with $\delta = 8$; and (6) for SimRank, $\gamma = 0.8$.

Evaluation: Average relevance performance (distance)



Evaluation: Average relevance performance (neighbors)



Evaluation: prediction overlap

❖ How much similar are the predictions made by the different methods?

Why?

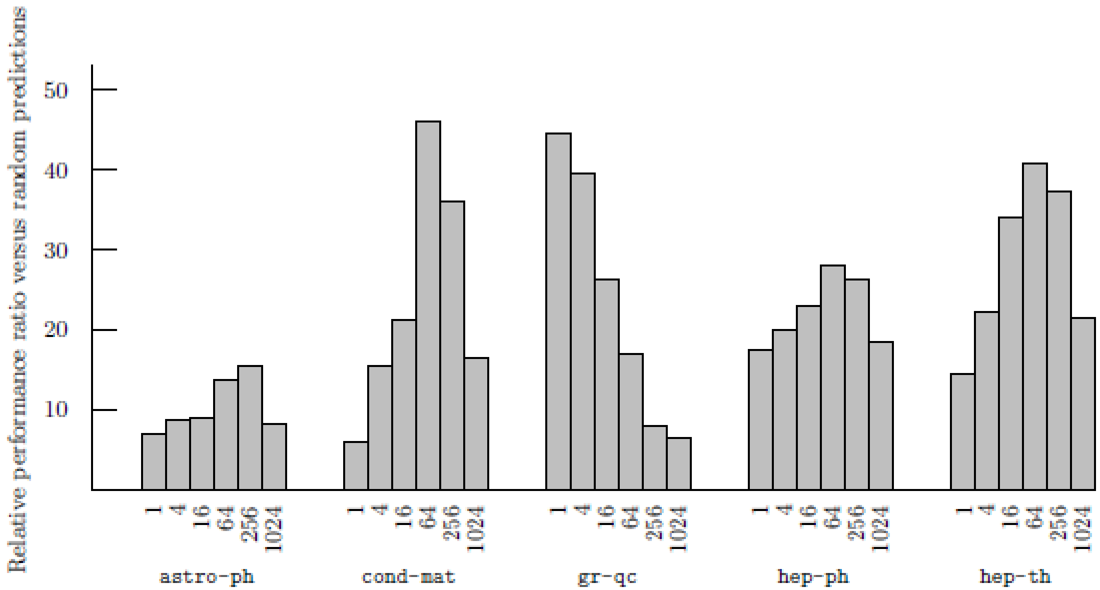
	Adamic/Adar	Katz clustering	common neighbors	hitting time	Jaccard's coefficient	weighted Katz	low-rank inner product	rooted Pagerank	SimRank	unseen bigrams
Adamic/Adar	1150	638	520	193	442	1011	905	528	372	486
Katz clustering		1150	411	182	285	630	623	347	245	389
common neighbors			1150	135	506	494	467	305	332	489
hitting time				1150	87	191	192	247	130	156
Jaccard's coefficient					1150	414	382	504	845	458
weighted Katz						1150	1013	488	344	474
low-rank inner product							1150	453	320	448
rooted Pagerank								1150	678	461
SimRank									1150	423
unseen bigrams										1150

correct

	Adamic/Adar	Katz clustering	common neighbors	hitting time	Jaccard's coefficient	weighted Katz	low-rank inner product	rooted Pagerank	SimRank	unseen bigrams
Adamic/Adar	92	65	53	22	43	87	72	44	36	49
Katz clustering		78	41	20	29	66	60	31	22	37
common neighbors			69	13	43	52	43	27	26	40
hitting time				40	8	22	19	17	9	15
Jaccard's coefficient					71	41	32	39	51	43
weighted Katz						92	75	44	32	51
low-rank inner product							79	39	26	46
rooted Pagerank								69	48	39
SimRank									66	34
unseen bigrams										68

Evaluation: datasets

❖ How much does the performance of the different methods depends on the dataset?



- (rank) On 4 of the 5 datasets best at an intermediate rank
  On qr-qc, best at rank 1, does it have a “simpler” structure”?
- On hep-ph, preferential attachment the best
- Why is astro-ph “difficult”?

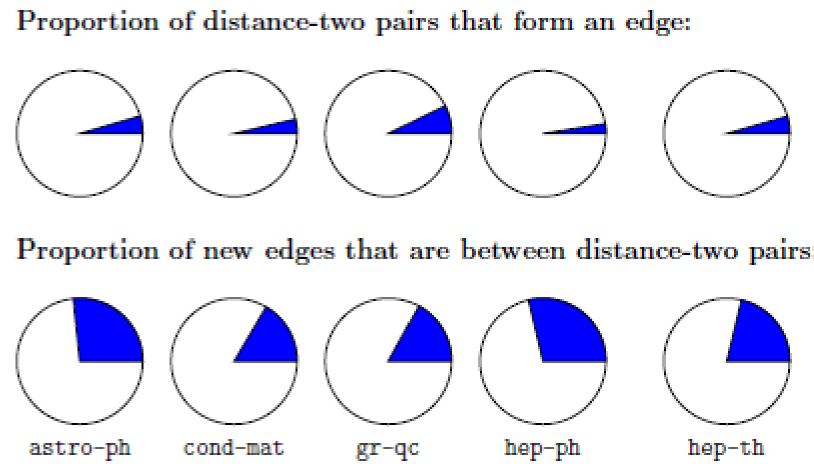
The culture of physicists and physics collaboration

Evaluation: small world

The shortest path even in unrelated disciplines is often very short

Basic classifier on graph distances does not work

Evaluation: restricting to distance three



Many pairs of authors separated by a graph distance of 2 will not collaborate and

Many pairs who collaborate at distance greater than 2

Disregard all distance 2 pairs

	astro-ph	cond-mat	gr-qc	hep-ph	hep-th
# pairs at distance two	33862	5145	935	37687	7545
# new collaborations at distance two	1533	190	68	945	335
# new collaborations	5751	1150	400	3294	1576

predictor		astro-ph	cond-mat	gr-qc	hep-ph	hep-th
graph distance (all distance-three pairs)		2.8	5.4	7.7	4.0	8.6
preferential attachment		3.2	2.6	8.6	4.7	1.4
SimRank	$\gamma = 0.8$	5.9	14.3	10.6	7.6	21.9
hitting time		4.4	10.1	13.7	4.5	4.7
hitting time—normed by stationary distribution		2.0	2.5	0.0	2.5	6.6
commute time		3.8	5.9	21.1	5.9	6.6
commute time—normed by stationary distribution		2.6	0.8	1.1	4.8	4.7
rooted PageRank	$\alpha = 0.01$	4.6	12.7	21.1	6.5	12.6
	$\alpha = 0.05$	5.3	13.5	21.1	8.7	16.6
	$\alpha = 0.15$	5.4	11.8	18.0	10.7	19.9
	$\alpha = 0.30$	5.8	13.5	8.4	11.6	19.9
	$\alpha = 0.50$	6.3	15.2	7.4	12.7	19.9
Katz (weighted)	$\beta = 0.05$	1.5	5.9	11.6	2.3	2.7
	$\beta = 0.005$	5.5	14.3	28.5	4.2	12.6
	$\beta = 0.0005$	6.2	13.5	27.5	4.2	12.6
Katz (unweighted)	$\beta = 0.05$	2.3	12.7	30.6	9.0	12.6
	$\beta = 0.005$	9.1	11.8	30.6	5.1	17.9
	$\beta = 0.0005$	9.2	11.8	30.6	5.1	17.9
Low-rank approximation:	rank = 1024	2.3	2.5	9.5	4.0	6.0
Inner product	rank = 256	4.8	5.9	5.3	9.9	10.6
	rank = 64	3.8	12.7	5.3	7.1	11.3
	rank = 16	5.3	6.7	6.3	6.8	15.3
	rank = 4	5.1	6.7	32.7	2.0	4.7
	rank = 1	6.1	2.5	32.7	4.2	8.0
Low-rank approximation:	rank = 1024	4.1	6.7	6.3	5.9	13.3
Matrix entry	rank = 256	3.8	8.4	3.2	8.5	19.9
	rank = 64	2.9	11.8	2.1	4.0	10.0
	rank = 16	4.4	8.4	4.2	5.9	16.6
	rank = 4	4.9	6.7	27.5	2.0	4.7
	rank = 1	6.1	2.5	32.7	4.2	8.0
Low-rank approximation:	rank = 1024	4.3	6.7	28.5	5.9	13.3
Katz ($\beta = 0.005$)	rank = 256	3.6	8.4	3.2	8.5	20.6
	rank = 64	2.8	11.8	2.1	4.2	10.6
	rank = 16	5.0	8.4	5.3	5.9	15.9
	rank = 4	5.2	6.7	28.5	2.0	4.7
	rank = 1	0.3	2.5	32.7	4.2	8.0
unseen bigrams	common neighbors, $\delta = 8$	5.8	6.7	14.8	4.2	23.9
(weighted)	common neighbors, $\delta = 16$	7.9	9.3	28.5	5.1	19.3
	Katz ($\beta = 0.005$), $\delta = 8$	5.2	10.1	22.2	2.8	17.9
	Katz ($\beta = 0.005$), $\delta = 16$	6.6	10.1	29.6	3.7	15.3
unseen bigrams	common neighbors, $\delta = 8$	5.4	5.1	13.7	4.5	21.3
(unweighted)	common neighbors, $\delta = 16$	6.3	8.4	25.3	4.8	21.9
	Katz ($\beta = 0.005$), $\delta = 8$	4.1	7.6	22.2	2.0	17.3
	Katz ($\beta = 0.005$), $\delta = 16$	4.3	4.2	28.5	3.1	16.6
clustering:	$\rho = 0.10$	3.2	4.2	31.7	7.1	8.6
Katz ($\beta_1 = 0.001, \beta_2 = 0.1$)	$\rho = 0.15$	4.6	4.2	32.7	7.6	6.6
	$\rho = 0.20$	2.3	5.9	7.4	4.5	8.0
	$\rho = 0.25$	2.0	11.8	6.3	6.8	5.3

Evaluation: the breadth of data

Three additional datasets

1. Proceedings of STOC and FOCS
2. Papers for Citeseer
3. All five of the arXiv sections

Common neighbors vs Random

STOC/FOCS	arXiv sections	combined arXiv sections	Citeseer
6.1	18.0—46.9	71.2	147.0

✓ Suggests that is easier to predict links *within communities*

Extensions

- Improve performance. Even the best (Katz clustering on gr-qc) correct on only about 16% of its prediction

- Improve efficiency on very large networks (approximation of distances)

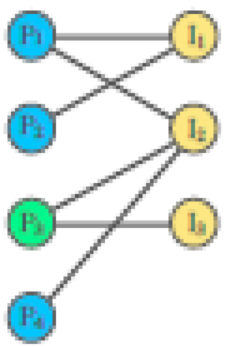
- Treat more recent collaborations as more important

- Additional information (paper titles, author institutions, etc)
  To some extent latently present in the graph

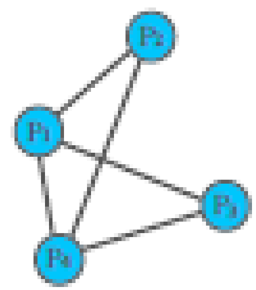
Extensions

❖ Consider bipartite graph (e.g., some form of an affiliation network)

author paper



Coauthor graph



Using Supervised Learning

Given a collection of records (*training set*)

Each record contains
a set of *attributes (features)* + the *class attribute*.

Find a *model* for class attribute as a function of the values of other attributes.

Goal: previously unseen records should be assigned a class as accurately as possible.

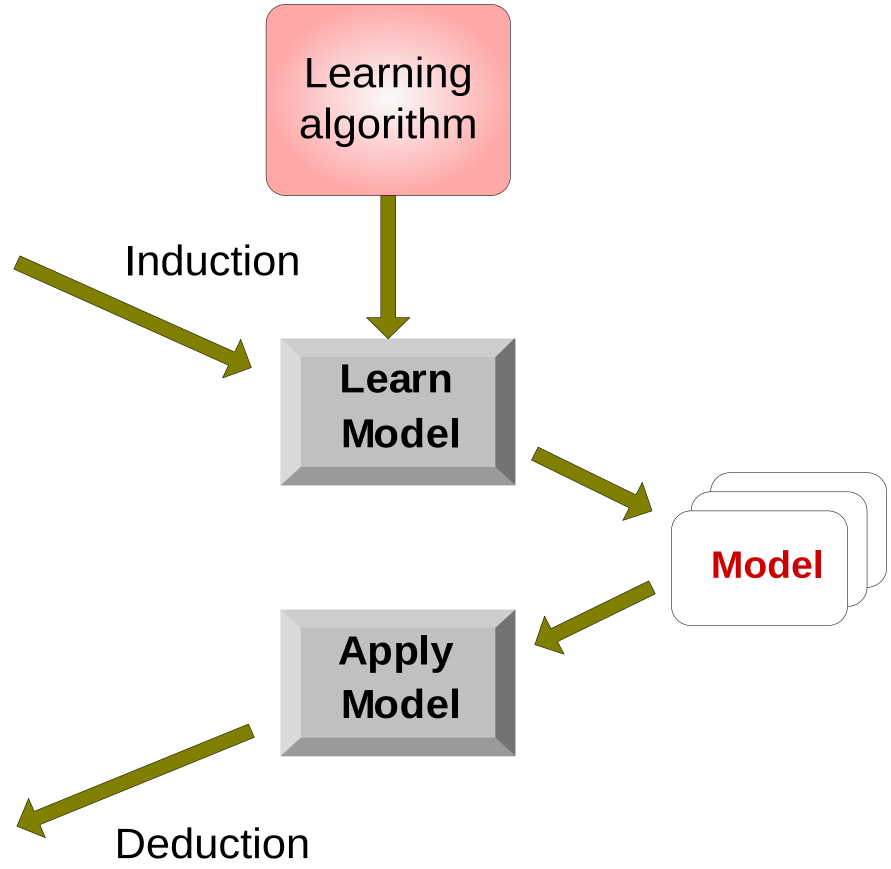
A test set is used to determine the accuracy of the model.

Usually, the given data set is divided into training and test sets, with training set used to build the model and test set used to validate it.

Illustrating the Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set



Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set

Classification Techniques

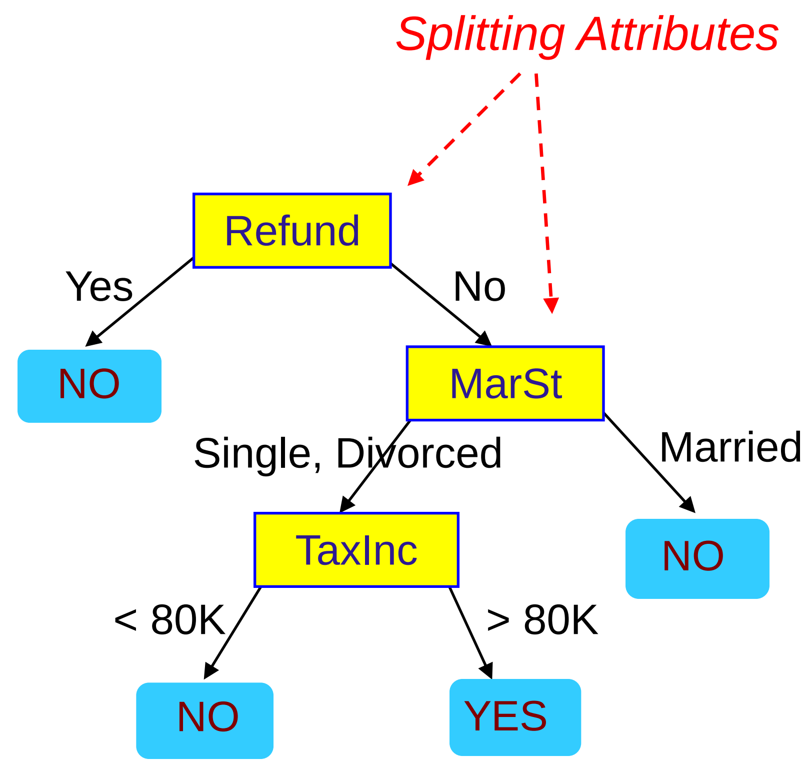
- Decision Tree based Methods
- Rule-based Methods
- Memory based reasoning
- Neural Networks
- Naïve Bayes and Bayesian Belief Networks
- Support Vector Machines

Example of a Decision Tree

categorical categorical continuous class

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	**No**
2	No	Married	100K	**No**
3	No	Single	70K	**No**
4	Yes	Married	120K	**No**
5	No	Divorced	95K	**Yes**
6	No	Married	60K	**No**
7	Yes	Divorced	220K	**No**
8	No	Single	85K	**Yes**
9	No	Married	75K	**No**
10	No	Single	90K	**Yes**

Training Data



Model: Decision Tree

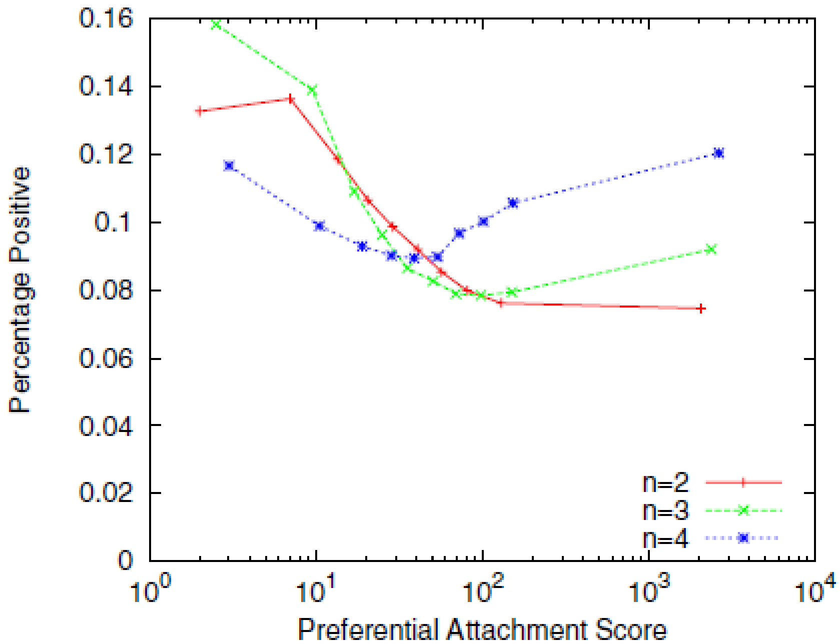
Classification for Link Prediction

Class?
Features (predictors)?

Name	Parameters	HPLP	HPLP+
In-Degree(i)	-	✓	✓
In-Volume(i)	-	✓	✓
In-Degree(j)	-	✓	✓
In-Volume(j)	-	✓	✓
Out-Degree(i)	-	✓	✓
Out-Volume(i)	-	✓	✓
Out-Degree(j)	-	✓	✓
Out-Volume(j)	-	✓	✓
Common Nbrs(i,j)	-	✓	✓
Max. Flow(i,j)	$l = 5$	✓	✓
Shortest Paths(i,j)	$l = 5$	✓	✓
PropFlow(i,j)	$l = 5$	✓	✓
Adamic/Adar(i,j)	-		✓
Jaccard's Coef(i,j)	-		✓
Katz(i,j)	$l = 5$, $\beta = 0.005$		✓
Pref Attach(i,j)	-		✓

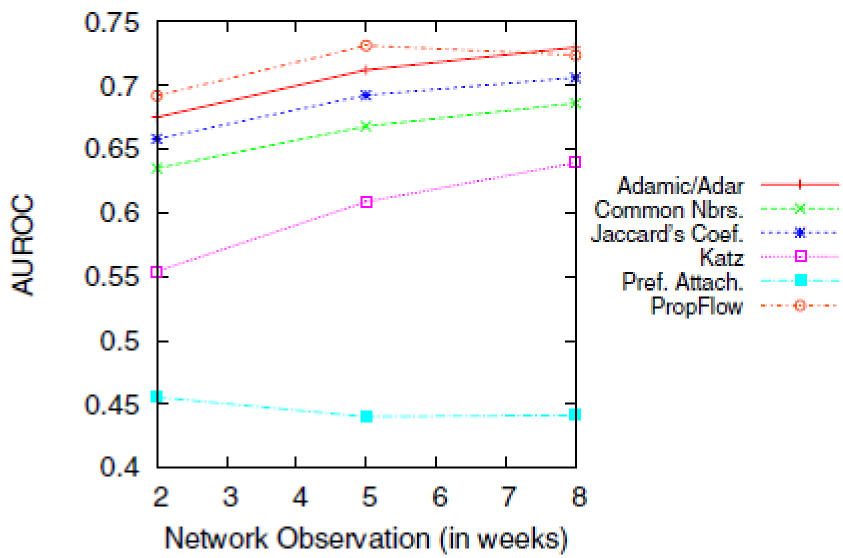
PropFlow: random walks, stops at l or when cycle

Using Supervised Learning: why?



- Even training on a single feature may outperform ranking (restriction to n-neighborhoods)
- Dependencies between features

How to split data



- Observations in [t1, t2] split at tx
- Large tx => better quality of features
- But less positives

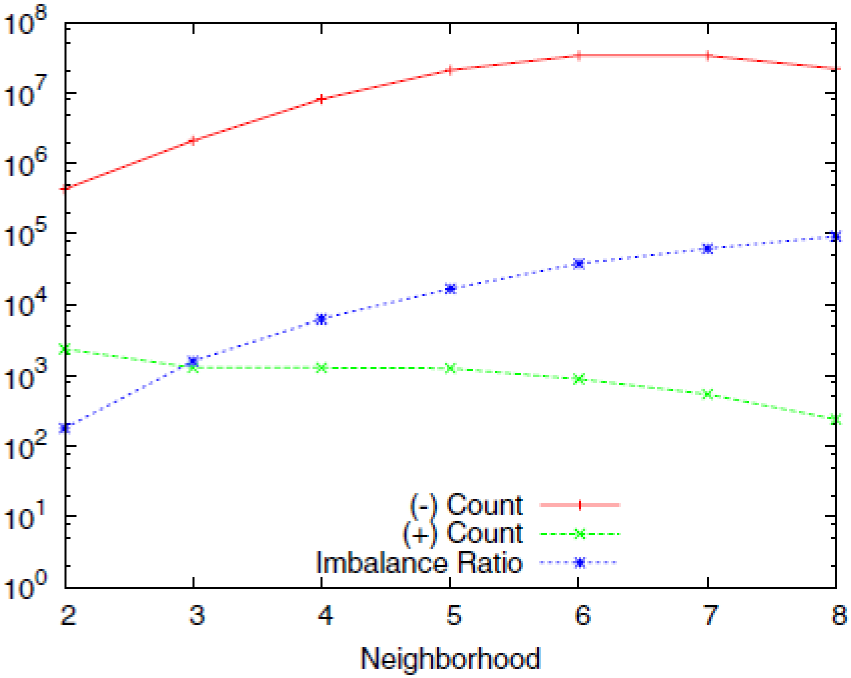
Imbalance

- Sparse networks: $|E| = k\ |V|$ for constant $k << |V|$

The class imbalance ration for link prediction in a sparse network is $\Omega(|V|/1)$ when at most $|V|$ nodes are added

Missing links is $|V|^2$
Positives V

Treat each neighborhood as a separate problem



Metrics for Performance Evaluation

Confusion Matrix:

	PREDICTED CLASS		
ACTUAL CLASS		Class=Yes	Class=No
	Class=Yes	TP	FN
	Class=No	FP	TN

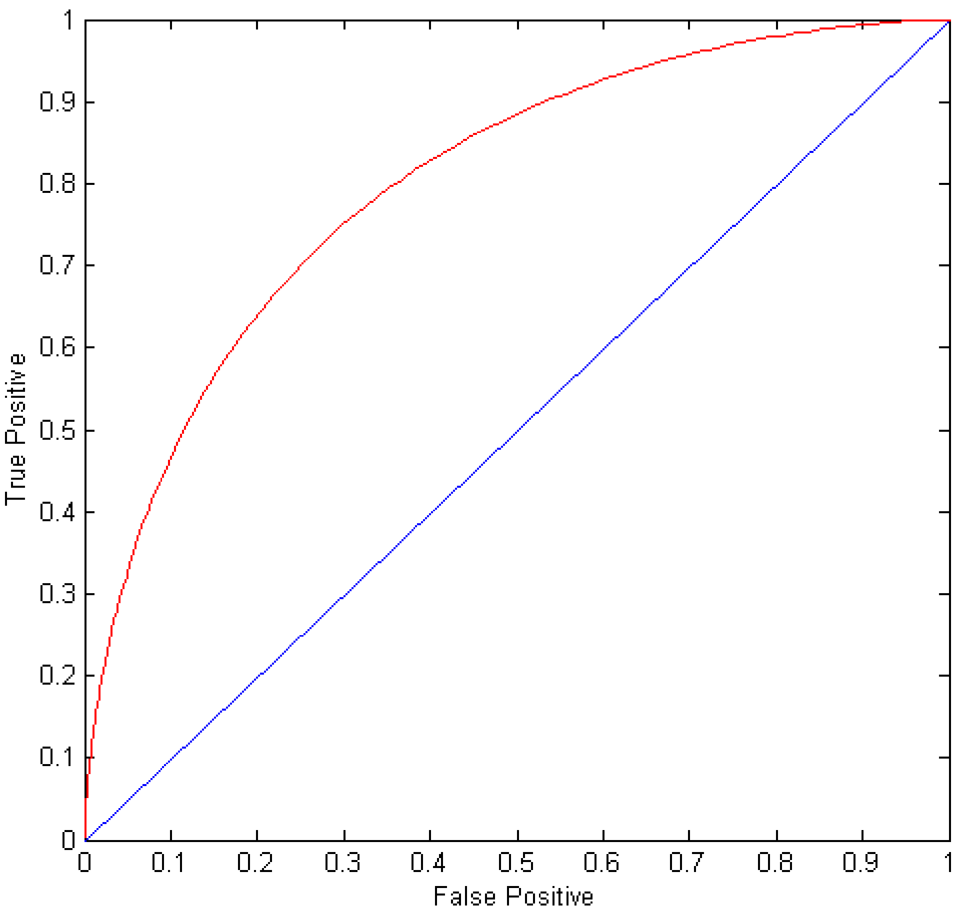
$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

ROC Curve

(TP,FP):

- (0,0): declare everything to be negative class
- (1,1): declare everything to be positive class
- (1,0): ideal

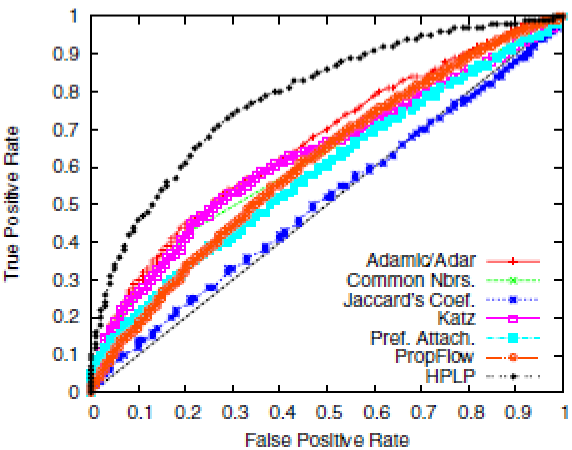
- Diagonal line:
  - Random guessing
  - Below diagonal line:
    - prediction is opposite of tl true class



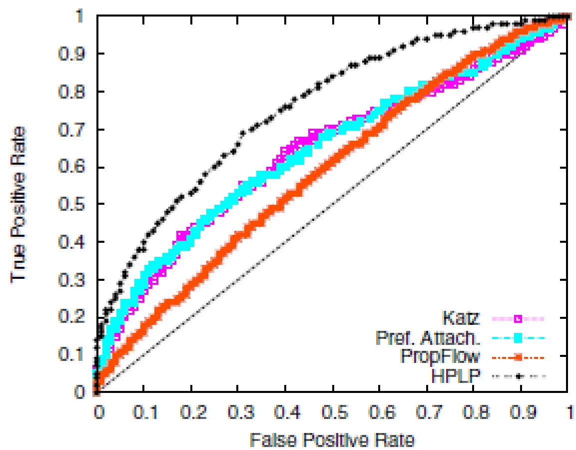
AUC: area under the ROC

Results

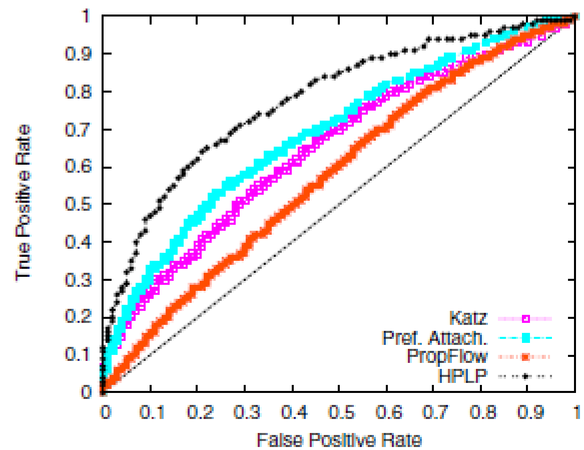
Ensemble of classifiers: Random Forest



(d) condmat $n = 2$



(e) condmat $n = 3$



(f) condmat $n = 4$

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