Online Social Networks and Media

Network Measurements

Measuring Networks

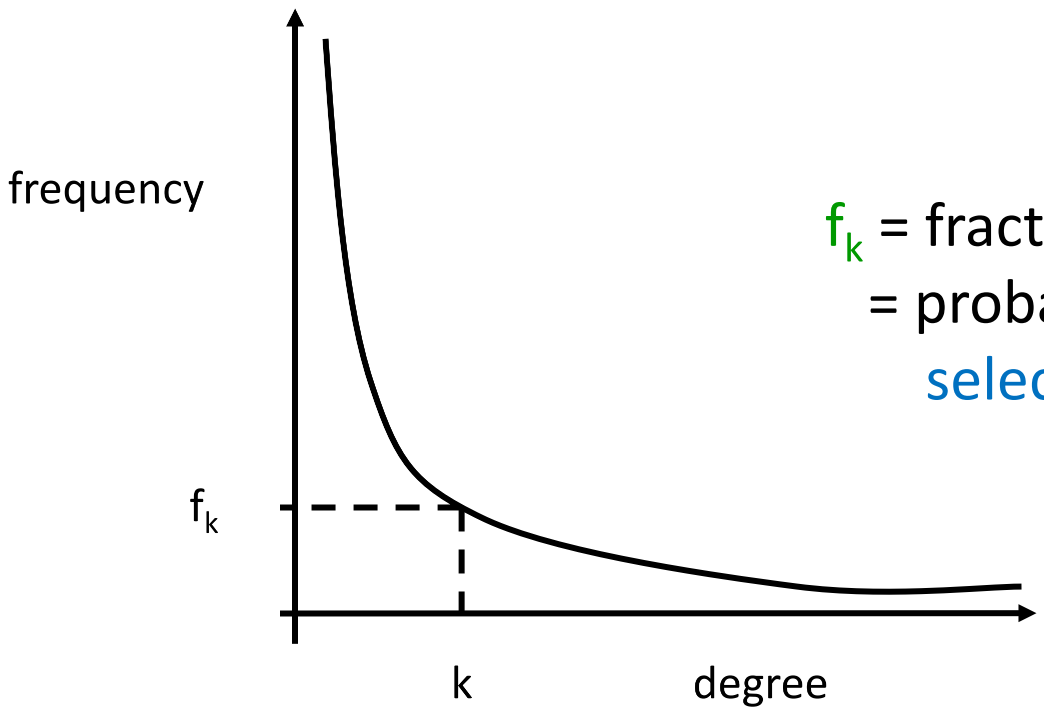
- Degree distributions and power-laws
- Clustering Coefficient
- Small world phenomena
- Components
- Motifs
- Homophily

The basic random graph model

- The measurements on real networks are usually compared against those on "random networks"

- The basic $G_{n,p}$ (Erdös-Renyi) random graph model:
  - n : the number of vertices
  - $0 \leq p \leq 1$
  - for each pair (i,j), generate the edge (i,j) independently with probability p
  - Expected degree of a node: $z = np$

Degree distributions



f_k = fraction of nodes with degree k
= probability of a randomly selected node to have degree k

- Problem: find the probability distribution that best fits the observed data

Power-law distributions

- The degree distributions of most real-life networks follow a power law

$$p(k) = Ck^{-\alpha}$$

- Right-skewed/Heavy-tail distribution
  - there is a non-negligible fraction of nodes that has very high degree (hubs)
  - scale-free: no characteristic scale, average is not informative

- In stark contrast with the random graph model!
  - Poisson degree distribution, z=np

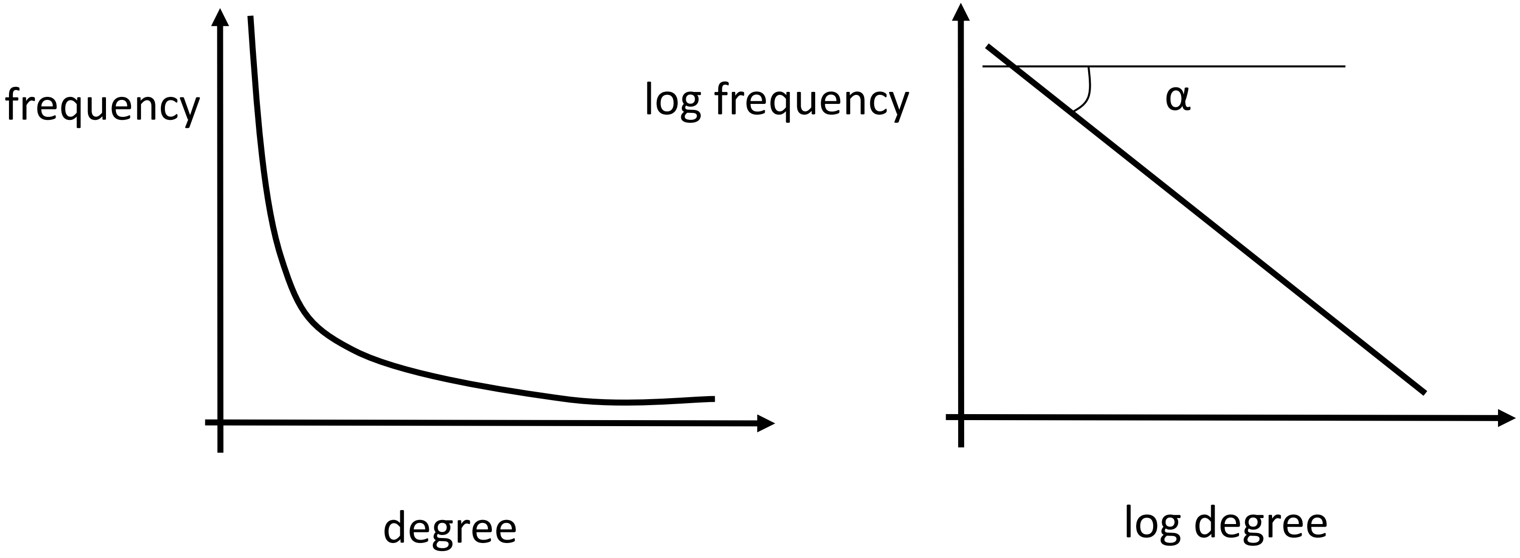
$$p(k) = \frac{z^k}{k!}e^{-z}$$

  - Concentrated around the mean
  - the probability of very high degree nodes is exponentially small

Power-law signature

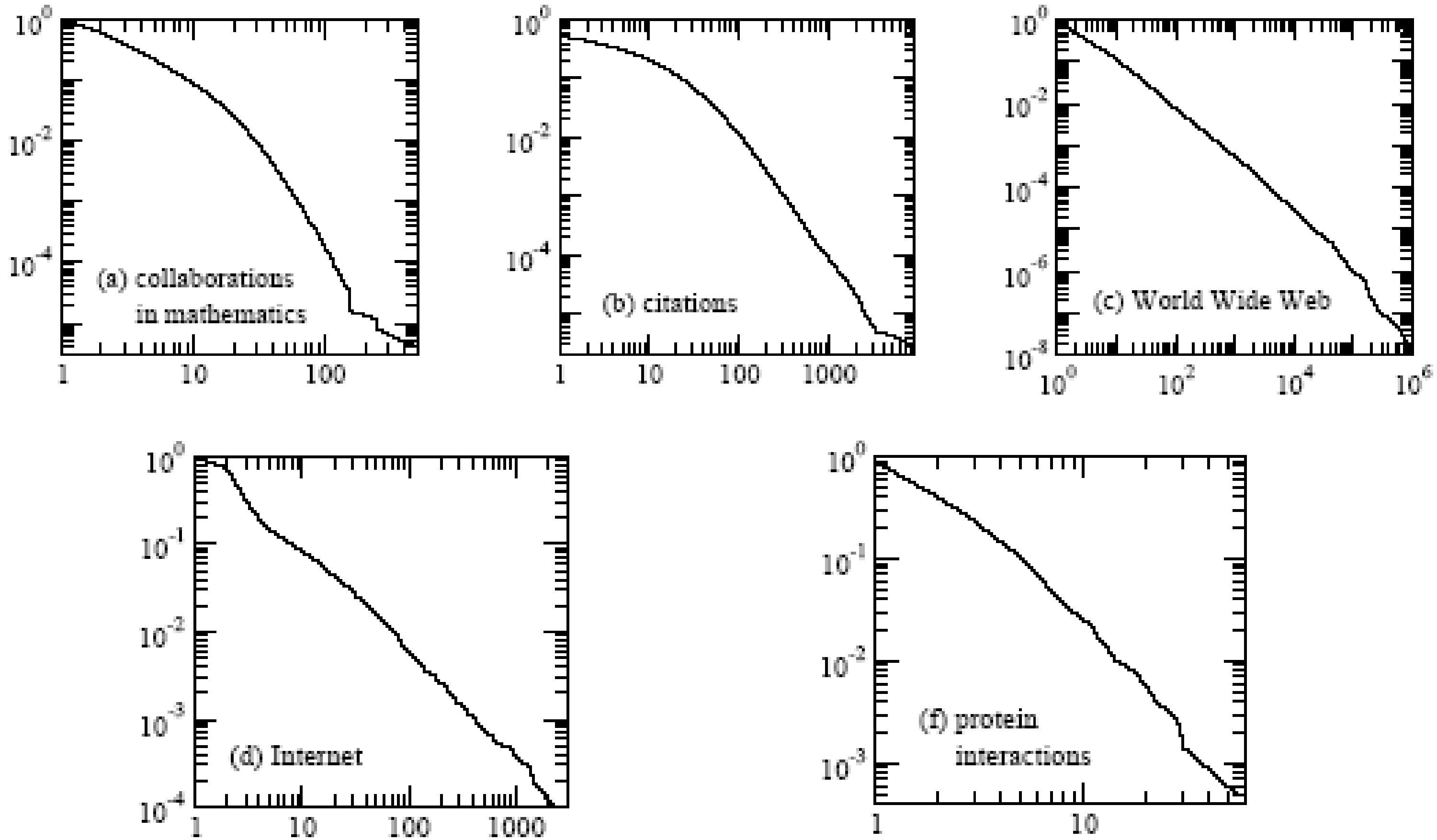
- Power-law distribution gives a line in the log-log plot

$$\log p(k) = -\alpha \log k + \log C$$



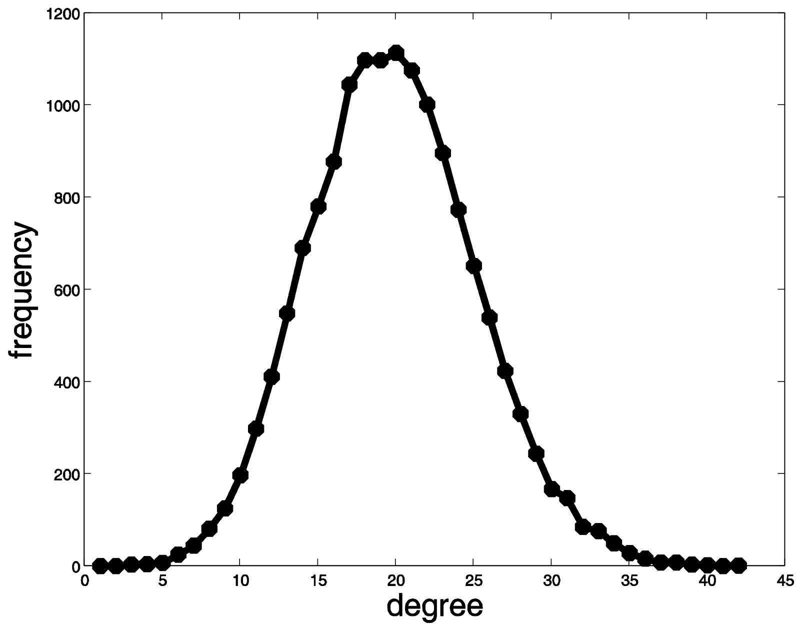
- α : power-law exponent (typically $2 \le \alpha \le 3$)

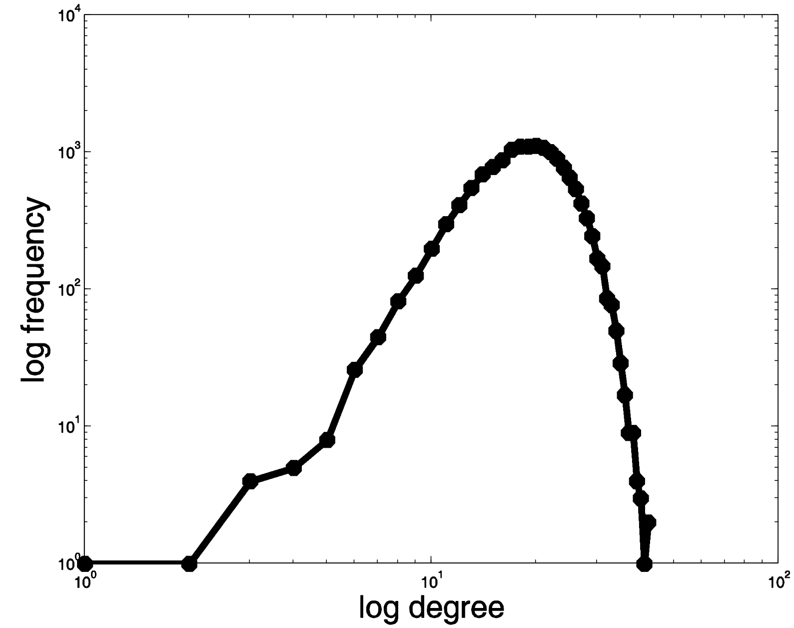
Examples



Taken from [Newman 2003]

A random graph example





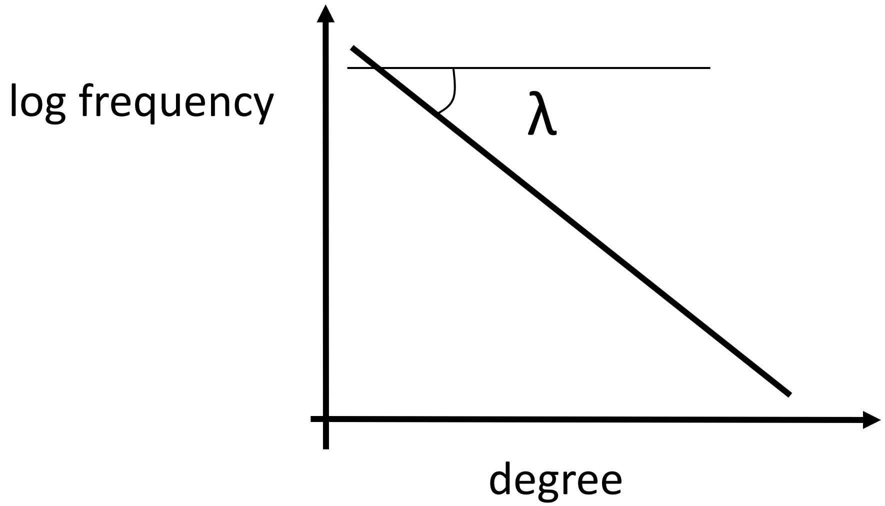
Exponential distribution

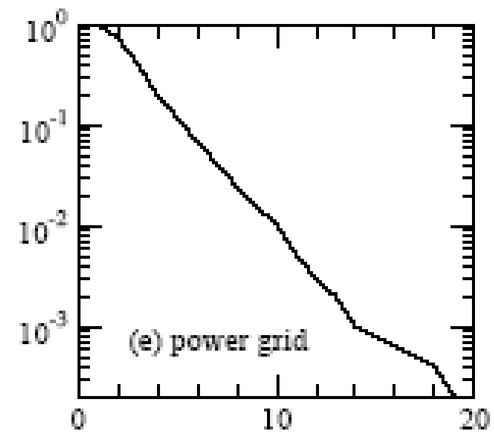
- Observed in some technological or collaboration networks

$$p(k) = \lambda e^{-\lambda k}$$

- Identified by a line in the log-linear plot

$$\log p(k) = -\lambda k + \log \lambda$$

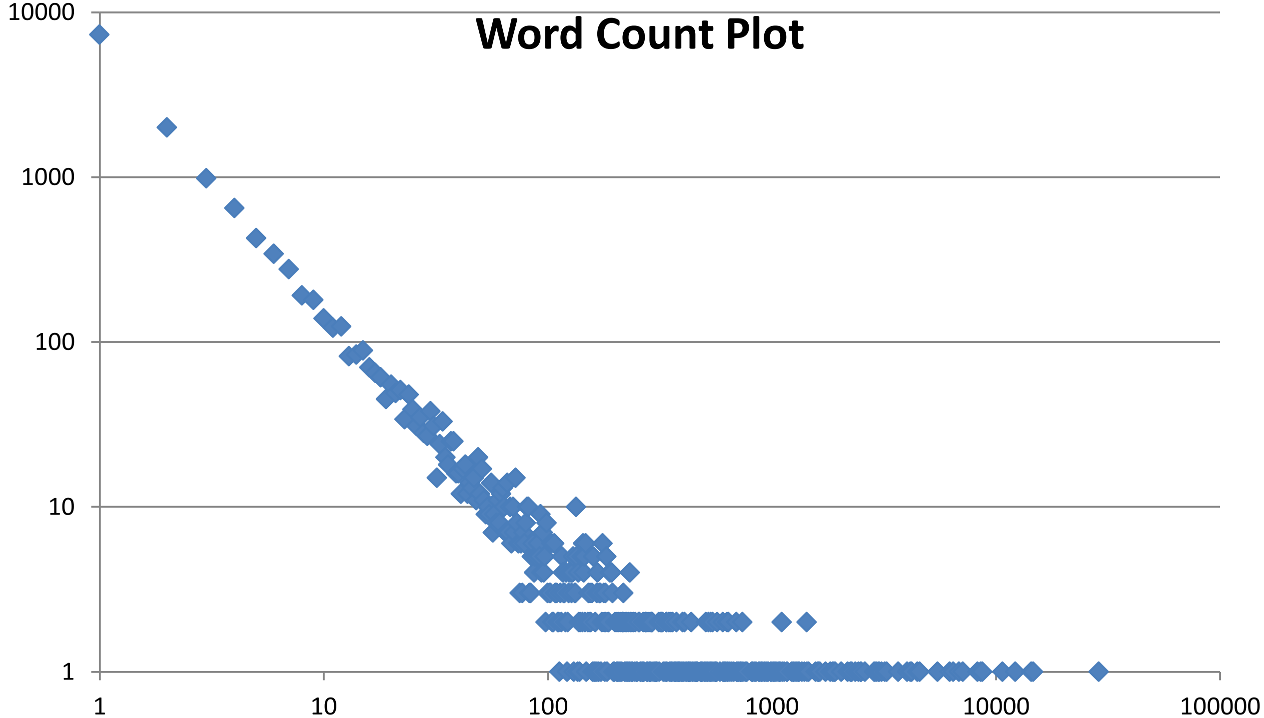




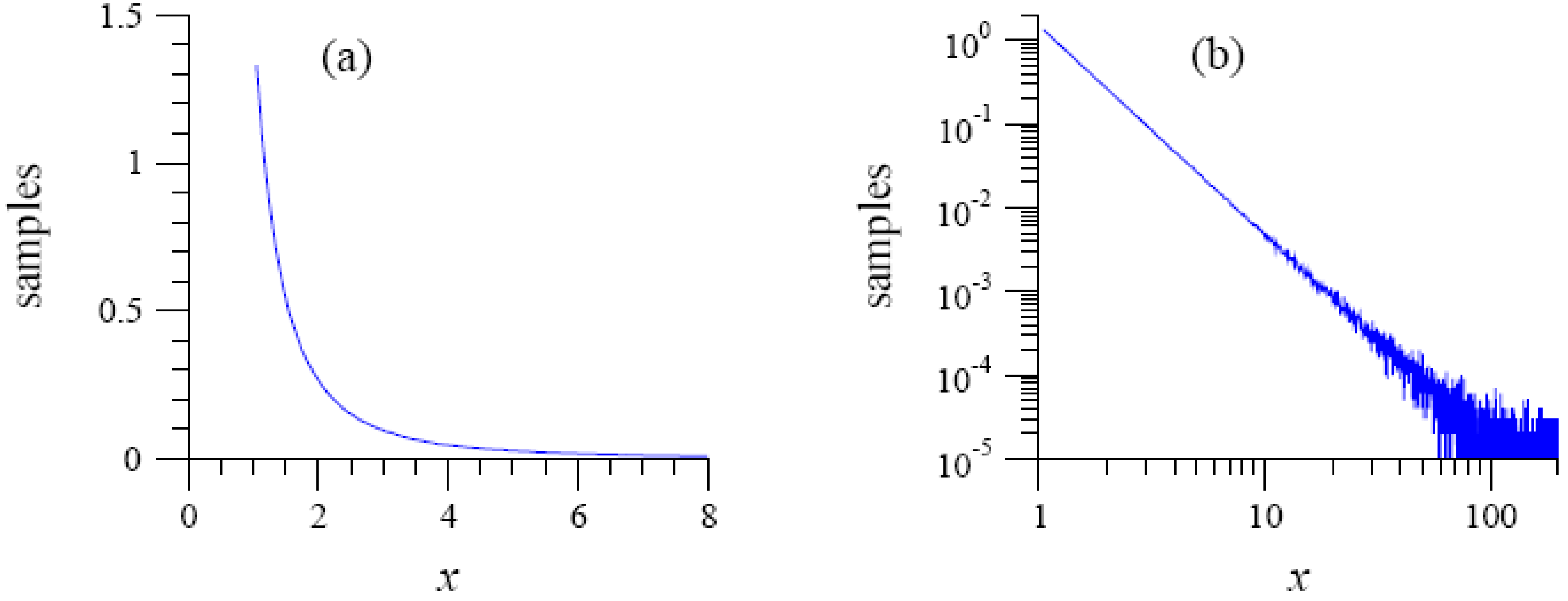
Measuring power-laws

- How do we create these plots? How do we measure the power-law exponent?
- Collect a set of measurements:
  - E.g., the degree of each page, the number of appearances of each word in a document, the size of solar flares(continuous)
- Create a value histogram
  - For discrete values, number of times each value appears
  - For continuous values (but also for discrete):
    - Break the range of values into bins of equal width
    - Sum the count of values in the bin
    - Represent the bin by the mean (median) value
- Plot the histogram in log-log scale
  - Bin representatives vs Value in the bin

Discrete Counts



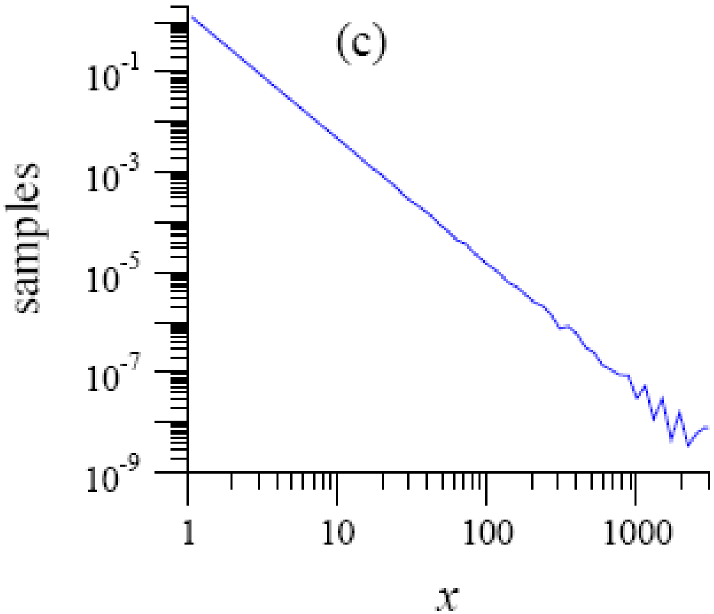
Measuring power laws



Simple binning produces a noisy plot

Logarithmic binning

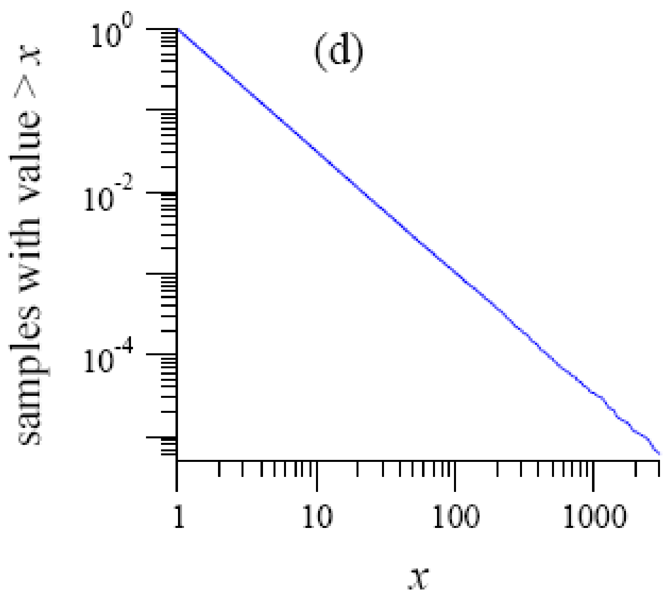
- Exponential binning
  - Create bins that grow exponentially in size
  - In each bin divide the sum of counts by the bin length (number of observations per bin unit)

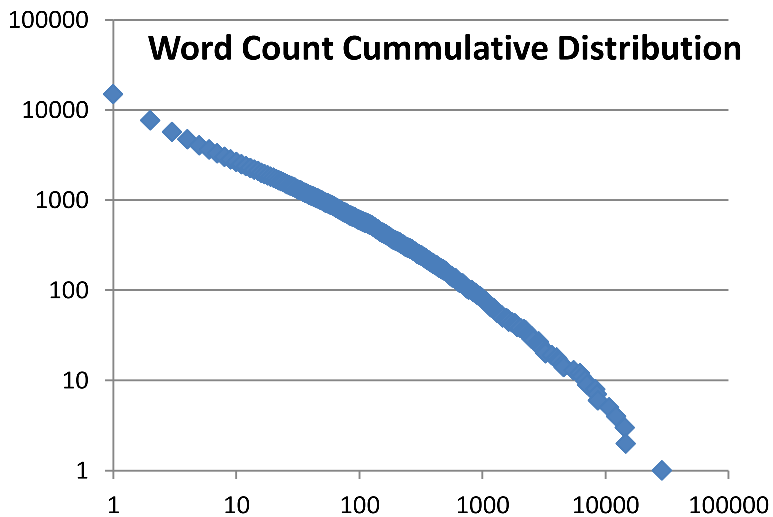


Still some noise at the tail

Cumulative distribution

- Compute the cumulative distribution
  - P[X≥x]: fraction (or number) of observations that have value at least x
  - It also follows a power-law with exponent α-1





Pareto distribution

- A random variable follows a Pareto distribution if

$$P[X \geq x] = C' x^{-\beta} \qquad\qquad x \geq x_{min}$$

- Power law distribution with exponent $\alpha = 1 + \beta$

Zipf plot

- There is another easy way to see the power-law, by doing the Zipf plot
  - Order the values in decreasing order
  - Plot the values against their rank in log-log scale
    - i.e., for the r-th value x_r, plot the point $(\log(r),\log(x_r))$
  - If there is a power-law you should see something like a straight line

Zipf's Law

- A random variable X follows Zipf's law if the r-th largest value x_r satisfies

$$x_r \approx r^{-\gamma}$$

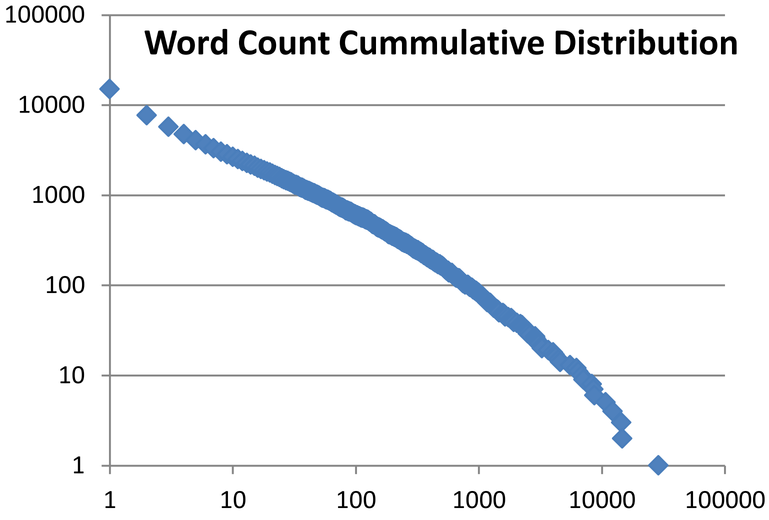
- Same as Pareto distribution

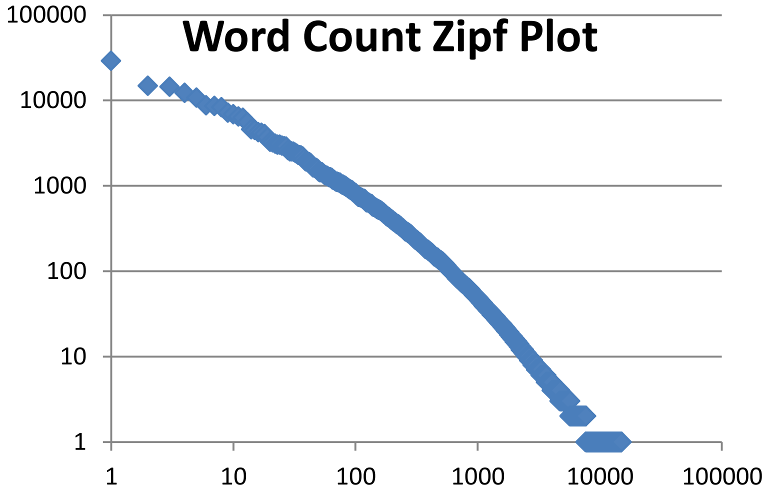
$$P[X \geq x] \approx x^{-1/\gamma}$$

- X follows a power-law distribution with $\alpha=1+1/\gamma$

- Named after Zipf, who studied the distribution of words in English language and found Zipf law with exponent 1

Zipf vs Pareto





Computing the exponent

- Maximum likelihood estimation
  - Assume that the set of data observations **x** are produced by a power-law distribution with some exponent α
    - Exact law: $p(x) = \frac{\alpha-1}{x_{min}}\left(\frac{x}{x_{min}}\right)^{-\alpha}$
  - Find the exponent that maximizes the probability P(α|**x**)

$$\alpha = 1 + n\left[\sum_{i=1}^{n} \ln \frac{x_i}{x_{min}}\right]^{-1}$$

Collective Statistics (M. Newman 2003)

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	−0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	−0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	−0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	−0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	−0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	−0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	−0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	−0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	−0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	−0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	−0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	−0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	−0.226	416, 421

TABLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices n; total number of edges m; mean degree z; mean vertex–vertex distance ℓ; exponent α of degree distribution if the distribution follows a power law (or "–" if not; in/out-degree exponents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r, Sec. III.F. The last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

Power Laws - Recap

- A (continuous) random variable X follows a power-law distribution if it has density function

$$p(x) = Cx^{-\alpha}$$

- A (continuous) random variable X follows a Pareto distribution if it has cumulative function

$$P[X \geq x] = Cx^{-\beta}$$

power-law with α=1+β

- A (discrete) random variable X follows Zipf's law if the frequency of the r-th largest value satisfies

$$p_r = Cr^{-\gamma}$$

power-law with α=1+1/γ

Average/Expected degree

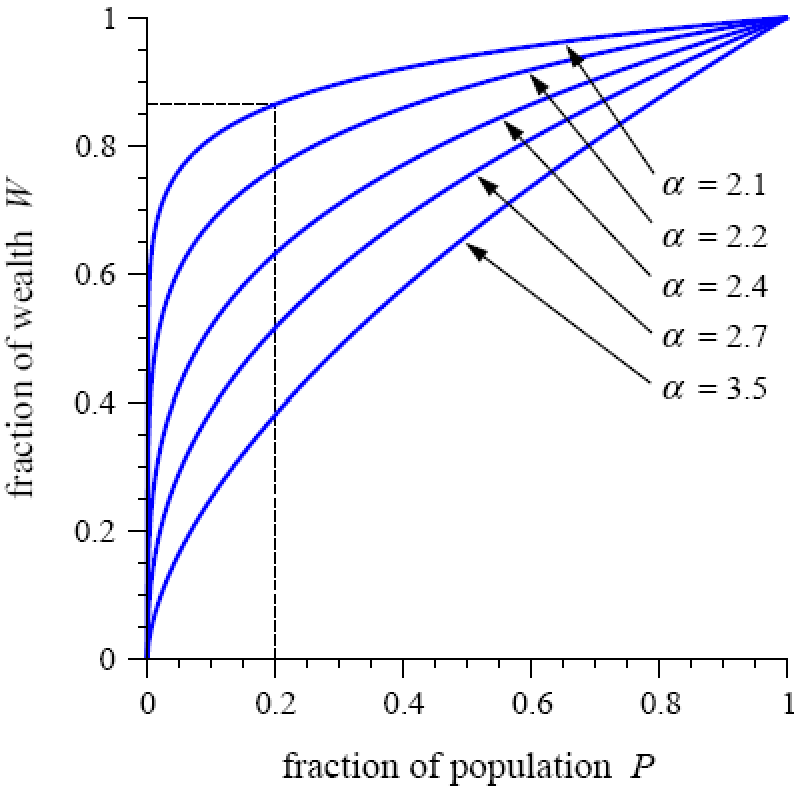
- For power-law distributed degree
  - if $\alpha \geq 2$, it is a constant

$$E[X] = \frac{\alpha - 1}{\alpha - 2} x_{min}$$

  - if $\alpha < 2$, it diverges
    - The expected value goes to infinity as the size of the network grows
- The fact that $\alpha \geq 2$ for most real networks guarantees a constant average degree as the graph grows

The 80/20 rule

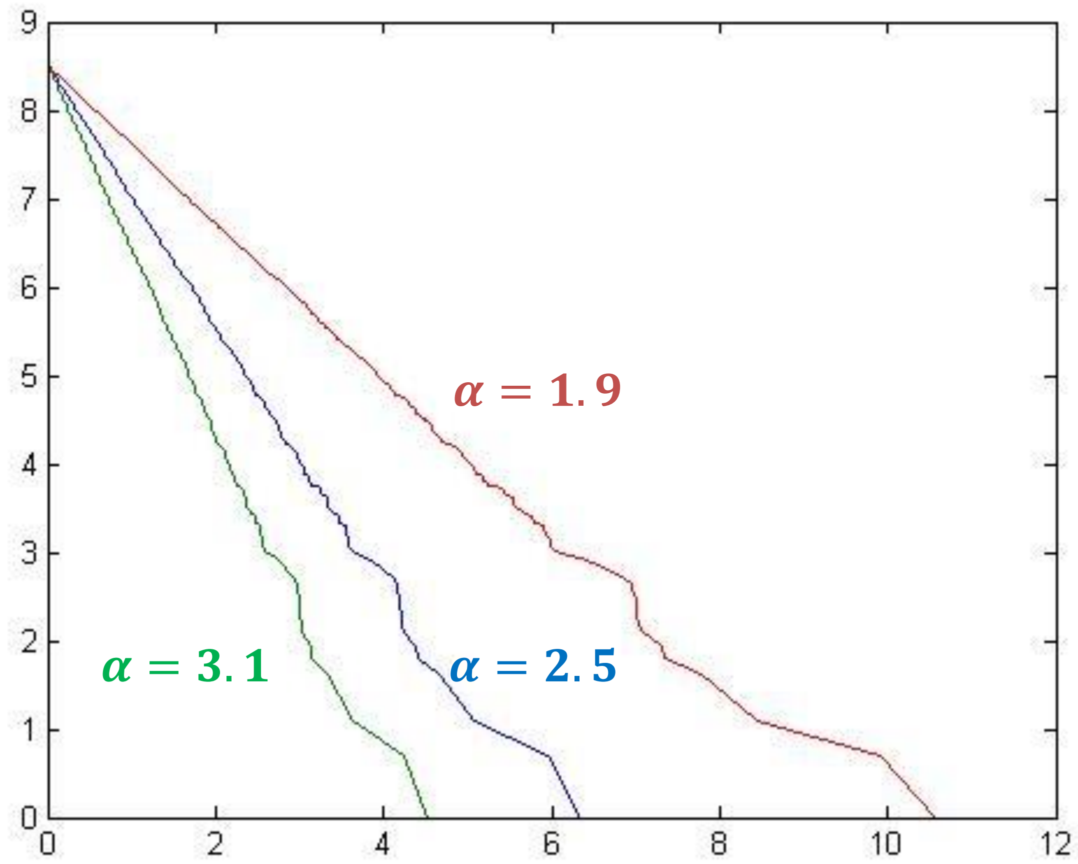
- Top-heavy: Small fraction of values collect most of distribution mass



- This phenomenon becomes more extreme when $\alpha < 2$
- 1% of values has 99% of mass
- E.g. name distribution

The effect of exponent

As the exponent increases the probability of observing an extreme value decreases



Generating power-law values

- A simple trick to generate values that follow a power-law distribution:
  - Generate values r uniformly at random within the interval [0,1]
  - Transform the values using the equation
    $$x = x_{min}(1 - r)^{-1/(\alpha-1)}$$
  - Generates values distributed according to power-law with exponent α

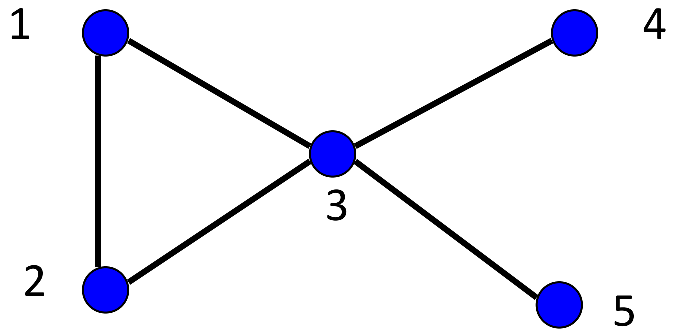
Clustering (Transitivity) coefficient

- Measures the density of triangles (local clusters) in the graph
- Two different ways to measure it:

$$C^{(1)} = \frac{\sum_i \text{triangles centeredat node i}}{\sum_i \text{triples centeredat node i}}$$

- The ratio of the means

Example



$$C^{(1)} = \frac{3}{1+1+6} = \frac{3}{8}$$

Clustering (Transitivity) coefficient

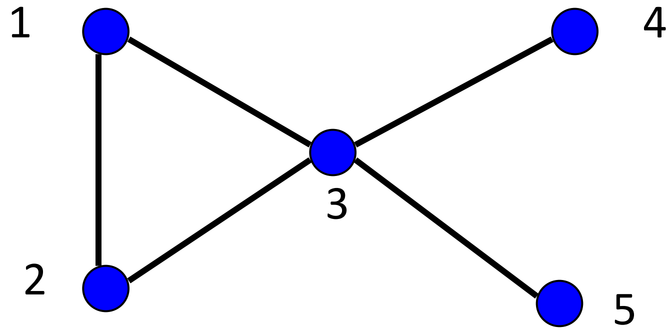
- Clustering coefficient for node i

$$C_i = \frac{\text{triangles centered at node } i}{\text{triples centered at node } i}$$

$$C^{(2)} = \frac{1}{n} C_i$$

- The mean of the ratios

Example



$$C^{(2)} = \frac{1}{5}\left(1 + 1 + 1/6\right) = \frac{13}{30}$$

$$C^{(1)} = \frac{3}{8}$$

- The two clustering coefficients give different measures
- $C^{(2)}$ increases with nodes with low degree

Collective Statistics (M. Newman 2003)

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	−0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	−0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	−0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	−0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	−0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	−0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	−0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	−0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	−0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	−0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	−0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	−0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	−0.226	416, 421

TABLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices n; total number of edges m; mean degree z; mean vertex–vertex distance ℓ; exponent α of degree distribution if the distribution follows a power law (or "–" if not; in/out-degree exponents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r, Sec. III.F. The last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

Clustering coefficient for random graphs

- The probability of two of your neighbors also being neighbors is p, independent of local structure
  - clustering coefficient C = p
  - when the average degree z=np is constant C =O(1/n)

Table 1: Clustering coefficients, C, for a number of different networks; n is the number of node, z is the mean degree. Taken from [146].

Network	n	z	C measured	C for random graph
Internet [153]	6,374	3.8	0.24	0.00060
World Wide Web (sites) [2]	153,127	35.2	0.11	0.00023
power grid [192]	4,941	2.7	0.080	0.00054
biology collaborations [140]	1,520,251	15.5	0.081	0.000010
mathematics collaborations [141]	253,339	3.9	0.15	0.000015
film actor collaborations [149]	449,913	113.4	0.20	0.00025
company directors [149]	7,673	14.4	0.59	0.0019
word co-occurrence [90]	460,902	70.1	0.44	0.00015
neural network [192]	282	14.0	0.28	0.049
metabolic network [69]	315	28.3	0.59	0.090
food web [138]	134	8.7	0.22	0.065

Small worlds

- Millgram’s experiment: Letters were handed out to people in Nebraska to be sent to a target in Boston
- People were instructed to pass on the letters to someone they knew on first-name basis
- The letters that reached the destination followed paths of length around 6
- Six degrees of separation: (play of John Guare)

- Also:
  - The Kevin Bacon game
  - The Erdös number

- Small world project: http://smallworld.columbia.edu/index.html

Measuring the small world phenomenon

- d_{ij} = shortest path between i and j
- Diameter:

$$d = \max_{i,j} d_{ij}$$

- Characteristic path length:

$$\ell = \frac{1}{n(n-1)/2} \sum_{i>j} d_{ij}$$

- Harmonic mean

Problem if no path between two nodes

$$\ell^{-1} = \frac{1}{n(n-1)/2} \sum_{i>j} d_{ij}^{-1}$$

- Also, distribution of all shortest paths

Collective Statistics (M. Newman 2003)

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW `nd.edu`	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	−0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	−0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	−0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	−0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	−0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	−0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	−0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	−0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	−0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	−0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	−0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	−0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	−0.226	416, 421

TABLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices n; total number of edges m; mean degree z; mean vertex–vertex distance ℓ; exponent α of degree distribution if the distribution follows a power law (or "–" if not; in/out-degree exponents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r, Sec. III.F. The last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

Small worlds in real networks

- For all real networks there are (on average) short paths between nodes of the network.
  - Largest path found in the IMDB actor network: 7
- Is this interesting?
  - Random graphs also have small diameter (d=logn/loglogn when z=ω(logn))
- Short paths are not surprising and should be combined with other properties
  - ease of navigation
  - high clustering coefficient

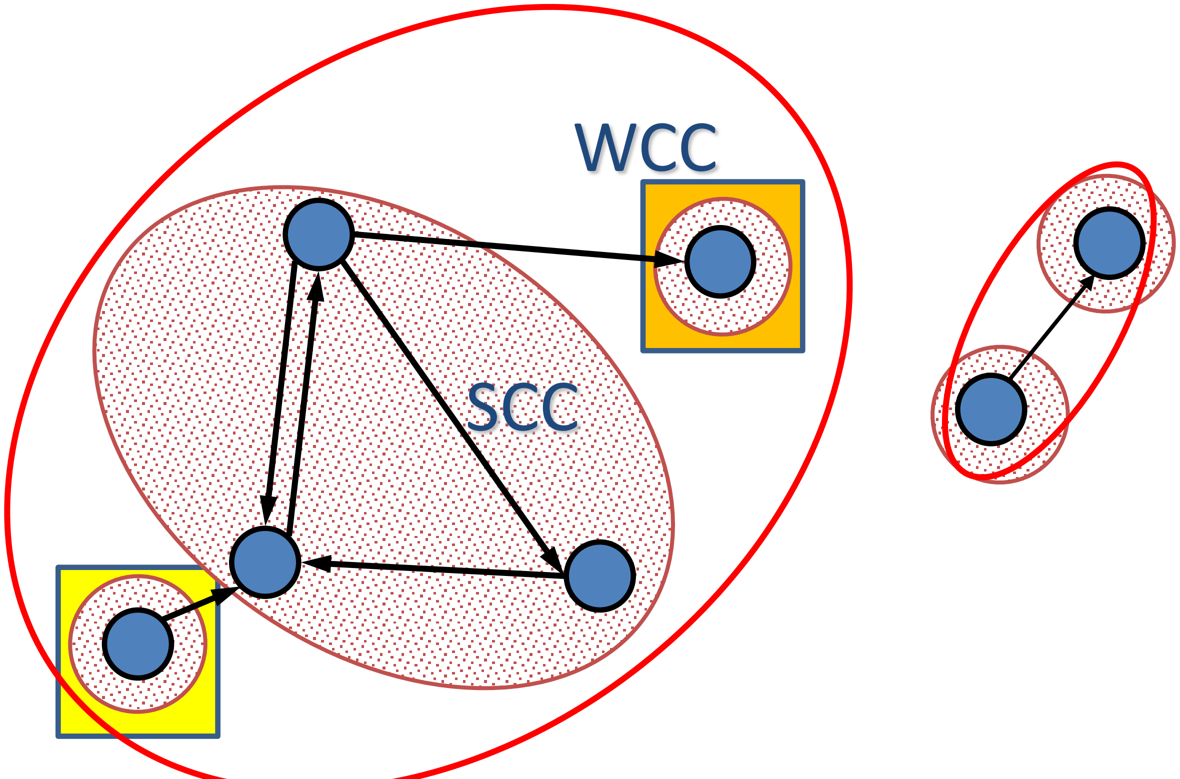
Connected components

- For undirected graphs, the size and distribution of the connected components
  - is there a giant component?
  - Most known real undirected networks have a giant component

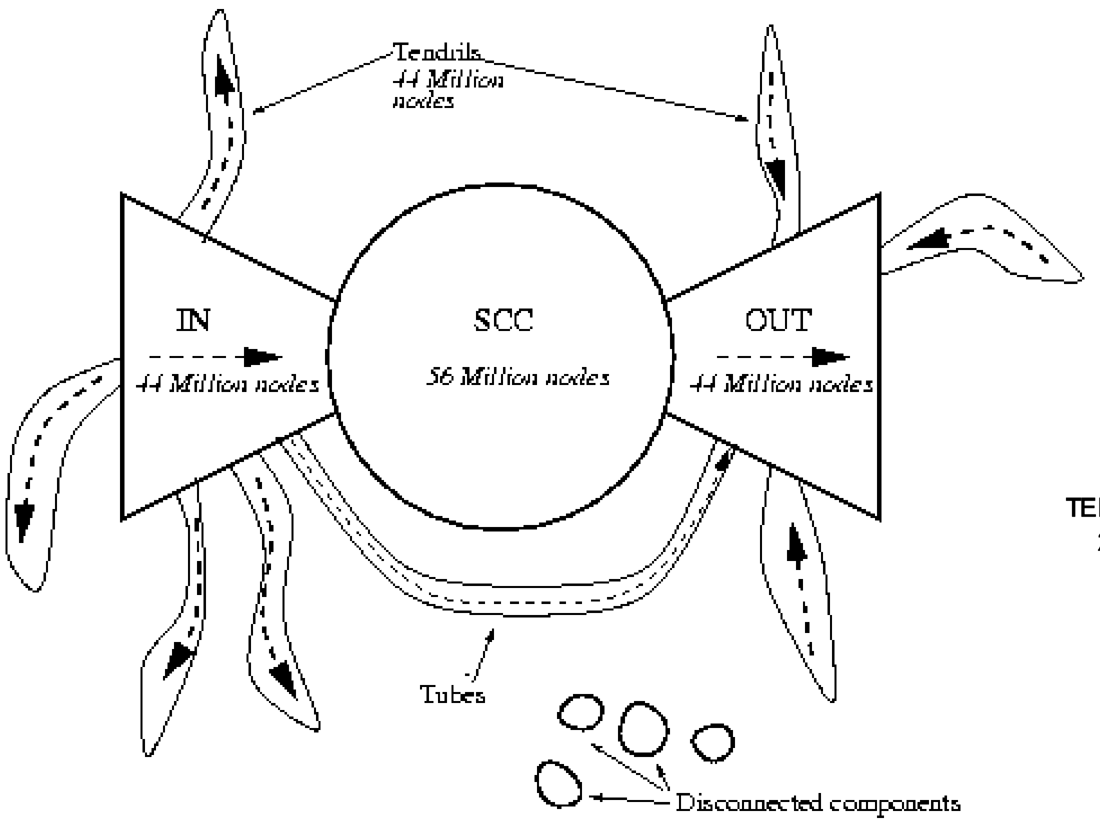
- For directed graphs, the size and distribution of strongly and weakly connected components

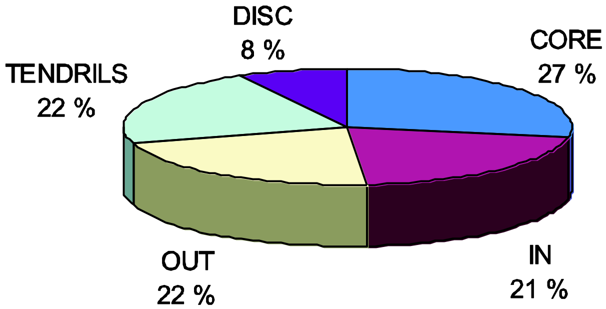
Connected components – definitions

- Weakly connected components (WCC)
  - Set of nodes such that from any node can go to any node via an undirected path
- Strongly connected components (SCC)
  - Set of nodes such that from any node can go to any node via a directed path.
  - IN: Nodes that can reach the SCC (but not in the SCC)
  - OUT: Nodes reachable by the SCC (but not in the SCC)



The bow-tie structure of the Web

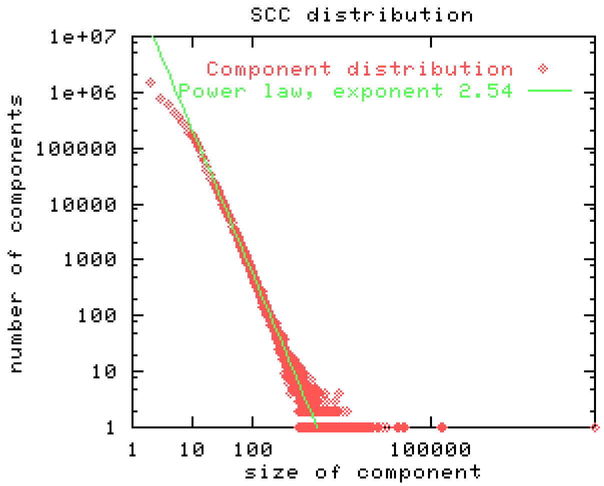


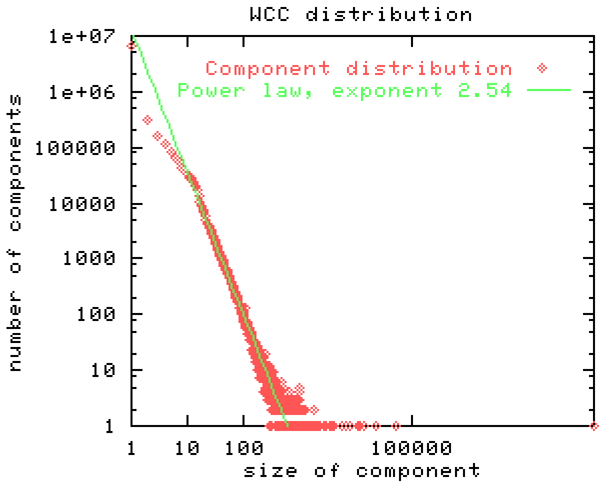


The largest weakly connected component contains 90% of the nodes

SCC and WCC distribution

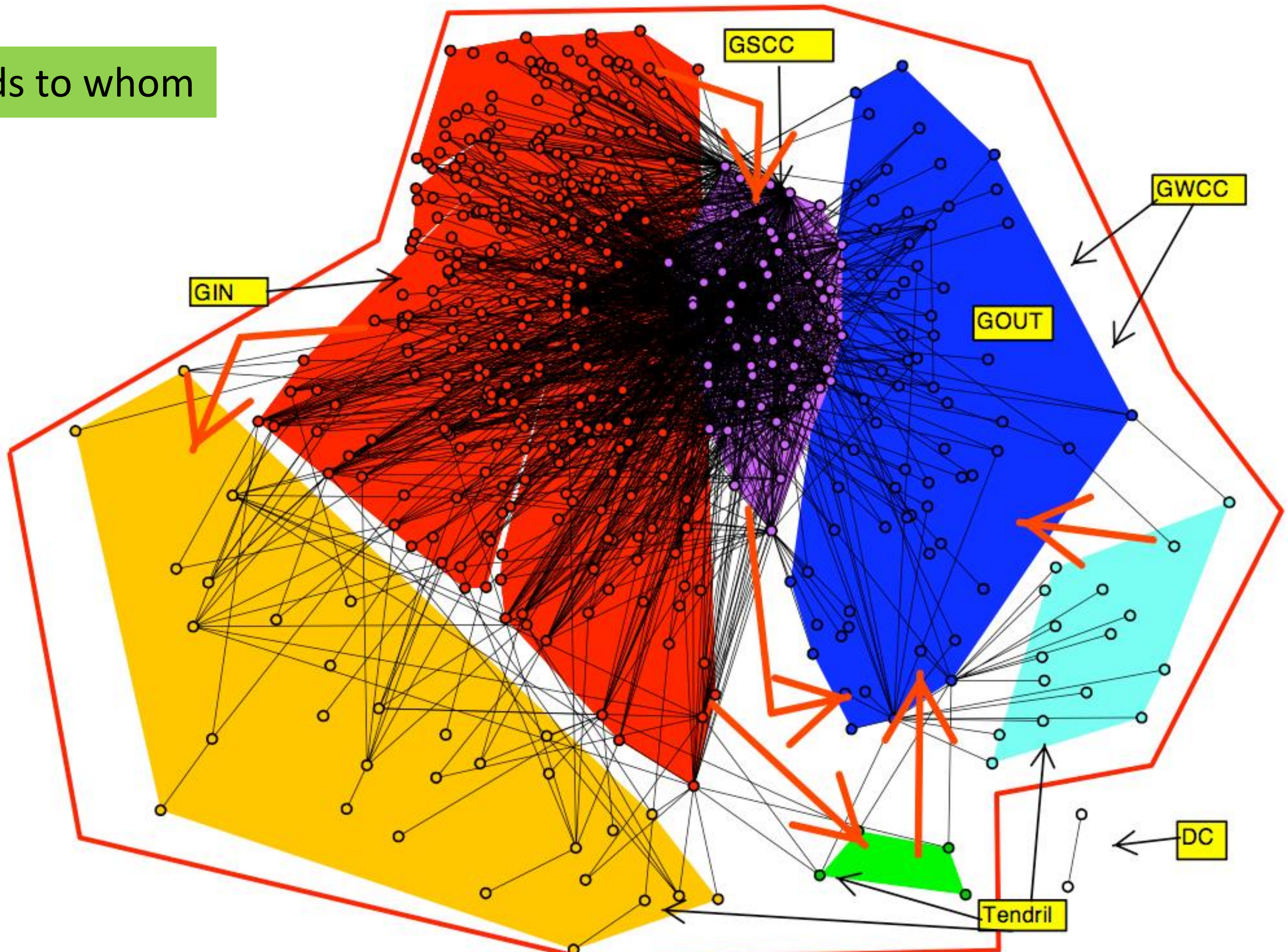
- The SCC and WCC sizes follows a power law distribution
  - the second largest SCC is significantly smaller





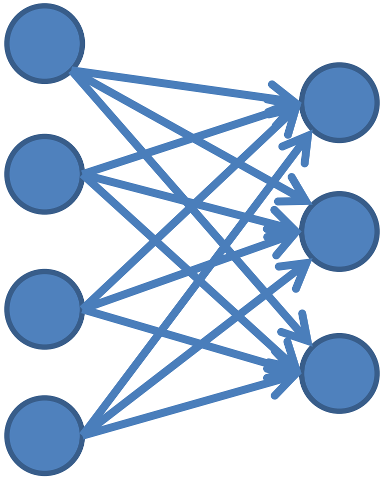
Another bow-tie

Who lends to whom



Web Cores

- Cores: Small complete bipartite graphs (of size 3x3, 4x3, 4x4)
  - Similar to the triangles for undirected graphs
- Found more frequently than expected on the Web graph
- Correspond to communities of enthusiasts (e.g., fans of japanese rock bands)



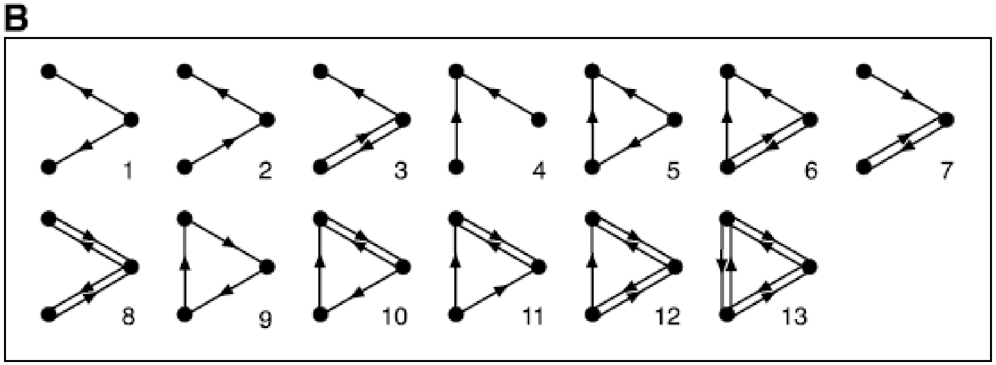
Motifs

- Most networks have the same characteristics with respect to global measurements
  - can we say something about the local structure of the networks?

- Motifs: Find small subgraphs that over-represented in the network

Example

- Motifs of size 3 in a directed graph

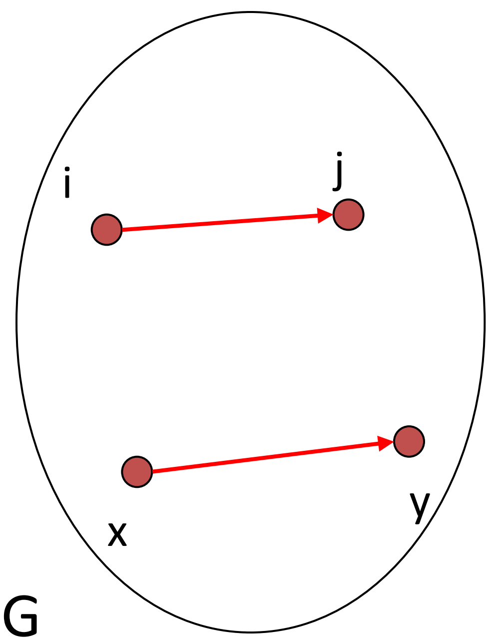


Finding interesting motifs

- Sample a part of the graph of size S
- Count the frequency of the motifs of interest
- Compare against the frequency of the motif in a random graph with the same number of nodes and the same degree distribution

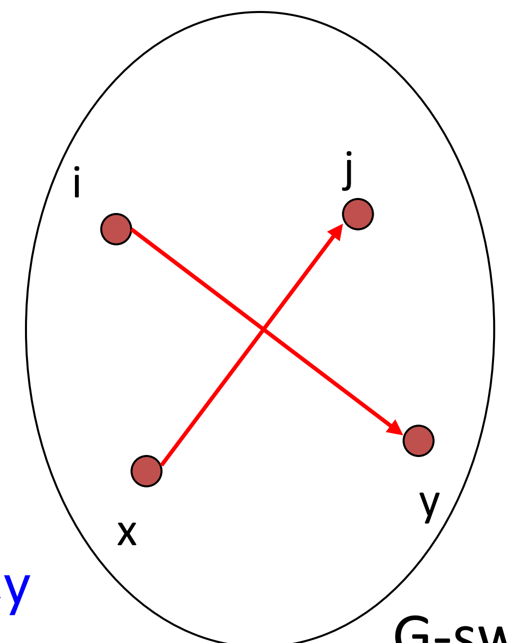
Generating a random graph

- Find edges (i,j) and (x,y) such that edges (i,y) and (x,j) do not exist, and swap them
  - repeat for a large enough number of times



G

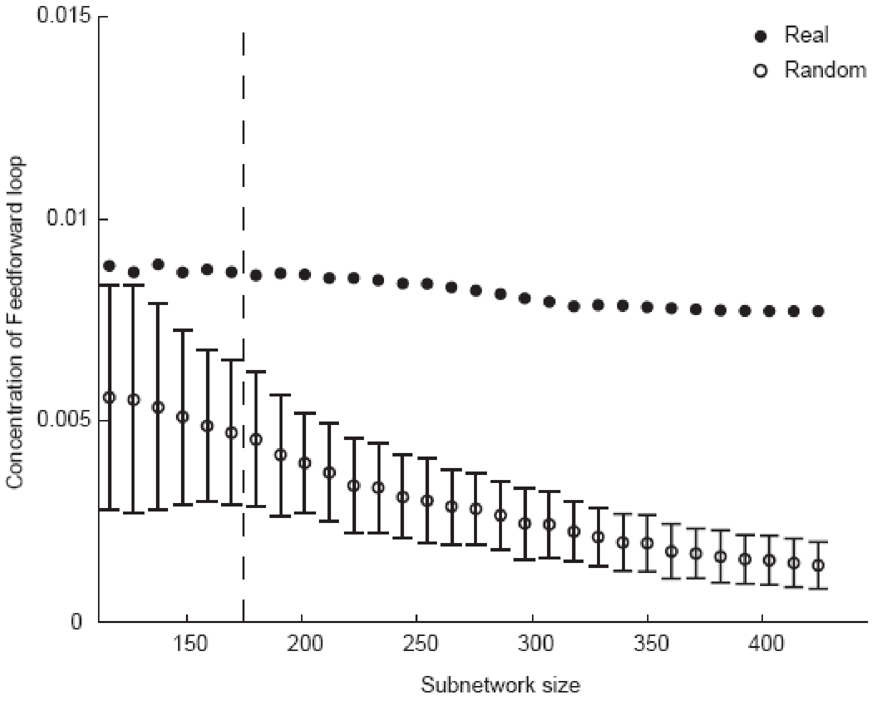
degrees of i,j,x,y are preserved

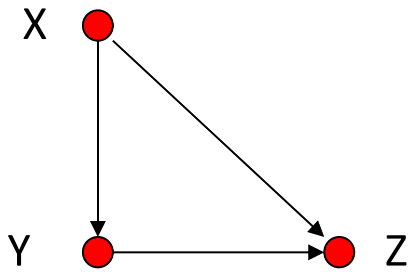


G-swapped

The feed-forward loop

- Over-represented in gene-regulation networks
  - a signal delay mechanism

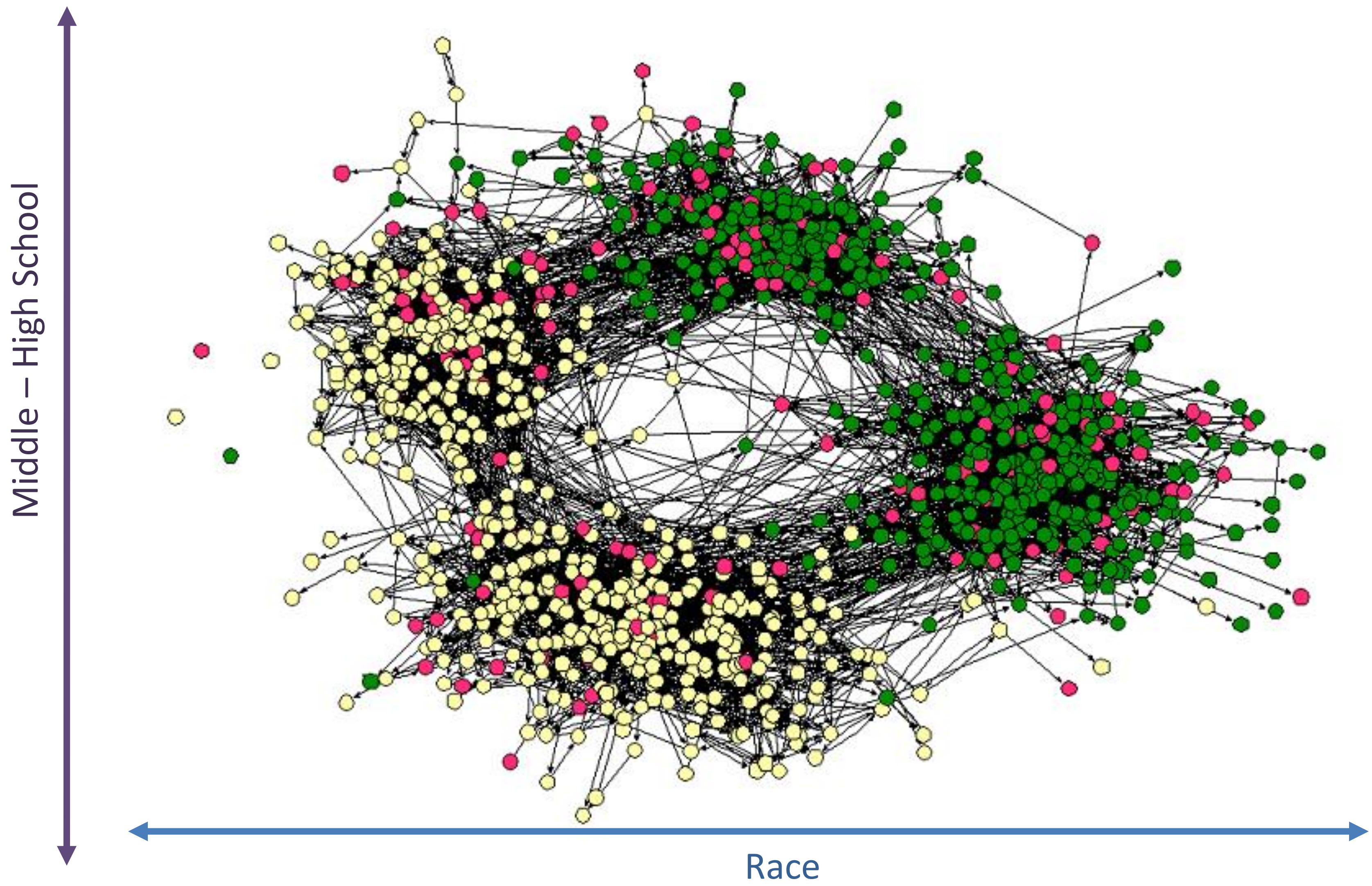




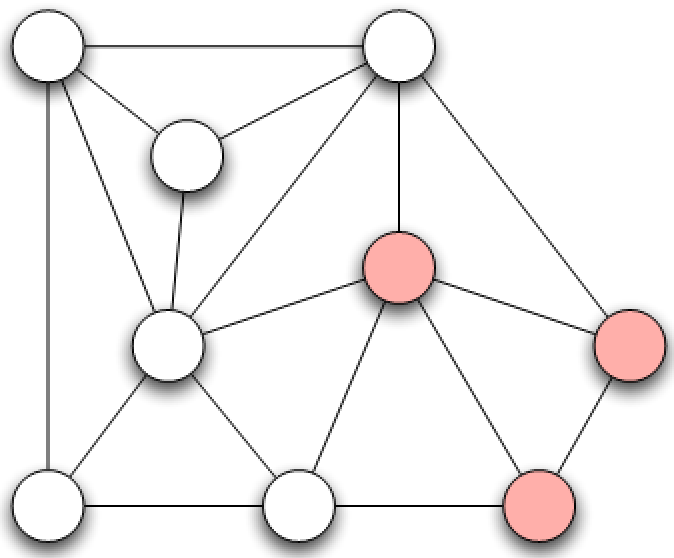
Milo et al. 2002

Homophily

- Love of the same: People tend to have friends with common interests
  - Students separated by race and age



Measuring Homophily



If the fraction of cross-gender edges is significantly less than expected, then there is evidence for homophily

gender male with probability p
gender female with probability q

Probability of cross-gender edge?

$$\frac{\#cross\_gender\_edges}{\#edges} << 2pq$$

Measuring Homophily

- “significantly” less than
- Inverse homophily
- Characteristics with more than two values:
  - Number of heterogeneous edges (edge between two nodes that are different)

Mechanisms Underlying Homophily:
Selection and Social Influence

Selection: tendency of people to form friendships with others who are like then

Socialization or Social Influence: the existing social connections in a network are influencing the individual characteristics of the individuals

Social Influence <u>as the inverse </u>of Selection

Mutable & immutable characteristics

The Interplay of Selection and Social Influence

Longitudinal studies in which the social connections and the behaviors within a group are tracked over a period of time

Why?

- Study teenagers, scholastic achievements/drug use (peer pressure and selection)
- Relative impact?
- Effect of possible interventions (example, drug use)

The Interplay of Selection and Social Influence

Christakis and Fowler on obesity, 12,000 people over a period of 32-years

People more similar on obesity status to the network neighbors than if assigned randomly

Why?
(i) Because of selection effects, choose friends of similar obesity status,
(ii) Because of confounding effects of homophily according to other characteristics that correlate with obesity
(iii) Because changes in the obesity status of person's friends was exerting an influence that affected her

(iii) As well -> "contagion" in a social sense

Tracking Link Formation in Online Data: interplay between selection and social influence

- Underlying social network
- Measure for behavioral similarity

Wikipedia
Node: Wikipedia editor who maintains a user account and user talk page
Link: if they have communicated with one writing on the user talk page of the other

Editor's behavior: set of articles she has edited

Neighborhood overlap in the bipartite affiliation network of editors and articles consisting only of edges between editors and the articles they have edited

$$\frac{|N_A \cap N_B|}{|N_A \cup N_B|}$$

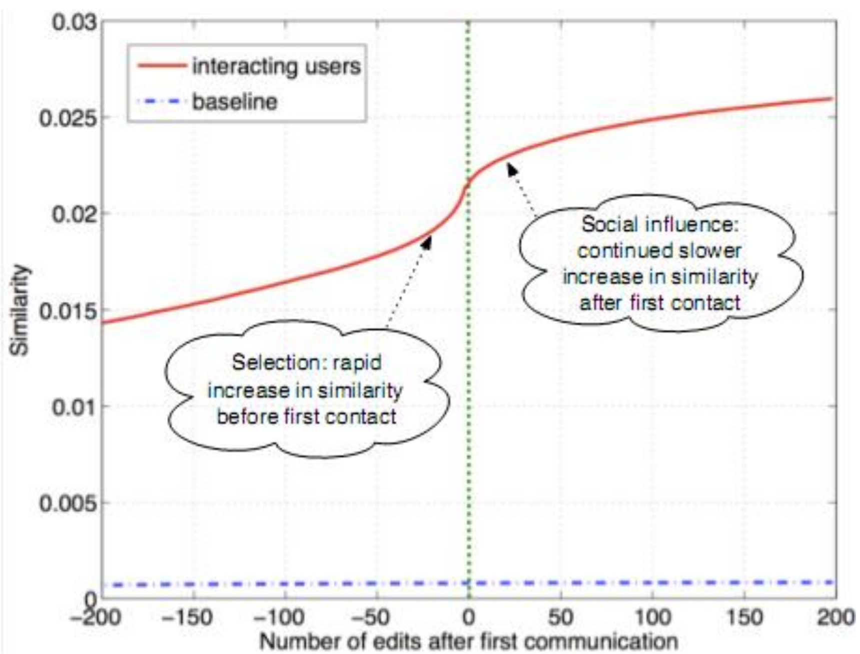
FACT: Wikipedia editors who have communicated are significantly more similar in their behavior than pairs of Wikipedia editors who have not (homomphily), **why?**
Selection (editors form connections with those have edited the same articles) vs Social Influence (editors are led to the articles of people they talk to)

Tracking Link Formation in Online Data: interplay between selection and social influence

Actions in Wikipedia are time-stamped
For each pair of editors A and B who have ever communicated,

- ○ Record their similarity over time
- ○ Time 0 when they first communicated -- Time moves in discrete units, advancing by one “tick” whenever either A or B performs an action on Wikipedia
- ○ Plot one curve for each pair of editors

Average, single plot: average level of similarity relative to the time of first interaction



Similarity is clearly increasing both before and after the moment of first interaction (both selection and social influence)
Not symmetric around time 0 (particular role on similarity): Significant increase before they meet
Blue line shows similarity of a random pair (non-interacting)

References

- M. E. J. Newman, The structure and function of complex networks, SIAM Reviews, 45(2): 167-256, 2003
- R. Albert and A.-L. Barabási, Statistical mechanics of complex networks, *Reviews of Modern Physics* **74**, 47-97 (2002).
- S. N. Dorogovstev and J. F. F. Mendez, Evolution of Networks: From Biological Nets to the Internet and WWW.
- Michalis Faloutsos, Petros Faloutsos and Christos Faloutsos. On Power-Law Relationships of the Internet Topology. ACM SIGCOMM 1999.
- E. Ravasz, A. L. Somera, D. A. Mongru, Z. N. Oltvai, and A.-L. Barabási, Hierarchical organization of modularity in metabolic networks, *Science* **297**, 1551-1555 (2002).
- R Milo, S Shen-Orr, S Itzkovitz, N Kashtan, D Chklovskii & U Alon, Network Motifs: Simple Building Blocks of Complex Networks. Science, 298:824-827 (2002).
- R Milo, S Itzkovitz, N Kashtan, R Levitt, S Shen-Orr, I Ayzenshtat, M Sheffer & U Alon, Superfamilies of designed and evolved networks. Science, 303:1538-42 (2004).