DATA MINING
LECTURE 9

Minimum Description Length

Information Theory

Co-Clustering

MINIMUM DESCRIPTION LENGTH

Occam's razor

- Most data mining tasks can be described as creating a model for the data
  - E.g., the EM algorithm models the data as a mixture of Gaussians, the K-means models the data as a set of centroids.
- What is the right model?

- Occam's razor: All other things being equal, the simplest model is the best.
  - A good principle for life as well

Occam's Razor and MDL

- What is a simple model?

- Minimum Description Length Principle: Every model provides a (lossless) encoding of our data. The model that gives the shortest encoding (best compression) of the data is the best.
  - Related: Kolmogorov complexity. Find the shortest program that produces the data (uncomputable).
  - MDL restricts the family of models considered

  - Encoding cost: cost of party A to transmit to party B the data.

Minimum Description Length (MDL)

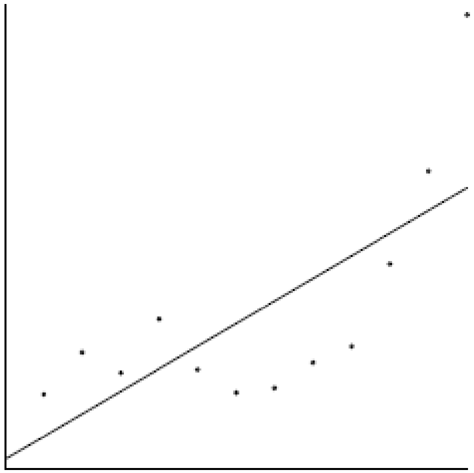
- The description length consists of two terms
  - The cost of describing the model (model cost)
  - The cost of describing the data given the model (data cost).
  - L(D) = L(M) + L(D|M)

- There is a tradeoff between the two costs
  - Very complex models describe the data in a lot of detail but are expensive to describe the model
  - Very simple models are cheap to describe but it is expensive to describe the data given the model

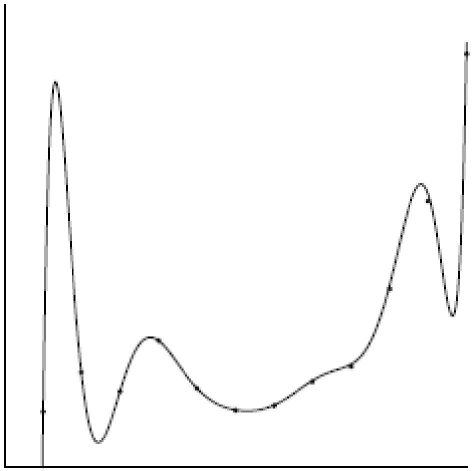
- This is generic idea for finding the right model
  - We use MDL as a blanket name.

Example

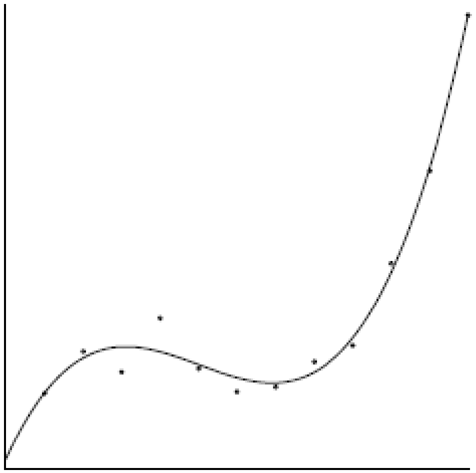
- Regression: find a polynomial for describing a set of values
  - Model complexity (model cost): polynomial coefficients
  - Goodness of fit (data cost): difference between real value and the polynomial value



Minimum model cost
High data cost



High model cost
Minimum data cost



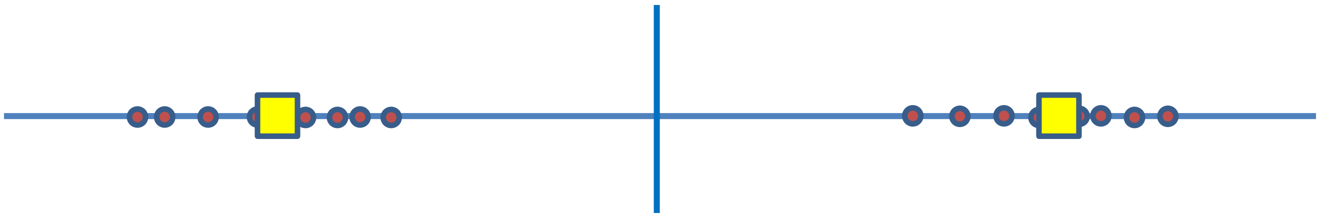
Low model cost
Low data cost

MDL avoids overfitting automatically!

Source: Grunwald et al. (2005) *Tutorial on MDL.*

Example

- Suppose you want to describe a set of integer numbers
  - Cost of describing a single number is proportional to the value of the number x (e.g., logx).
  - How can we get an efficient description?



- Cluster integers into two clusters and describe the cluster by the centroid and the points by their distance from the centroid
  - Model cost: cost of the centroids
  - Data cost: cost of cluster membership and distance from centroid

- What are the two extreme cases?

MDL and Data Mining

- Why does the shorter encoding make sense?
  - Shorter encoding implies regularities in the data
  - Regularities in the data imply patterns
  - Patterns are interesting

- Example

  00001000010000100001000010000100001000010000100010000100001

  - Short description length, just repeat 12 times 00001

  0100111001010011011010100001110101111011011010101110010011100

  - Random sequence, no patterns, no compression

Is everything about compression?

- Jürgen Schmidhuber: A theory about creativity, art and fun
  - Interesting Art corresponds to a novel pattern that we cannot compress well, yet it is not too random so we can learn it
  - Good Humor corresponds to an input that does not compress well because it is out of place and surprising
  - Scientific discovery corresponds to a significant compression event
    - E.g., a law that can explain all falling apples.

- Fun lecture:
  - Compression Progress: The Algorithmic Principle Behind Curiosity and Creativity

Issues with MDL

- What is the right model family?
  - This determines the kind of solutions that we can have
    - E.g., polynomials
    - Clusterings
- What is the encoding cost?
  - Determines the function that we optimize
  - Information theory

INFORMATION THEORY

A short introduction

Encoding

- Consider the following sequence

AAABBBAAACCCABACAABBAACCABAC

- Suppose you wanted to encode it in binary form, how would you do it?

50% A
25% B
25% C

A is 50% of the sequence
We should give it a shorter representation

$A \rightarrow 0$
$B \rightarrow 10$
$C \rightarrow 11$

This is actually provably the best encoding!

Encoding

- Prefix Codes: no codeword is a prefix of another

$A \rightarrow 0$
$B \rightarrow 10$
$C \rightarrow 11$

Uniquely directly decodable

For every code we can find a prefix code of equal length

- Codes and Distributions: There is one to one mapping between codes and distributions
  - If P is a distribution over a set of elements (e.g., {A,B,C}) then there exists a (prefix) code C where $L_C(x) = -\lceil \log P(x) \rceil, x \in \{A, B, C\}$
  - For every (prefix) code C of elements {A,B,C}, we can define a distribution $P(x) = 2^{-C(x)}$

- The code defined has the smallest average codelength!

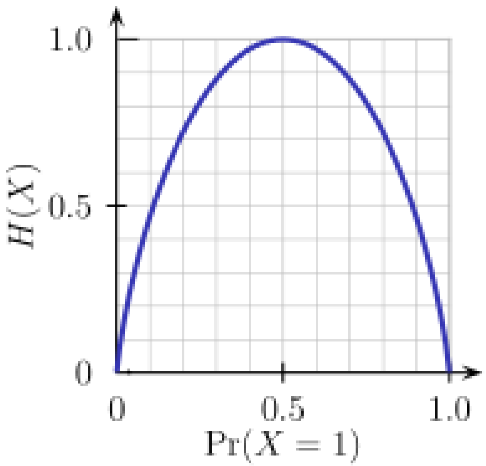
Entropy

- Suppose we have a random variable X that takes n distinct values
$$X = \{x_1, x_2, \dots, x_n\}$$
that have probabilities $\mathrm{P(X)} = \{p_1, \dots, p_n\}$

- This defines a code C with $L_C(x_i) = -\lceil \log p_i \rceil$. The average codelength is
$$-\sum_{i=1}^{n} p_i \lceil \log p_i \rceil$$
- This (more or less) is the entropy $H(X)$ of the random variable X
$$H(X) = -\sum_{i=1}^{n} p_i \log p_i$$
- Shannon's theorem: The entropy is a lower bound on the average codelength of any code that encodes the distribution P(X)
  - When encoding N numbers drawn from P(X), the best encoding length we can hope for is $N * H(X)$
  - Reminder: Lossless encoding

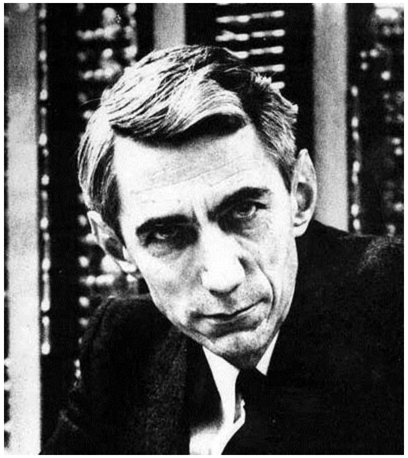
Entropy

$$H(X) = -\sum_{i=1}^{n} p_i \log p_i$$



- What does it mean?
- Entropy captures different aspects of a distribution:
  - The compressibility of the data represented by random variable X
    - Follows from Shannon's theorem
  - The uncertainty of the distribution (highest entropy for uniform distribution)
    - How well can I predict a value of the random variable?
  - The information content of the random variable X
    - The number of bits used for representing a value is the information content of this value.

Claude Shannon



Father of Information Theory

Envisioned the idea of communication of information with 0/1 bits

Introduced the word "bit"

The word entropy was suggested by Von Neumann

- Similarity to physics, but also
- "nobody really knows what entropy really is, so in any conversation you will have an advantage"

Some information theoretic measures

- Conditional entropy H(Y|X): the uncertainty for Y given that we know X

$$H(Y|X) = -\sum_{x} p(x) \sum_{y} p(y|x) \log p(y|x)$$

$$= -\sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)}$$

- Mutual Information I(X,Y): The reduction in the uncertainty for Y (or X) given that we know X (or Y)

$$I(X,Y) = H(Y) - H(Y|X) = H(X) - H(X|Y)$$

Some information theoretic measures

- Cross Entropy: The cost of encoding distribution P, using the code of distribution Q

$$-\sum_x P(x) \log Q(x)$$

- KL Divergence KL(P||Q): The increase in encoding cost for distribution P when using the code of distribution Q

$$KL(P||Q) = -\sum_x P(x) \log Q(x) + \sum_x P(x) \log P(x)$$

  - Not symmetric
  - Problematic if Q not defined for all x of P.

Some information theoretic measures

- Jensen-Shannon Divergence JS(P,Q): distance between two distributions P and Q
  - Deals with the shortcomings of KL-divergence

- If M = ½ (P+Q) is the mean distribution

$$JS(P,Q) = \frac{1}{2} KL(P||M) + \frac{1}{2} KL(Q||M)$$

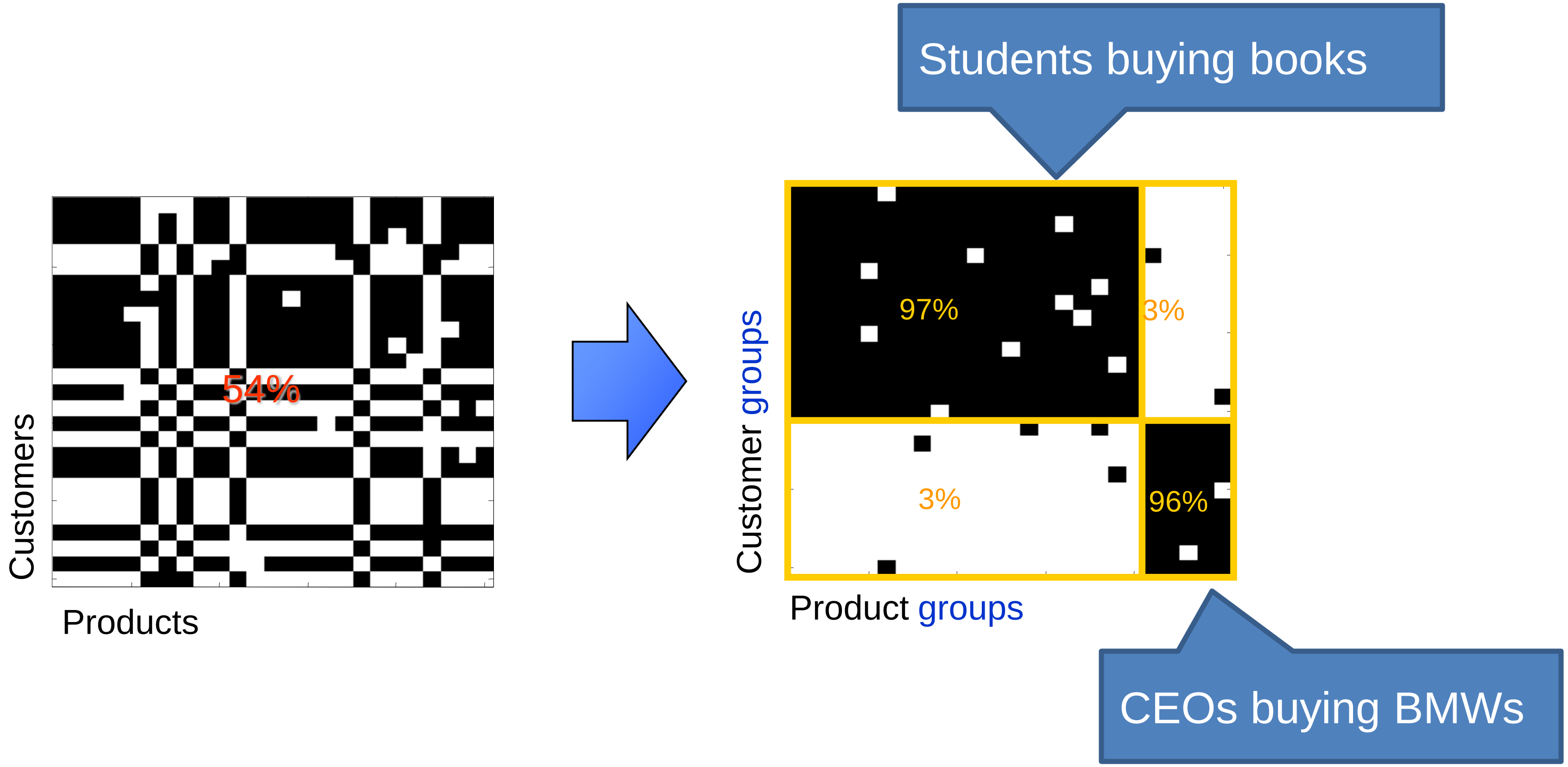
- Jensen-Shannon is a metric

USING MDL FOR CO-CLUSTERING (CROSS-ASSOCIATIONS)

Thanks to Spiros Papadimitriou.

Co-clustering

- Simultaneous grouping of rows and columns of a matrix into homogeneous groups



Co-clustering

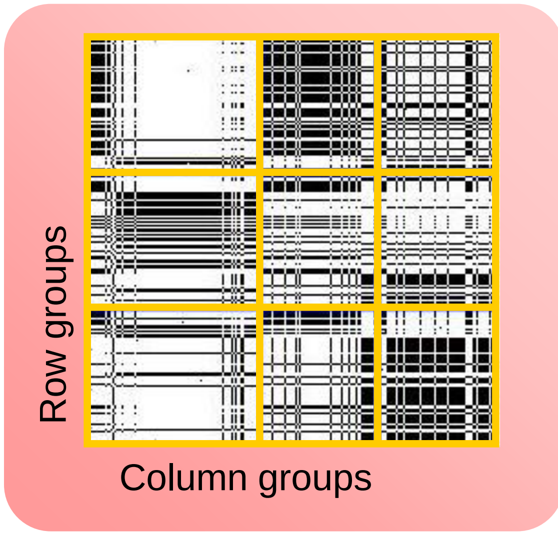
- **Step 1**: How to define a “good” partitioning?

  Intuition and formalization

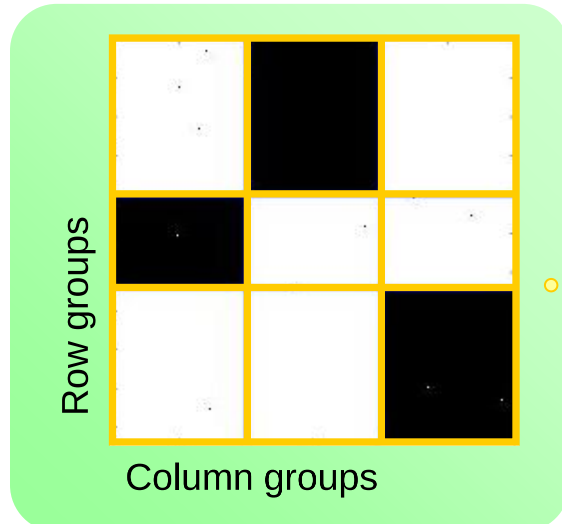
- **Step 2**: How to find it?

Co-clustering

Intuition



versus



Why is this better?

Good Clustering

1. Similar nodes are grouped together
2. As few groups as necessary

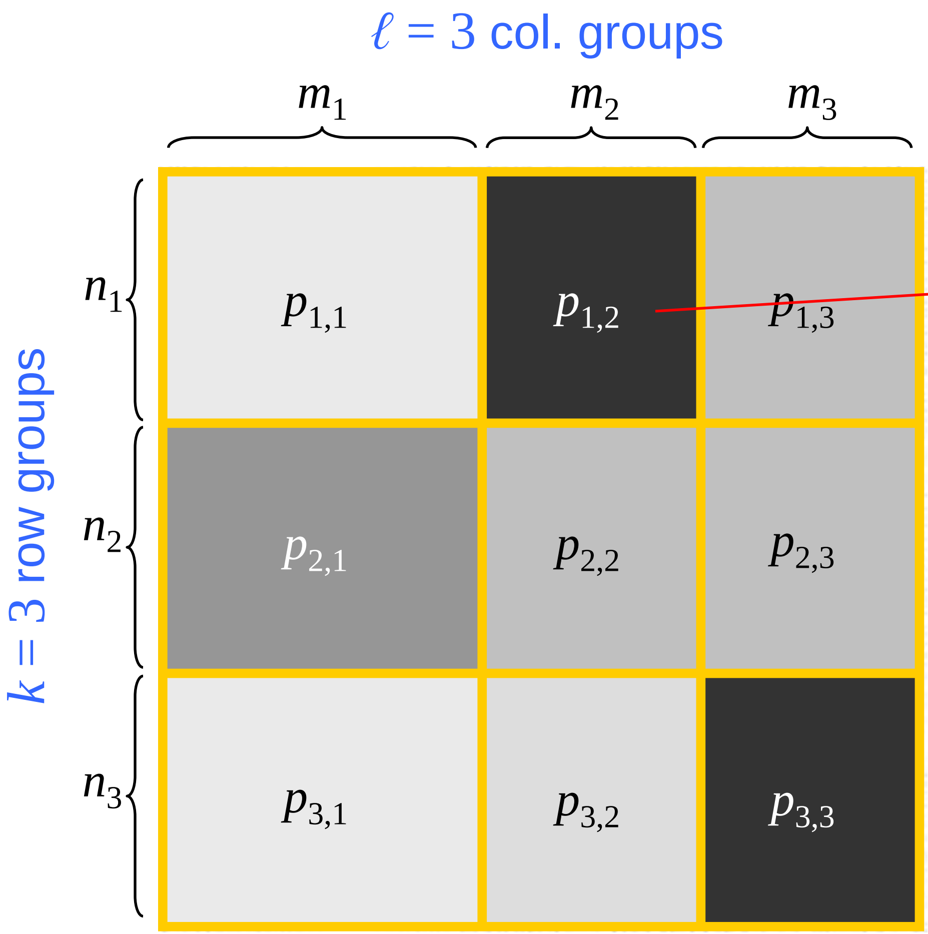
A *few*, *homogeneous* blocks

Good Compression

implies

Co-clustering

MDL formalization—Cost objective



$n \times m$ matrix

$$p_{i,j} := \frac{e_{i,j}}{n_i m_j}$$

→ density of ones

$n_1 m_2 \, H(p_{1,2})$ bits for (1,2)

block size ← $n_1 m_2$; $H(p_{1,2})$ → entropy

$\sum_{i,j} n_i m_j \, H(p_{i,j})$ bits total

data cost

+

model cost

$$nH\left(\frac{n_1}{n}, \ldots, \frac{n_k}{n}\right) + mH\left(\frac{m_1}{m}, \ldots, \frac{m_\ell}{m}\right)$$

row-partition description; col-partition description

$$+ \log^* k + \log^* \ell + \sum_{i,j} \lceil \log n_i m_j \rceil$$

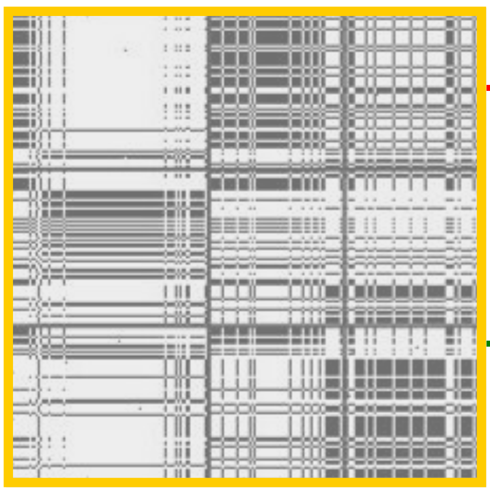
transmit #partitions; transmit #ones $e_{i,j}$

Co-clustering

MDL formalization—Cost objective

one row group
one col group

n row groups
m col groups



high →

low →

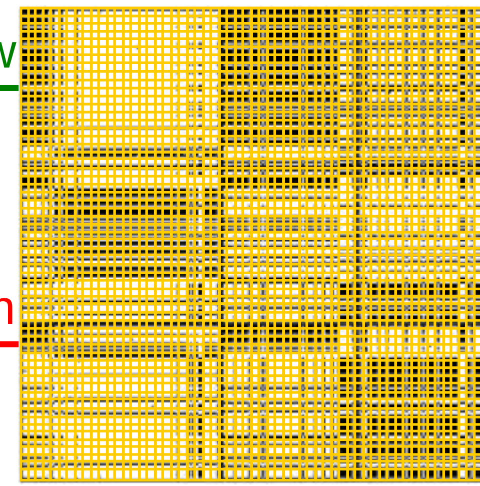
code cost
(block contents)

+

description cost
(block structure)

← low

← high

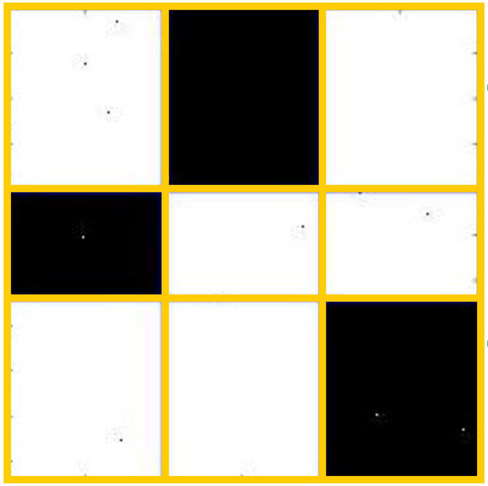


Co-clustering

MDL formalization—Cost objective

$k = 3$ row groups ✓

$\ell = 3$ col groups



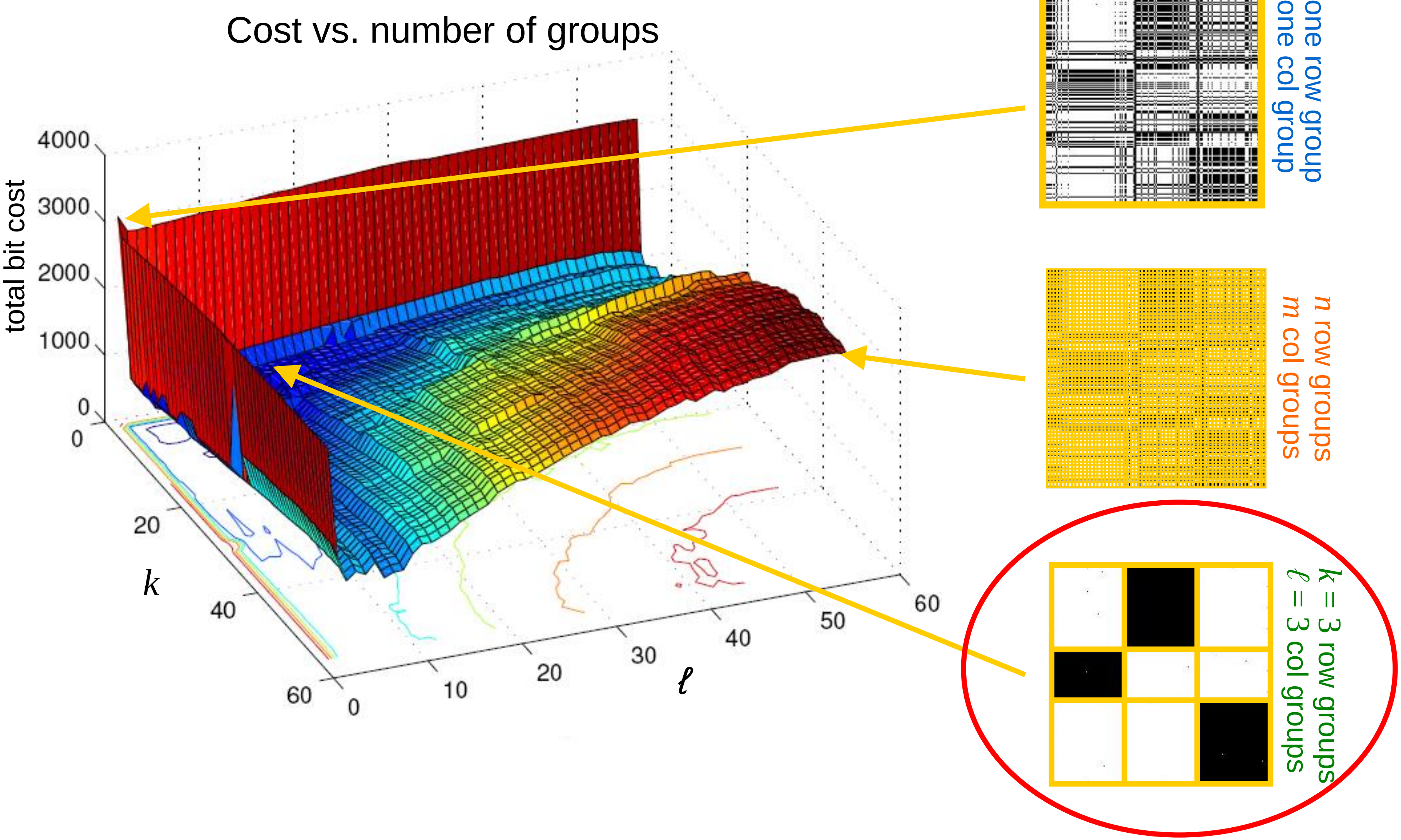
low → code cost (block contents)

+

low → description cost (block structure)

Co-clustering

MDL formalization—Cost objective



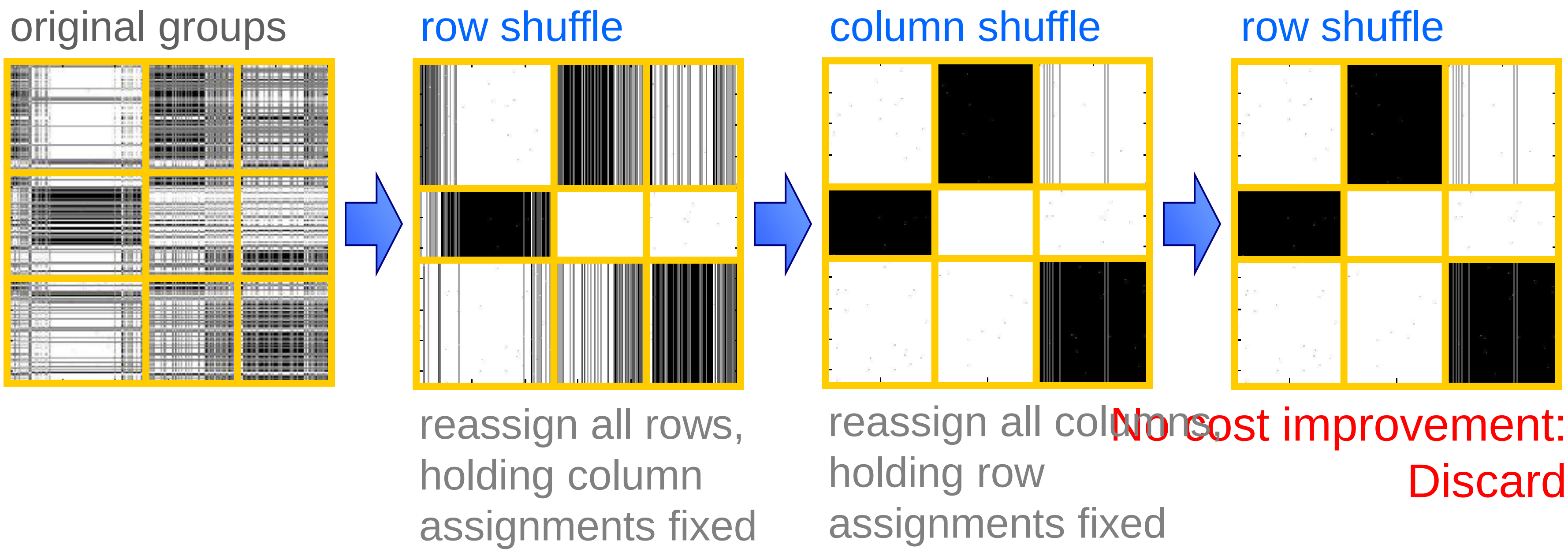
Co-clustering

- **Step 1**: How to define a “good” partitioning?
  Intuition and formalization

- **Step 2**: How to find it?

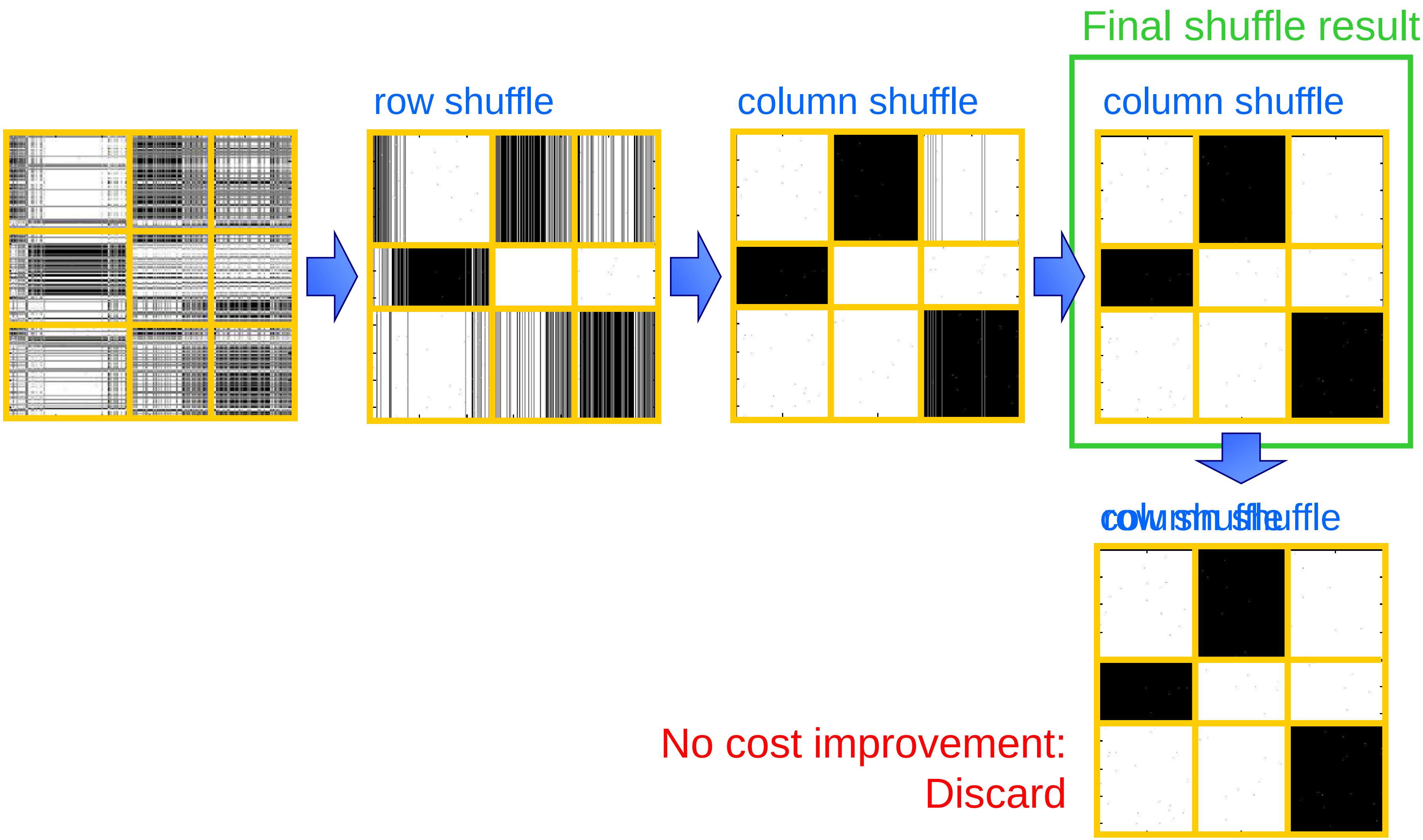
Search for solution

Overview: assignments w/ fixed number of groups (shuffles)



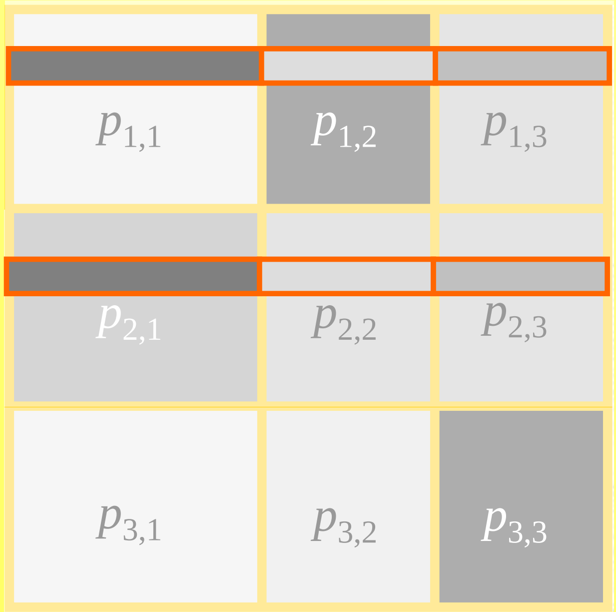
Search for solution

Overview: assignments w/ fixed number of groups (shuffles)



Search for solution

Shuffles



Similarity ("KL-divergences")
of row fragments
to blocks of a row group

Assign to second row-group

...teration

...ach part
... that, for all

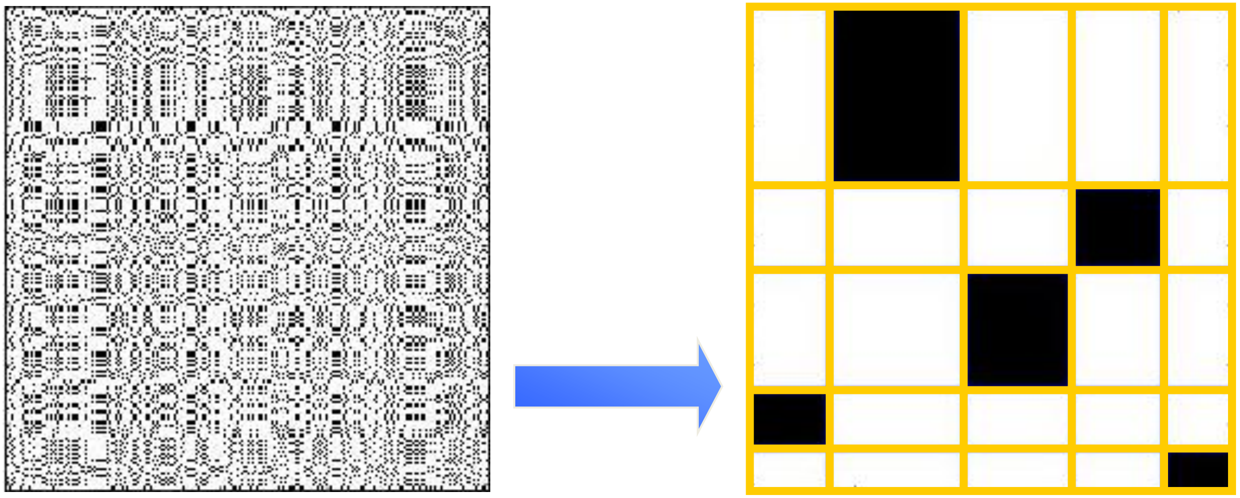
$$- \sum_{j=1}^{\ell} \left(\nu_j \log p_{i^*,j} + (n - \nu_j) \log(1 - p_{i^*,j}) \right)$$

$$\leq - \sum_{j=1}^{\ell} \left(\nu_j \log p_{i,j} + (n - \nu_j) \log(1 - p_{i,j}) \right)$$

Search for solution

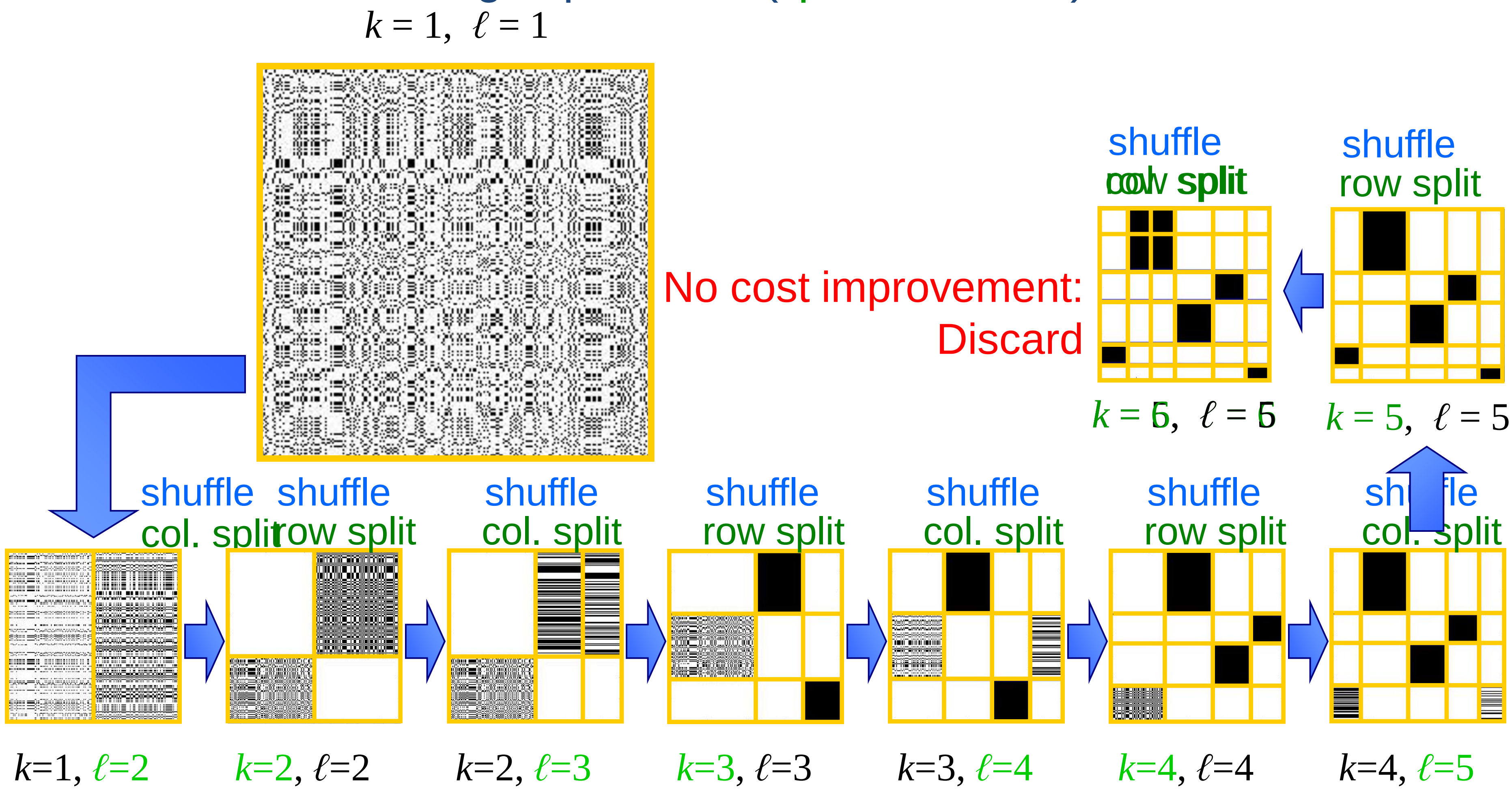
Overview: number of groups k and ℓ (splits & shuffles)

$k = 5, \;\ \ell = 5$



Search for solution

Overview: number of groups k and ℓ (splits & shuffles)

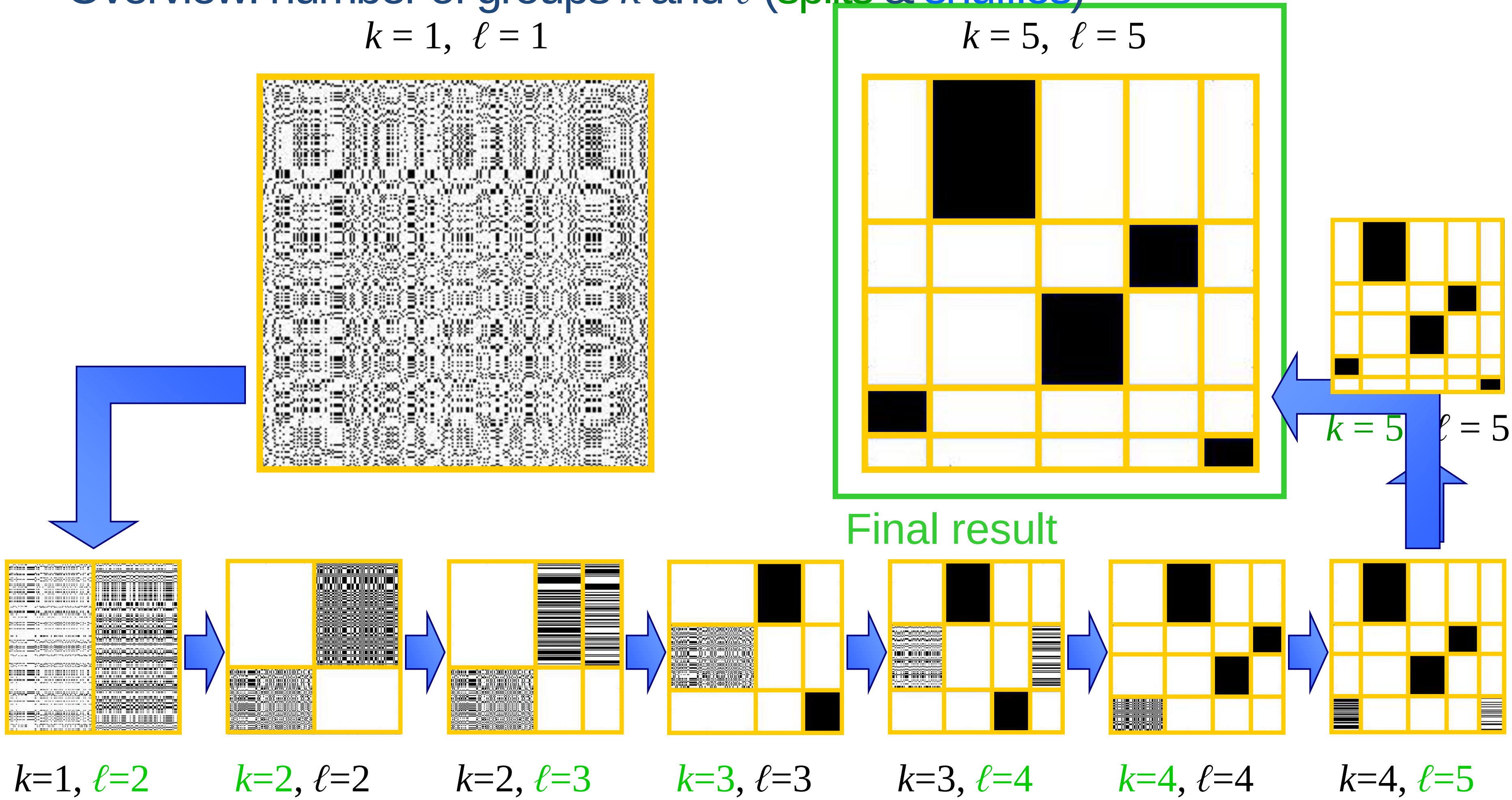


Split:
Increase k or ℓ

Shuffle:
Rearrange rows or cols

Search for solution

Overview: number of groups k and ℓ (splits & shuffles)

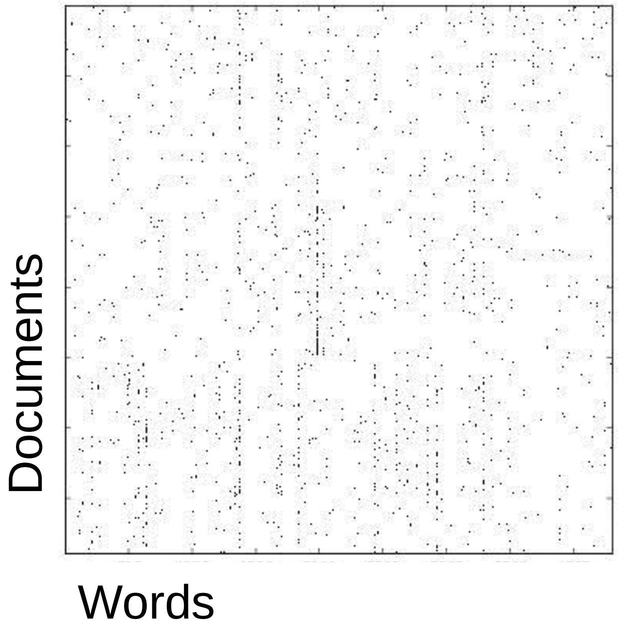


Split:
Increase k or ℓ

Shuffle:
Rearrange rows or cols

Co-clustering

CLASSIC



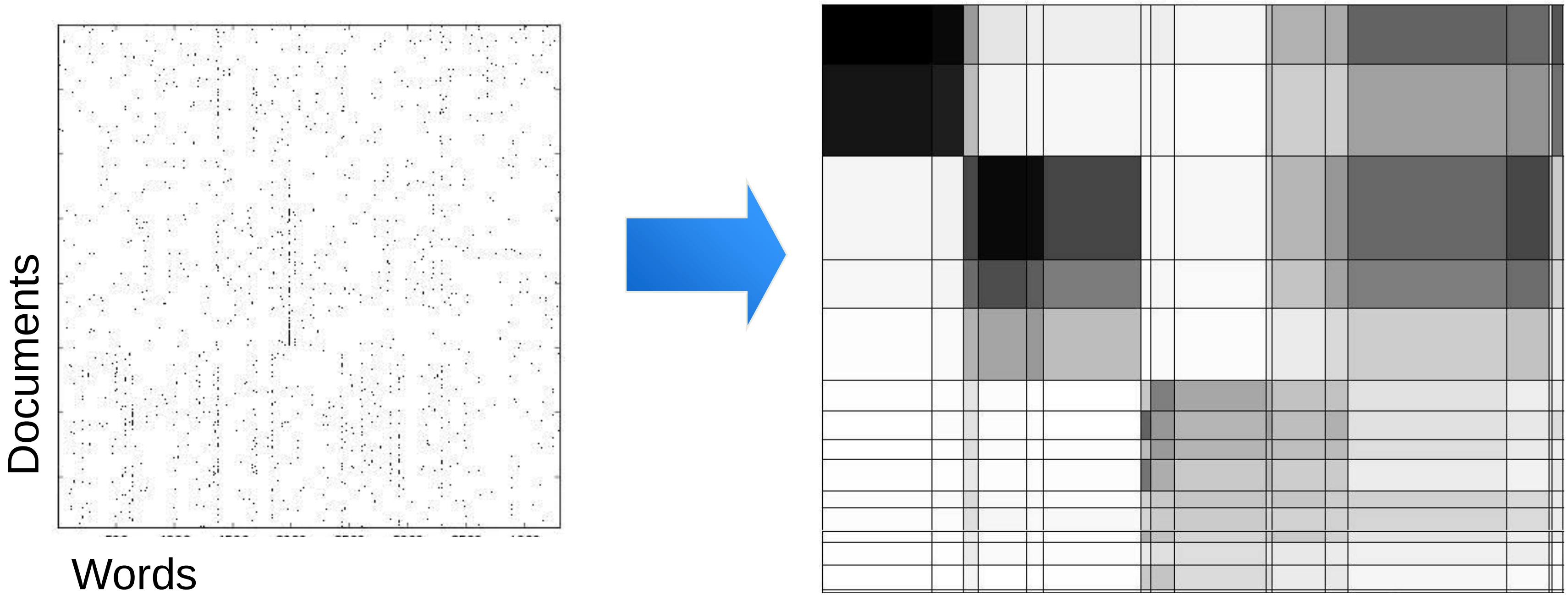
CLASSIC corpus

- 3,893 documents
- 4,303 words
- 176,347 “dots” (edges)

Combination of 3 sources:

- MEDLINE (medical)
- CISI (info. retrieval)
- CRANFIELD (aerodynamics)

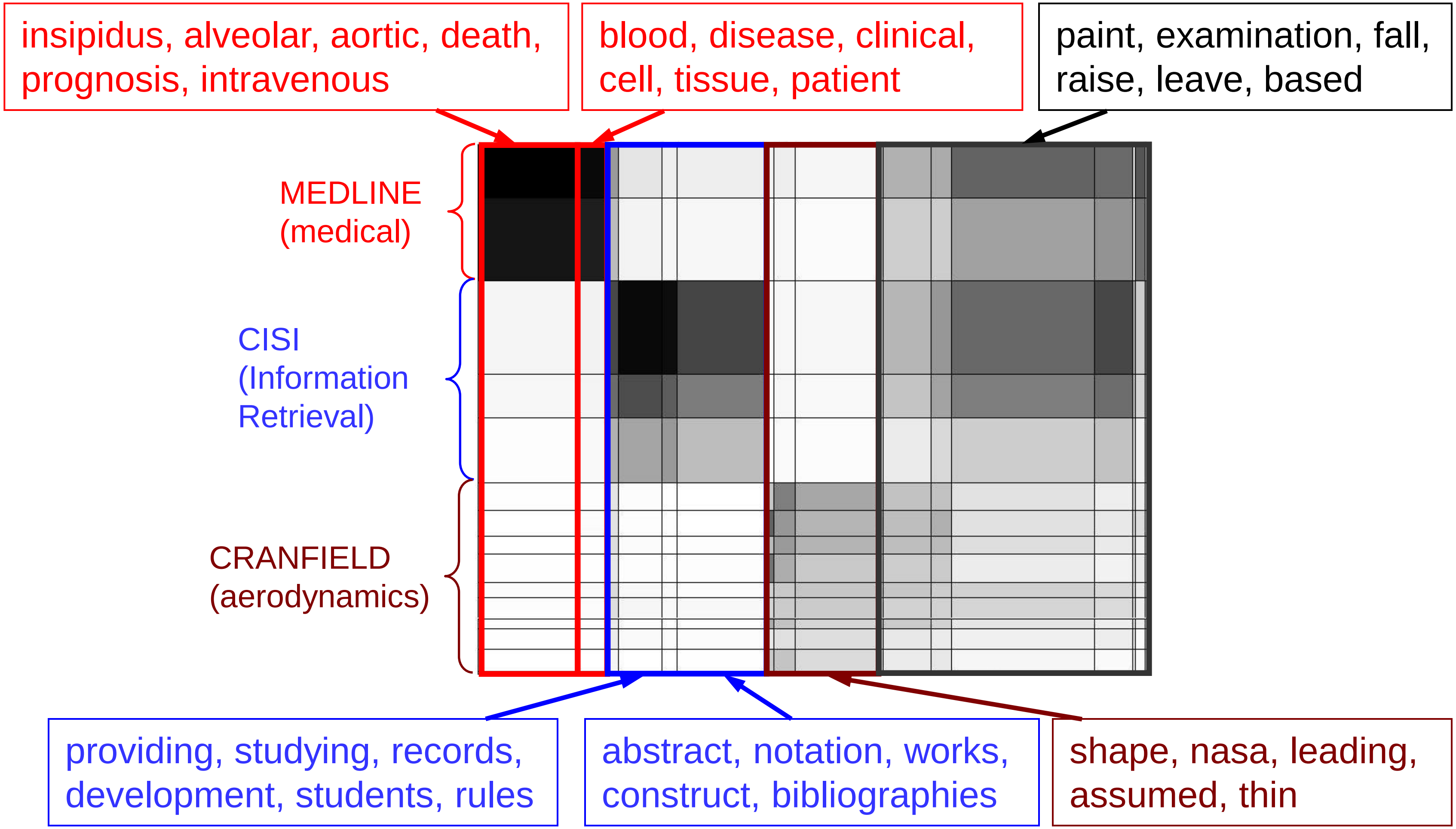
Graph co-clustering
CLASSIC



"CLASSIC" graph of documents & words:
$k = 15$, $\ell = 19$

Co-clustering

CLASSIC



“CLASSIC” graph of documents & words:
$k = 15,\ \ell = 19$

Co-clustering

CLASSIC

Document cluster #	Document class			Precision
	CRANFIELD	CISI	MEDLINE	
1	0	1	390	0.997
2	0	0	610	1.000
3	2	676	9	0.984
4	1	317	6	0.978
5	3	452	16	0.960
6	207	0	0	1.000
7	188	0	0	1.000
8	131	0	0	1.000
9	209	0	0	1.000
10	107	2	0	0.982
11	152	3	2	0.968
12	74	0	0	1.000
13	139	9	0	0.939
14	163	0	0	1.000
15	24	0	0	1.000
Recall	0.996	0.990	0.968	

0.999 (clusters 1–2)

0.975 (clusters 3–5)

0.987 (clusters 6–15)

0.94-1.00

0.97-0.99