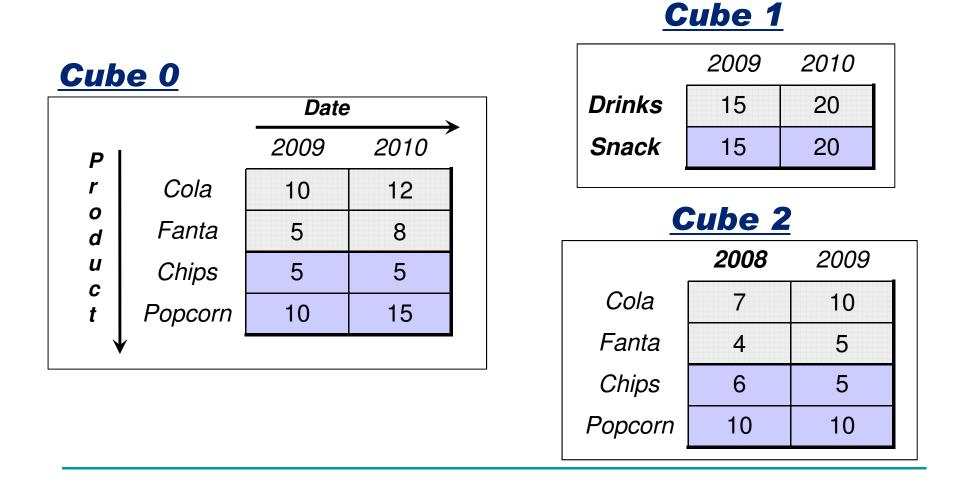
Similarity Measures for Multidimensional Data

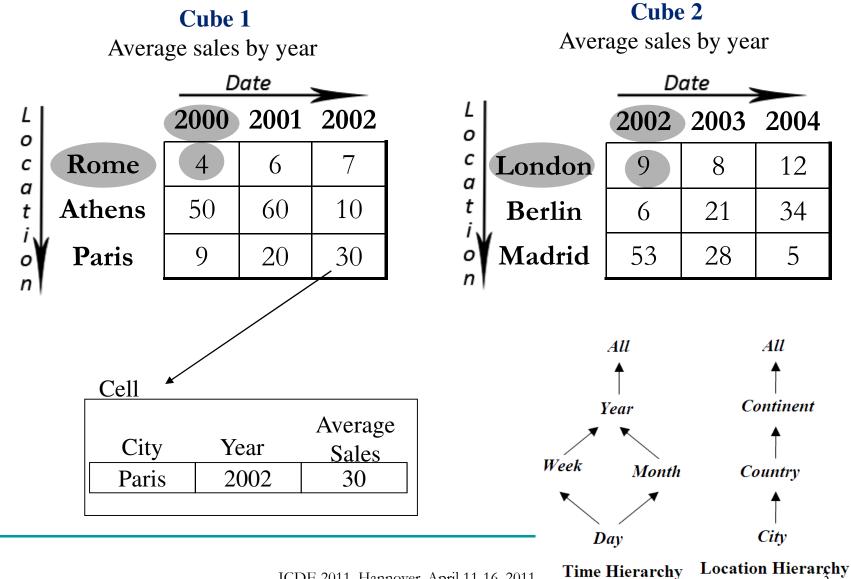


University of Ioannina Dept. of Computer Science Eftychia Baikousi Georgios Rogkakos Panos Vassiliadis

Cube 1 or Cube 2 most Similar to Cube 0 ?



Motivating Example



ICDE 2011, Hannover, April 11-16, 2011

Contents

Background & Related Work

Distance Functions

- □ between 2 values of a dimension
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 - User study between 2 sets of points in m/d space (cubes)

Background

Fundamentals

- Distance Measures
- Hausdorff
- Controversy on Metric Axioms
- Distances on Graphs
 - Highway Hierarchies
 - Semantic Similarity between Words

Distance Measures

• A distance measure is called a *metric* when :

 $\Box \ d(i,j) \ge 0 \ \& \ d(i,j) = d(j,i) \ \& \ d(i,i) = 0 \ \& \ d(i,j) \le d(i,k) + d(j,k)$

Categorization

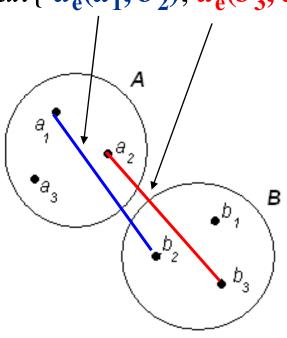
- interval-scaled variables (Euclidean, Minkowski, Manhattan)
- binary variables (Jaccard)
- categorical variables

Hausdorff distance

Example:

 $d_{\rm H}(A,B) = max\{d_{\rm s}(A,B), d_{\rm s}(B,A)\} = max\{d_{\rm e}(a_1, b_2), d_{\rm e}(b_3, a_2)\}$

- □ *d*_e denotes the Euclidian distance
- *d*_s denotes the max distance of the set of minimum distances.



Controversy on Metric Axioms

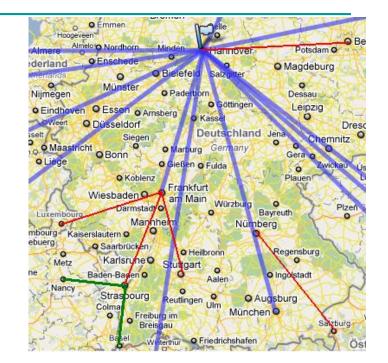
 Properties of metrics are convenient for Mathematicians/Computer Scientists

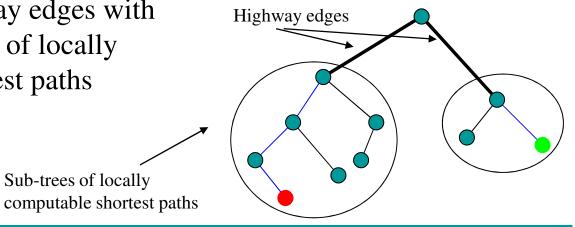
However

 Human perception does not comply with properties of metrics

Highway Hierarchies

- highways in road maps
 - The shortest paths among 2 points in a road network consists of
 - small roads locally
 - a highway road
 - Hierarchy: highway edges with attached sub-trees of locally computable shortest paths





Distances on Graphs

Semantic Similarity between Words

- Word similarity measures
- Semantic hierarchies
- □ 2 datasets (pairs of words)
 - One for constructing their method
 - The other to test it

Distances for Collections of Structured Data

- Relax operator
- Diff operator
- Distance between two relational databases under the same schema

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Distance functions between 2 values of a dimension

Locally computable

Hierarchical

Highway

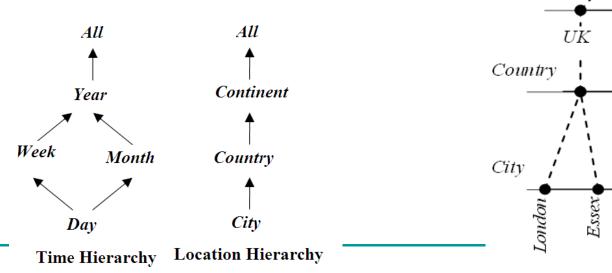
Distance functions between 2 values of a dimension

Locally computable distance functions

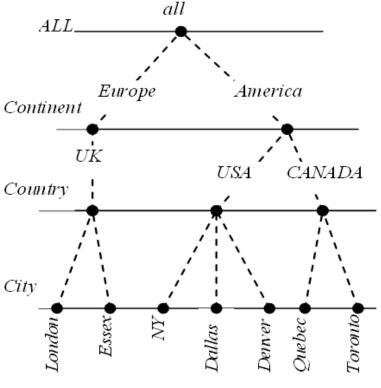
explicit assignment
based on the values x and y
based on Attribute values

Hierarchical distance functions

- W.r.t. an aggregation function
- W.r.t. Hierarchy Path
- *Percentage* distance functions
- *Highway* distance functions



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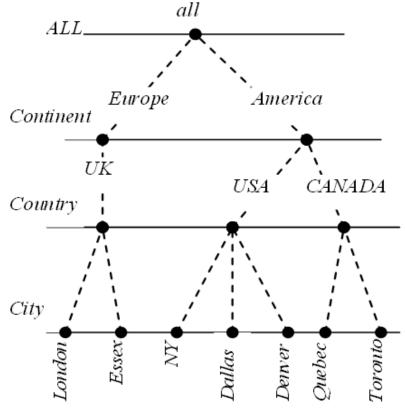
16

Distance functions w.r.t. an aggregation function • $x \in L_i$, $L_L \prec L_i$ • $desc_{L_L}^{L_i}(x)$ set of its descendants ALL____

$$x_{aggr} = f_{aggr}(desc_{L_{L}}^{L_{i}}(x))$$
$$y_{aggr} = f_{aggr}(desc_{L_{L}}^{L_{j}}(y))$$

•
$$f_{aggr}$$
: count, min, max, avg, sum

$$dist(x, y) = g(x_{aggr}, y_{aggr})$$



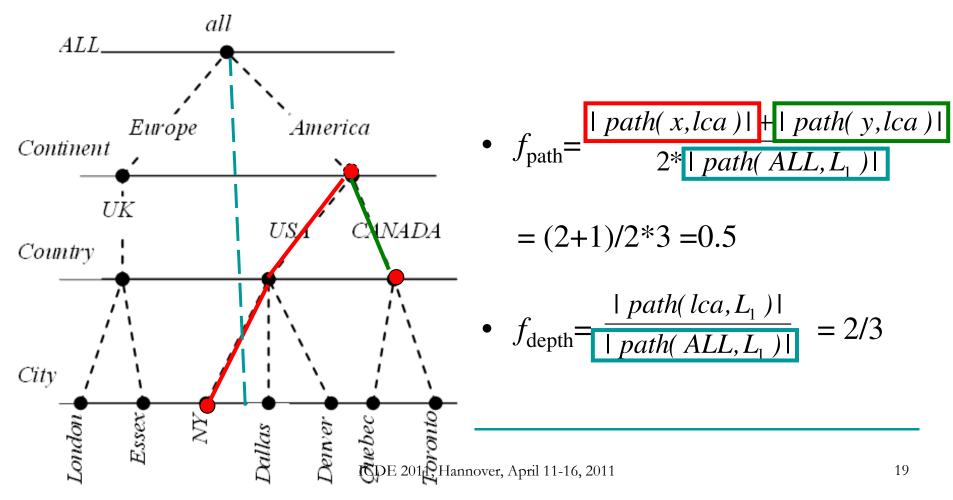
Distance Functions w.r.t. Hierarchy Path

- Assume 2 values x and y s.t. $x \in L_x$ and $y \in L_y$
- lca(x,y): the *Lowest Common Ancestor* of x and y

$$d_{\text{path}}(x, y) = \left(\frac{w_x *| path(x, lca) | + w_y *| path(y, lca) |}{(w_x + w_y) *| path(ALL, L_1) |} \right)$$
$$d_{\text{depth}}(x, y) = \left(\frac{| path(lca, L_1) |}{| path(ALL, L_1) |} \right)$$

Example w.r.t. Hierarchy Path

• x= 'NY', y= 'Canada' lca(x,y) = 'America'



Percentage distance functions

dist(x, y) = $1 - \frac{|desc_{L_i}^{L_x}(x)|}{|desc_{L_i}^{L_y}(y)|}$, only when y is an ancestor of x

the percentage of occurrences over the values of the hierarchy

• Europe America
• Europe America

$$1 - \frac{| desc_{City}^{Country} ('USA') |}{| desc_{City}^{Continent} ('America') |} = \frac{2}{5}$$
where L_{i} is the detailed level L_{city}

Highway Distance Functions

- Every level L grouped into k groups,
- $r_{\rm k}$ the representative
 - distance between two representatives can be thought of as a highway

$$d(x, y) = d(x, \boldsymbol{r}_{x}) + d(\boldsymbol{r}_{x}, \boldsymbol{r}_{y}) + d(y, \boldsymbol{r}_{y})$$

- \square r_x , r_y : representatives of the groups of x, y
- representative selected w.r.t an ancestor or a descendant

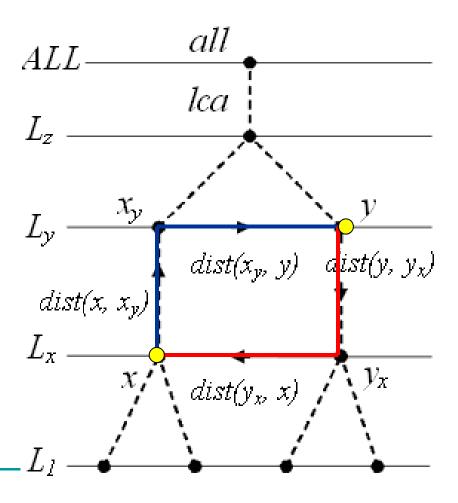
Highway Distance Functions

•
$$\mathbf{r}_{\mathbf{x}}$$
: is an ancestor

$$d(x, y) = d(x, x_y) + d(x_y, y)$$

• $\mathbf{r}_{\mathbf{y}}$: is an descendant

$$d(x, y) = d(x, y_x) + d(y_x, x)$$



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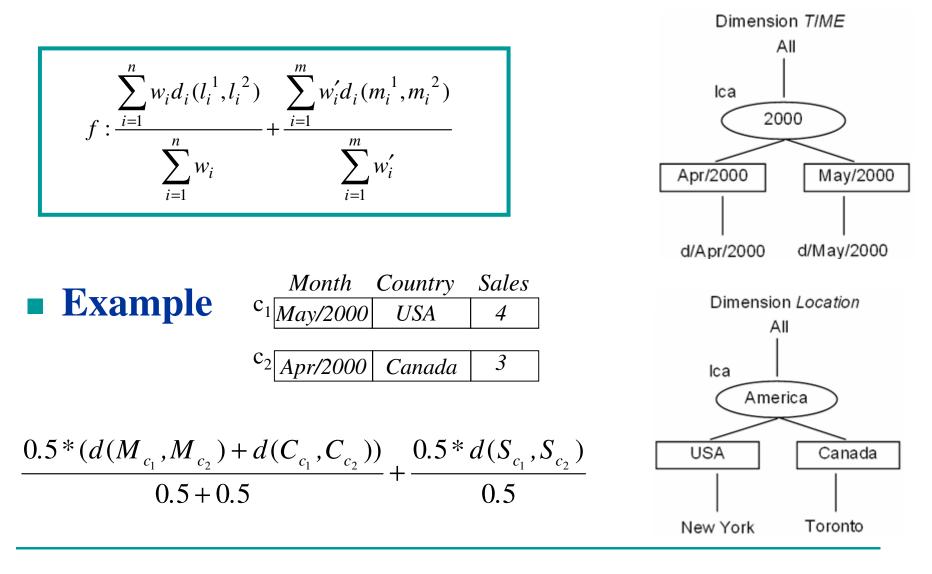
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Distance functions between 2 points in the multidimensional space

- Assume two cells from a cube
 - $c_1 = (l_1^{-1}, l_2^{-1}, \dots, l_n^{-1}, m_1^{-1}, m_2^{-1}, \dots, m_m^{-1})$ $c_2 = (l_1^{-2}, l_2^{-2}, \dots, l_n^{-2}, m_1^{-2}, m_2^{-2}, \dots, m_m^{-2})$
- $dist(c_1, c_2)$ can be expressed w.r.t.
 - their level coordinates $d_i(L_i^1, L_i^2)$ and
 - their measure values $d_i(M_i^1, M_i^2)$

$$dist(c_1, c_2) = f(d_i(L_i^1, L_i^2), d_i(M_i^1, M_i^2))$$

Weighted Sum



$$L_{p} = \sqrt[p]{\sum_{i=1}^{n} (d_{i}(l_{i}^{1}, l_{i}^{2}))^{p}} + \sqrt[p]{\sum_{i=1}^{m} (d_{i}(m_{i}^{1}, m_{i}^{2}))^{p}} \quad \text{p-norm}$$

Minimum Partial distance

• cells $c_1 = (l_1^{1}, l_2^{1}, ..., l_n^{1}, m_1^{1}, m_2^{1}, ..., m_m^{1})$ $c_2 = (l_1^{2}, l_2^{2}, ..., l_n^{2}, m_1^{2}, m_2^{2}, ..., m_m^{2})$

$$dist(c_1, c_2) = \min_{d_i} \{ d_i(l_i^1, l_i^2) \} + \min_{d_i} \{ d_i(m_i^1, m_i^2) \}$$

Proportion of common coordinates

$$\frac{count(l_i^1 = l_i^2 \forall i \in \{1, 2, ..., n\})}{n} + \frac{count(m_i^1 = m_i^2 \forall i \in \{1, 2, ..., m\})}{m}$$

- \square *n*: number of level values, *m*: number of measures
- the number of level values same for both cells
- the number of measures that have the same value for both cells

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Distance functions between 2 sets of points in m/d space

• Cubes: *C* of *l* cells and *C*' of *k* cells

•
$$c = (l_1, l_2, ..., l_n, m_1, m_2, ..., m_m)$$

• $c' = (l_1', l_2', ..., l_n', m_1', m_2', ..., m_m')$

•
$$dist(C, C')=f(dist(c, c'))$$

 \Box f: a function of the partial distances dist(c, c')

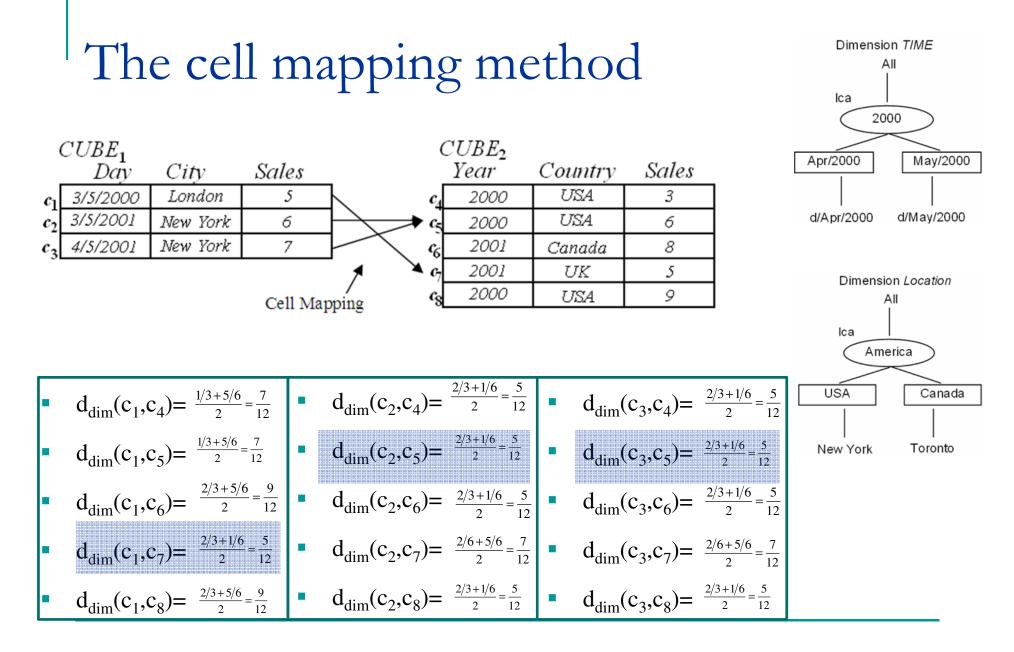
	$CUBE_1$		
	Day	City	Sales
c_1	3/5/2000	London	5
c_2	3/5/2001	New York	6
c_3	4/5/2001	New York	7

($CUBE_2$		
	Year	Country	Sales
c_4	2000	USA	3
c_5	2000	USA	6
c ₆	2001	Canada	8
c 7	2001	UK	5
c_8	2000	USA	9

 $\sim r r n r$

The <u>Cell Mapping</u> method

- Map a cell in a cube to the "closest possible representative" cell in another cube
- Compute all dimension value distances between every cell of 1st cube with every cell of 2nd cube
- The *Mapped* cell of 2nd cube: The cell with the less distance from a cell of 1st cube

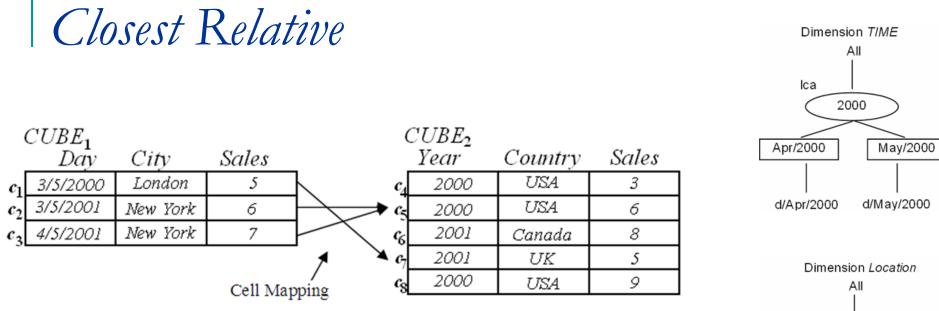


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Closest Relative

$dist(C,C') = \frac{\sum_{i=1}^{k} (dist(c_i,c'))}{k} \quad \forall c' | dist_{dim}(c_i,c') = min\{dist_{dim}(c_i,c')\}}$

- *dist*_{dim}: the distance between two cells according to their dimension values
- Each one of the *k* cells from cube *C* is mapped to the cell of the cube *C*' that has the minimum $dist_{dim}$ from it.



• cells c_1 , c_2 , c_3 , mapped to cells c_7 , c_5 , and c_5

 $d(c_1, c_7) = 5/12$, $d(c_2, c_5) = 5/12$, $d(c_3, c_5) = 5/12$

Dimensions : *f*_{path}, cells: *weighted sum*,

$$d(CUBE_1, CUBE_2) = \frac{d(c_1, c_7) + d(c_2, c_5) + d(c_3, c_5)}{3}$$

• H(C, C') = max(h(C, C'), h(C', C))

- h(C, C'): directed Hausdorff
 - measures the max distance of a cube C to the "nearest" cell of the other cube C'

$$h(C,C') = \max_{c \in C} \{\min_{c' \in C'} \{dist(c,c')\}\}$$

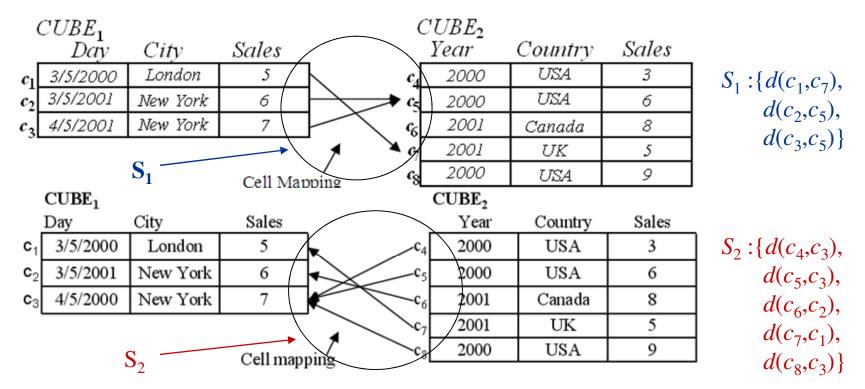
- dist(c, c') distance between two cells c and c'
- Includes bidirectional cell mapping method

Hausdorff computation

- Two sets of mapped cells
- For each set
 - for every pair of mapped cells
 - compute their distance considering their measures as well
- Obtain two sets of min distances between cells
 - a) from C to C'
 - b) from *C*' to *C*
 - □ For each set pick the greatest distance

Pick the greater of the two greatest distances





• $d(CUBE_1, CUBE_2) = max\{max\{S_1\}, max\{S_2\}\} = max\{5/12, 5/12\} = 5/12$

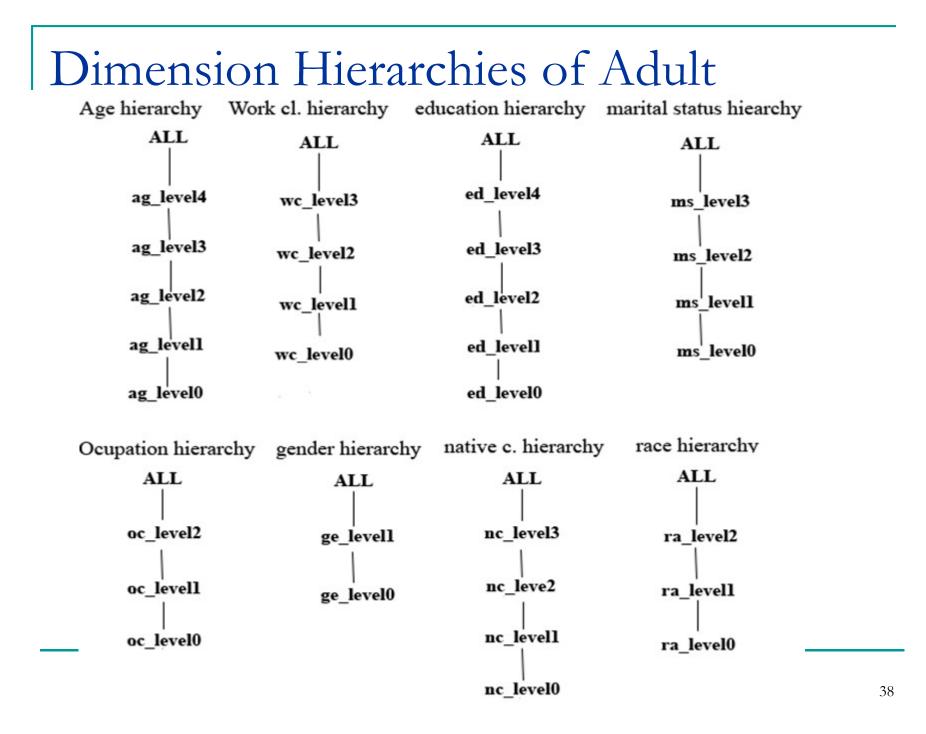
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User study between 2 values of a dimension

- 15 users users_all
 10 users_cs, 5 users_non
- Dataset: '*Adult*'

Table	Value Type	# Tuples	# Dim. Levels
Adult fact		30418	-
Age Dim.	Numeric	72	5
Education Dim.	Categorical	16	5
Gender Dim.	Categorical	2	2
Marital Status Dim.	Categorical	7	4
Native Country Dim.	Categorical	41	4
Occupation Dim.	Categorical	14	3
Race Dim.	Categorical	5	3
Work Class Dim.	Categorical	7	4



Experimental setting

• **Purpose** of the experiment:

- which distance function between two values of a dimension is best in regards to the user preferences
- Each user was given 14 scenarios
- Each scenario contains:
 - a reference cube
 - a set of *variant* cubes
- variant cubes: slightly altering the reference cube
- The 14 scenarios included different kinds of cubes
 value types, levels of granularity

Variant cubes

altering

- granularity level for one dimension
- □ value range of the reference cube
- Example
 - reference cube
 - dimension levels Age_level1, WorkClass_level1
 - age interval [52, 56].
 - 1st type modification: change dimension level (e.g.,*age_level1* to *age_level2*)
 - 2nd type modification: change the age interval to [22, 26] or to [17, 26].

ag_level1	wc_level1
52-56	Gov
52-56	Private
52-56	Self-emp
52-56	Without-pay

ag_level2	wc_level1
47-56	Gov
47-56	Private
47-56	Self-emp
47-56	Without-pay

ag_level1	wc_level1
47-51	Gov
47-51	Private
47-51	Self-emp
47-51	Without-pay

Sample scenario

Reference Cube

Cube4

ag_level1	wc_level1	ra_level1
52-56	Gov	White
52-56	Private	Colored
47-51 52-56	Self-emp	White
52-56	Without-pay	White

Variant Cubes

Cube4_4		
ag_level1	wc_level1	ra_level1
52-56	Gov	White
52-56 52-56	Private	Colored
52-56	Self-emp	White
52-56	Without-pay	White

Cube4_5ag_level1wc_level1ra_level127-31GovColored52-56PrivateColored47-51Self-empWhite52-56Without-payWhite

Cube4_6		
ag_level1	wc_level1	ra_level1
47-51	Self-emp	White
52-56	Without-pay	White

Cube4_8

ag_level2	wc_level1	ra_level1
47-56	Gov	White
47-56	Private	Colored
47-56	Self-emp	White
47-56	Without-pay	White

	Cube4_1 ag_level1	wc_level1	ra_level1
З	37-41	Gov	White
з	37-41	Private	White
	17-51	Self-emp	White
6	52-66	Without-pay	White

Cube4_2

ag_level1	wc_level1	ra_level1
52-56	Gov	White
47-51	Private	White
47-51	Self-emp	White
52-56	Without-pay	White

Cube4_3

ag_level1	wc_level1	ra_level1
37-41	Gov	White
37-41	Private	White
42-46	Self-emp	White
42-46	Without-pay	White

Cube4_7

ag_level1	wc_level2	ra_level1
52-56	With-Pay	White
52-56	With-Pay	Colored
47-51	With-Pay	White
52-56	Without-pay	White

Scenarios of User Study

- Each *variant cube*: most similar to the *reference cube* according to a distance function
- 14 scenarios organized as:
 - □ cubes with arithmetic type values (5 scenarios)
 - □ cubes with categorical type values (2 scenarios)
 - □ cubes with mixed type values (7 scenarios)

Notation of distance functions

Family	Abbr.	Distance function name	
Local	$\delta_{ m M}$	Manhattan	
Aggregation	$\delta_{ m Low,c}$	With respect to a lower level of hierarchy f_{aggr} =count	
	$\delta_{\mathrm{Low,m}}$	With respect to a lower level of hierarchy $f_{aggr} = max$	
Hierarchical Path	$\delta_{ m LCA,P}$	Lowest common ancestor through f_{path}	
	$\delta_{ m LCA,D}$	Lowest common ancestor through f_{depth}	
Percentage	$\delta_{_{\%}}$	Applying percentage function	
Highway	$\delta_{ m Anc}$	With respect to an ancestor x_y	
	$\delta_{ m Desc}$	With respect to a descendant y_x	
	$\delta_{ m H,Desc}$	Highway, selecting the representative from a descendant	
	$\delta_{ m H,Anc}$	Highway, selecting the representative from an ancestor	

• Top three most preferred distance functions

	Users_all	Users_cs	Users_non
δ _{lCA,P}	40.47%	38.57%	44.28%
δ _{Anc}	18.09%	20%	14.28%
$\delta_{\mathrm{H,Desc}}$	9.52%	10.71%	7.14%

• Most preferred function by users w.r.t value type

Value Type Users_a		Users_cs	Users_non
Arithmetic	δ_{Anc}	$\delta_{LCA,P}, \delta_{H,Desc}, \delta_{Anc}$	$\delta_{LCA,P}$
Categorical	$\delta_{\text{LCA,P}}$	$\delta_{LCA,P}$	$\delta_{LCA,P}$
Arithmetic & Categorical	δ_{Anc}	δ_{Anc}	$\delta_{LCA,P'} \delta_{Anc}$

winner distance function

per scenario

- *winner function*: is the most frequent function per scenario for all 15 users
- The most frequent winner function was $\delta_{LCA,P}$

Percentages

- □ 35.71% for the *Users_all* group
- 35,71% for the *Users_cs* group
- □ 57.14% for the *Users_non* group

Diversity and spread of user choices

Two major findings

- □ (a) All functions were picked by some user
- (b) certain functions appeared as user choices for all users of a user group
 - $\delta_{\text{LCA,P}}, \delta_{\text{H,Desc}} \text{ and } \delta_{\text{Anc}} \text{ for } Users_cs$
 - $\delta_{\text{LCA,P}}$, $\delta_{\text{Low,m}}$ and δ_{Anc} for *Users_non*

most preferred family of functions

	Local	Aggregation	Hierarchy Path	Percentage	Highway
Users_cs	1	9	69	9	52
Users_non	2	5	34	5	24
Users_all	3	14	103	14	76

Selection stability of users

- 13th and 14th scenarios replicas of 3rd and 10th scenario
 - □ 4 out of 5 *Users_non* users
 - □ 6 out of 10 *Users_cs* users
- selected the same function for **both** of the two replicas scenarios
- The rest of the users selected the same function for only one replica

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User study between 2 sets of points in M/D space

- which distance function between two cubes do the users prefer?
 - Closest Relative
 - □ Hausdorff
- Between dimensions $\delta_{LCA,P}$
- Between cells *weighted sum*

Scenarios of User Study

14 scenarios

- Each scenario contains 4 cubes (A, B, C, D)
 - Cube A: reference cube
 - B, C, D: variant cubes
 - one most similar to *A* according to the *Closest relative*
 - one most similar to *A* according to the *Hausdorff*
 - \Box remaining less similar to *A* for both functions
 - Users were asked <u>to order</u> the three cubes from the most similar to the less similar when compared to the cube A

Sample scenario

Α			
ag_level1	wc_level1	AVG(hours_per_week)	
27-31	Gov	41.636	
27-31	Private	42.2742	
27-31	Self-emp	46.3854	
27-31	Without-pay	65	

В

_ag_level1_wc_level1_AVG(hours_per_week)

37-41	Private	40.2509
62-66	Without-pay	32.7143

С

ag_level1 wc_level1 AVG(hours_per_week)

22-26	Gov	36.5979
22-26	Private	38.602
22-26	Self-emp	43.6528
22-26	Without-pay	40

	d(A,B)	d(A,C)	d(A,D)
Closest Relative	0.34126	0.19812	0.10799
Hausdorff	0.38151	0.25170	0.30385

D

ag_level1_wc_level1_AVG(hours_per_week)

27-31	Gov	41.636
32-36	Private	42.8008

Scenario groups

no_measures

- Cube distances computed ignoring measures
- not_equal
 - Cube distances computed with different weights between k dimensions and l measures

•
$$w_d = k/k + l, w_m = l/k + l$$

- equal
 - Cube distances computed with equal weights between dimensions and measures

 $W_d = W_m$

User Reliability & Stability

User Reliability

- 6^{th} scenario has cube *B* identical to cube *A*
 - 2 out of 39 users answered wrong
 - 37 valid users
- User Stability
 - 13th and 14th scenario were replicas of the 5th and 9th scenario
 - □ *User_ok* : same ordering for one scenario
 - □ *User_half_ok* : same first choice
 - User_Stable : User_ok for both replicas or User_ok and User_half_ok

User Stability

	User_OK		User_Half_OK		User_Stable	
	Frequency	Pct	Frequency	Pct	Frequency	Pct
13 th scenario	28	75%	5	13%	24	65%
14 th scenario	19	51%	8	21%	24	65%

Most frequent distance function

Most frequent function chosen as the first ordering in all scenarios

Over all scenarios	Frequency	Percentage
Hausdorff	154	38%
Closest relative	232	57%
Most distant cube	21	5%

Local scenario winner

• Local scenario winner function:

- function that was mostly selected as the first choice from the users in each scenario
- closest relative: 6 scenarios
- Hausdorff: 5 scenarios

Group winner function

Scenario Group	Scenario	Winning function	Winner function
no_measures	Scenario1	Closest relative	Closest relative
	Scenario2	Closest relative	
	Scenario3	Closest relative	
	Scenario4	Hausdorff	
not_equal	Scenario5	Hausdorff	Hausdorff
	Scenario7	Closest relative	
	Scenario8	Hausdorff	
equal	Scenario9	Hausdorff	Draw: both
	Scenario10	Hausdorff	
	Scenario11	Closest relative	
	Scenario12	Closest relative	

Conclusions

- Taxonomy of distances
- Distance between values of a dimension:
 - Most preferred function according to the path of the lowest common ancestor
- Distance between sets of points in a m/d space
 - Closest relative and Hausdorff
- Future work
 - More user studies
 - Combine texts

Thank you for your attention!



http://www.cs.uoi.gr/~ebaikou/publications/2011_ICDE/

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