# Similarity Measures for Multidimensional Data 

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## Cube 1 or Cube 2 most Similar to Cube 0 ?

Cube 1
Cube 0

| P |  | Date |  |
| :---: | :---: | :---: | :---: |
|  |  | 2009 | 2010 |
| $r$ | Cola | 10 | 12 |
| d | Fanta | 5 | 8 |
| $u$ | Chips | 5 | 5 |
| $t$ | Popcorn | 10 | 15 |



Cube 2

| Cola | 2008 | 2009 |
| :---: | :---: | :---: |
|  | 7 | 10 |
| Fanta | 4 | 5 |
| Chips | 6 | 5 |
| Popcorn | 10 | 10 |

## Motivating Example

Cube 1
Average sales by year


Cube 2
Average sales by year


Time Hierarchy Location Hierarçhy

## Contents

- Background \& Related Work
- Distance Functions
- between 2 values of a dimension
- between 2 points in the multidimensional space
$\square$ between 2 sets of points in the multidimensional space
- User Study Experiments
- User study between 2 values of a dimension
- User study between 2 sets of points in $\mathrm{m} / \mathrm{d}$ space (cubes)


## Background

- Fundamentals
- Distance Measures
- Hausdorff
- Controversy on Metric Axioms
- Distances on Graphs
- Highway Hierarchies
- Semantic Similarity between Words


## Distance Measures

- A distance measure is called a metric when :
- $d(i, j) \geq 0 \& d(i, j)=d(j, i) \& d(i, i)=0 \& d(i, j) \leq d(i, k)+d(j, k)$
- Categorization
- interval-scaled variables (Euclidean, Minkowski, Manhattan)
- binary variables (Jaccard)
- categorical variables


## Hausdorff distance

- Example:
$d_{\mathrm{H}}(A, B)=\max \left\{d_{\mathrm{s}}(A, B), d_{\mathrm{s}}(B, A)\right\}=\max \left\{\boldsymbol{d}_{\mathrm{e}}\left(\boldsymbol{a}_{\mathbf{1}}, \boldsymbol{b}_{\mathbf{2}}\right), \boldsymbol{d}_{\mathrm{e}}\left(\boldsymbol{b}_{3}, \boldsymbol{a}_{2}\right)\right\}$
- $d_{\mathrm{e}}$ denotes the Euclidian distance
- $d_{\mathrm{s}}$ denotes the max distance of the set of minimum distances.



# Controversy on Metric Axioms 

- Properties of metrics are convenient for Mathematicians/Computer Scientists


## However

- Human perception does not comply with properties of metrics


## Highway Hierarchies

- highways in road maps
- The shortest paths among 2 points in a road network consists of
- small roads locally
- a highway road

- Hierarchy: highway edges with attached sub-trees of locally computable shortest paths

Sub-trees of locally computable shortest paths


## Distances on Graphs

- Semantic Similarity between Words
- Word similarity measures
- Semantic hierarchies
- 2 datasets (pairs of words)
- One for constructing their method
- The other to test it


## Distances for Collections of Structured Data

- Relax operator
- Diff operator
- Distance between two relational databases under the same schema


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- User study between 2 values of a dimension
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## Contents

- Background \& Related Work
- Distance Functions
- between 2 values of a dimension
- between 2 points in the multidimensional space
- between 2 sets of points in the multidimensional space
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# Distance functions between 2 values of a dimension 

- Locally computable
- Hierarchical
- Highway


# Distance functions between 2 values of a dimension 

- Locally computable distance functions
- explicit assignment
$\square$ based on the values $x$ and $y$
- based on Attribute values


## Hierarchical distance functions

- W.r.t. an aggregation function
- W.r.t. Hierarchy Path
- Percentage distance functions
- Highway distance functions



## Distance functions w.r.t. an

 aggregation function- $x \in L_{\mathrm{i}}, \quad L_{\mathrm{L}} \prec L_{\mathrm{i}}$
- $\operatorname{desc}_{L_{\mathrm{L}}}^{L_{L_{1}}}(x)$ set of its descendants

$$
\begin{aligned}
& x_{\text {aggr }}=f_{\text {aggr }}\left(\operatorname{desc}_{L_{\mathrm{L}}}^{L_{\mathrm{L}}}(x)\right) \\
& y_{\text {aggr }}=f_{\text {aggr }}\left(\operatorname{desc}_{L_{\mathrm{L}}}^{L_{\mathrm{j}}}(y)\right)
\end{aligned}
$$

- $f_{\text {aggr }}$ : count, min, max, avg, sum
- $\operatorname{dist}(x, y)=g\left(x_{\text {aggr }}, y_{\text {aggr }}\right)$
- $g$ can be from the locally computable functions



## Distance Functions w.r.t. Hierarchy Path

- Assume 2 values $x$ and $y$ s.t.
- $x \in L_{\mathrm{x}}$ and $y \in L_{\mathrm{y}}$
- lca $(x, y)$ : the Lowest Common Ancestor of $x$ and $y$
- $d_{\text {path }}(x, y)=\left(\frac{w_{\mathrm{x}}{ }^{*}|\operatorname{path}(x, l c a)|+w_{y} *|\operatorname{path}(y, l c a)|}{\left.\left(w_{\mathrm{x}}+w_{\mathrm{y}}\right)^{*\left|\operatorname{path}\left(A L L, L_{1}\right)\right|}\right)}\right.$
- $d_{\text {depth }}(x, y)=\left(\frac{\left|\operatorname{path}\left(l c a, L_{1}\right)\right|}{\left|\operatorname{path}\left(A L L, L_{1}\right)\right|}\right)$


## Example w.r.t. Hierarchy Path

- $x=$ 'NY', $y=$ 'Canada' lca $(x, y)=$ 'America'



## Percentage distance functions

- $\operatorname{dist}(x, y)=1-\frac{\left|\operatorname{des} c_{L_{1}}^{L_{x}}(x)\right|}{\left|\operatorname{desc}_{L_{\mathrm{i}}}^{L_{y}}(y)\right|}$, only when y is an ancestor of x
- the percentage of occurrences over the values of the hierarchy
- Example: dist('USA','America')
where $L_{\mathrm{i}}$ is the detailed level $L_{\text {city }}$



## Highway Distance Functions

- Every level $L$ grouped into $k$ groups,
- $r_{\mathrm{k}}$ the representative
- distance between two representatives can be thought of as a highway

$$
d(x, y)=d\left(x, \boldsymbol{r}_{\mathrm{x}}\right)+d\left(\boldsymbol{r}_{\mathbf{x}}, \boldsymbol{r}_{\mathbf{y}}\right)+d\left(y, \boldsymbol{r}_{\mathbf{y}}\right)
$$

- $r_{x}, r_{y}$ : representatives of the groups of $x, y$
- representative selected w.r.t an ancestor or a descendant


## Highway Distance Functions

- $\mathbf{r}_{\mathbf{x}}$ : is an ancestor

$$
\mathrm{d}(\mathrm{x}, \mathrm{y})=\mathrm{d}\left(\mathrm{x}, \mathrm{x}_{\mathrm{y}}\right)+\mathrm{d}\left(\mathrm{x}_{\mathrm{y}}, \mathrm{y}\right)
$$

- $\mathbf{r}_{\mathbf{y}}$ : is an descendant

$$
d(x, y)=d\left(x, y_{x}\right)+d\left(y_{x}, x\right)
$$



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## Distance functions between 2 points

 in the multidimensional space- Assume two cells from a cube
- $c_{1}=\left(l_{1}{ }^{1}, l_{2}{ }^{1}, \ldots, l_{\mathrm{n}}{ }^{1}, m_{1}{ }^{1}, m_{2}{ }^{1}, \ldots, m_{\mathrm{m}}{ }^{1}\right)$
- $c_{2}=\left(l_{1}^{2}, l_{2}^{2}, \ldots, l_{\mathrm{n}}^{2}, m_{1}^{2}, m_{2}^{2}, \ldots, m_{\mathrm{m}}^{2}\right)$
- $\operatorname{dist}\left(c_{1}, c_{2}\right)$ can be expressed w.r.t.
- their level coordinates $d_{\mathrm{i}}\left(L_{\mathrm{i}}{ }^{1}, L_{\mathrm{i}}{ }^{2}\right)$ and
- their measure values $d_{\mathrm{i}}\left(M_{\mathrm{i}}{ }^{1}, M_{\mathrm{i}}{ }^{2}\right)$

$$
\operatorname{dist}\left(c_{1}, c_{2}\right)=f\left(d_{\mathrm{i}}\left(L_{\mathrm{i}}^{1}, L_{\mathrm{i}}^{2}\right), d_{\mathrm{i}}\left(M_{\mathrm{i}}^{1}, M_{\mathrm{i}}^{2}\right)\right)
$$

## Weighted Sum




$\mathrm{c}_{2}$| Apr/2000 | Canada | 3 |
| :--- | :--- | :--- |

$\frac{0.5 *\left(d\left(M_{c_{1}}, M_{c_{2}}\right)+d\left(C_{c_{1}}, C_{c_{2}}\right)\right)}{0.5+0.5}+\frac{0.5 * d\left(S_{c_{1}}, S_{c_{2}}\right)}{0.5}$


- Minkowski

$$
L_{p}=\sqrt[p]{\sum_{i=1}^{n}\left(d_{i}\left(l_{i}{ }^{1}, l_{i}{ }^{2}\right)\right)^{p}}+\sqrt[p]{\sum_{i=1}^{m}\left(d_{i}\left(m_{i}{ }^{1}, m_{i}{ }^{2}\right)\right)^{p}} \quad \text { p-norm }
$$

- Minimum Partial distance
- cells $c_{l}=\left(l_{1}{ }^{1}, l_{2}{ }^{1}, \ldots, l_{\mathrm{n}}{ }^{1}, m_{1}{ }^{1}, m_{2}{ }^{1}, \ldots, m_{\mathrm{m}}{ }^{1}\right)$

$$
c_{2}=\left(l_{1}^{2}, l_{2}^{2}, \ldots, l_{\mathrm{n}}^{2}, m_{1}^{2}, m_{2}^{2}, \ldots, m_{\mathrm{m}}^{2}\right)
$$

$$
\operatorname{dist}\left(c_{1}, c_{2}\right)=\min _{d_{i}}\left\{d_{i}\left(l_{i}^{1}, l_{i}^{2}\right)\right\}+\min _{d_{i}}\left\{d_{i}\left(m_{i}^{1}, m_{i}^{2}\right)\right\}
$$

- Proportion of common coordinates

$$
\frac{\operatorname{count}\left(l_{\mathrm{i}}^{1}=l_{\mathrm{i}}^{2} \forall i \in\{1,2, \ldots, n\}\right)}{n}+\frac{\operatorname{count}\left(m_{\mathrm{i}}^{1}=m_{\mathrm{i}}^{2} \forall i \in\{1,2, \ldots, m\}\right)}{m}
$$

- $n$ : number of level values, $m$ : number of measures
- the number of level values same for both cells

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## Distance functions between 2 sets of

 points in $\mathrm{m} / \mathrm{d}$ space- Cubes: $C$ of $l$ cells and $C^{\prime}$ of $k$ cells
- $c=\left(l_{1}, l_{2}, \ldots, l_{\mathrm{n}}, m_{1}, m_{2}, \ldots, m_{\mathrm{m}}\right)$
- $c^{\prime}=\left(l_{1}{ }^{\prime}, l_{2}^{\prime}, \ldots, l_{\mathrm{n}}{ }^{\prime}, m_{1}{ }^{\prime}, m_{2}{ }^{\prime}, \ldots, m_{\mathrm{m}}{ }^{\prime}\right)$
- $\operatorname{dist}\left(C, C^{\prime}\right)=f\left(\operatorname{dist}\left(c, c^{\prime}\right)\right)$
- $f$ : a function of the partial distances $\operatorname{dist}\left(c, c^{\prime}\right)$



## The Cell Mapping method

- Map a cell in a cube to the "closest possible representative" cell in another cube
- Compute all dimension value distances between every cell of $1^{\text {st }}$ cube with every cell of $2^{\text {nd }}$ cube
- The Mapped cell of $2^{\text {nd }}$ cube: The cell with the less distance from a cell of $1^{\text {st }}$ cube


## The cell mapping method

|  | $\begin{gathered} C U B E_{1} \\ D a y \end{gathered}$ | City | Sales | $\begin{gathered} C_{\text {Year }}^{2} \end{gathered}$ | Country | Sales |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $c_{1}$ | 3/5/2000 | London | 5 | 2000 | USA | 3 |
| $c_{2}$ | 3/5/2001 | New York | 6 | 2000 | USA | 6 |
| $c_{3} 4 / 5 / 2001$ |  | New York | 7 | 2001 | Canada | 8 |
|  |  |  |  | 2001 | UK | 5 |
|  |  |  | Cell | 2000 | USA | 9 |



Dimension Location


## Closest Relative

$$
\left.\operatorname{dist}\left(C, C^{\prime}\right)=\frac{\sum_{\mathrm{i}=1}^{\mathrm{k}}\left(\operatorname{dist}\left(c_{i}, c^{\prime}\right)\right)}{\mathrm{k}} \forall c^{\prime} \right\rvert\, \operatorname{dist}_{\operatorname{dim}}\left(c_{i}, c_{\mathrm{i}}^{\prime}\right)=\min \left\{\operatorname{dist}_{\operatorname{dim}}\left(c_{i}, c^{\prime}\right)\right\}
$$

- dist $_{\text {dim }}$ : the distance between two cells according to their dimension values
- Each one of the $k$ cells from cube $C$ is mapped to the cell of the cube $C^{\prime}$ that has the minimum dist $_{\mathrm{dim}}$ from it.


## Closest Relative

|  | $\begin{array}{r} C U B E_{1} \\ D a y \end{array}$ | City | Sales | $\begin{gathered} C U B E_{2} \\ \text { Year } \end{gathered}$ | Country | Sales |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $c_{1}$ | 3/5/2000 | London | 5 | 2000 | USA | 3 |
| $c_{2}$ | 3/5/2001 | New York | 6 | 2000 | USA | 6 |
| $c_{3}$ | 4/5/2001 | New York | 7 | 2001 | Canada | 8 |
|  |  |  |  | 2001 | UK | 5 |
|  |  |  | Cell | 2000 | USA | 9 |

- cells $c_{1}, c_{2}, c_{3}$, mapped to cells $c_{7}, c_{5}$, and $c_{5}$

$$
d\left(\mathrm{c}_{1}, \mathrm{c}_{7}\right)=5 / 12, d\left(\mathrm{c}_{2}, \mathrm{c}_{5}\right)=5 / 12 \quad, d\left(\mathrm{c}_{3}, \mathrm{c}_{5}\right)=5 / 12
$$

-Dimensions : $f_{\text {path }}$, cells: weighted sum,
$d\left(\right.$ CUBE $\left._{1}, C U B E_{2}\right)=\frac{d\left(c_{1}, c_{7}\right)+d\left(c_{2}, c_{5}\right)+d\left(c_{3}, c_{5}\right)}{3}$

## Hausdorff

$H\left(C, C^{\prime}\right)=\max \left(h\left(C, C^{\prime}\right), h\left(C^{\prime}, C\right)\right)$

- $h\left(C, C^{\prime}\right)$ : directed Hausdorff
- measures the max distance of a cube C to the "nearest" cell of the other cube C'
- $h\left(C, C^{\prime}\right)=\max _{\mathrm{c} \in \mathrm{C}}\left\{\min _{\mathrm{c}^{\prime} \in \mathrm{C}^{\prime}}\left\{\operatorname{dist}\left(c, c^{\prime}\right)\right\}\right\}$
- $\operatorname{dist}\left(c, c^{\prime}\right)$ distance between two cells $c$ and $c^{\prime}$
- Includes bidirectional cell mapping method


## Hausdorff computation

- Two sets of mapped cells
- For each set
- for every pair of mapped cells
- compute their distance considering their measures as well
- Obtain two sets of min distances between cells
a) from $C$ to $C^{\prime}$
b) from $C^{\prime}$ to $C$
- For each set pick the greatest distance
- Pick the greater of the two greatest distances


## Hausdorff



- $d\left(\right.$ CUBE $\left._{1}, C U B E_{2}\right)=$ $\max \left\{\max \left\{S_{1}\right\}, \max \left\{S_{2}\right\}\right\}=\max \{5 / 12,5 / 12\}=5 / 12$


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## User study between 2 values of a dimension

- 15 users users_all
- 10 users_cs, 5 users_non
- Dataset: ‘Adult'

| Table | Value Type | \# Tuples | \# Dim. Levels |
| :---: | :---: | :---: | :---: |
| Adult fact |  | 30418 | - |
| Age Dim. | Numeric | 72 | 5 |
| Education <br> Dim. | Categorical | 16 | 5 |
| Gender Dim. | Categorical | 2 | 2 |
| Marital <br> Status Dim. | Categorical | 7 | 4 |
| Native <br> Country Dim. | Categorical | 41 | 4 |
| Occupation <br> Dim. | Categorical | 14 | 3 |
| Race Dim. | Categorical | 5 | 3 |
| Work Class <br> Dim. | Categorical | 7 | 4 |

## Dimension Hierarchies of Adult

Age hierarchy Work cl. hierarchy education hierarchy marital status hiearchy


Ocupation hierarchy gender hierarchy native c. hierarchy race hierarchy


## Experimental setting

- Purpose of the experiment:
- which distance function between two values of a dimension is best in regards to the user preferences
- Each user was given 14 scenarios
- Each scenario contains:
- a reference cube
- a set of variant cubes
- variant cubes: slightly altering the reference cube
- The 14 scenarios included different kinds of cubes
- value types, levels of granularity


## Variant cubes

- altering
- granularity level for one dimension
- value range of the reference cube
ag_level1 wc_level1

| $52-56$ | Gov |
| :--- | :--- |
| $52-56$ | Private |
| $52-56$ | Self-emp |
| $52-56$ | Without-pay |

- Example
- reference cube
- dimension levels Age_level1, WorkClass_level1 - age interval [52, 56].
- $\quad 1^{\text {st }}$ type modification: change dimension level (e.g.,age_level1 to age_level2)
- $2^{\text {nd }}$ type modification: change the age interval to [22, 26] or to [17, 26].

| ag_level2 | wc_level1 |
| :--- | :--- |
| $47-56$ | Gov |
| $47-56$ | Private |
| $47-56$ | Self-emp |
| $47-56$ | Without-pay |


| ag_level1 | wc_level1 |
| :--- | :--- |
| $47-51$ | Gov |
| $47-51$ | Private |
| $47-51$ | Self-emp |
| $47-51$ | Without-pay |

## Sample scenario

- Reference Cube

| Cube4 <br> ag_level1 | wc_level1 | ra_level1 |
| :--- | :--- | :--- |
| $52-56$ | Gov | White |
| $52-56$ | Private | Colored |
| $47-51$ | Self-emp | White |
| $52-56$ | Without-pay | White |

- Variant Cubes

| $\uparrow$ | $\left\lvert\, \begin{aligned} & 52-56 \\ & 47-51 \\ & 52-56 \end{aligned}\right.$ | Self-emp <br> Without-pay | White <br> White |
| :---: | :---: | :---: | :---: |
|  | Cube4_6 ag_level1 | wc_level1 | ra_level1 |
|  | 47-51 | Self-emp | White |
|  | 52-56 | Without-pay | White |
|  | Cube4_8 <br> ag_level2 | wc_level1 | ra_level1 |
|  | 47-56 | Gov | White |
|  | 47-56 | Private | Colored |
|  | 47-56 | Self-emp | White |
|  | 47-56 | Without-pay | White |

Cube4_1

| ag_level1 | wc_level1 | ra_level1 |
| :--- | :--- | :--- |
| $37-41$ Gov White <br> $37-41$ Private White <br> $47-51$ Self-emp White <br> $62-66$ Without-pay White |  |  | 

Cube4_2

| ag_level1 | wc_level1 | ra_level1 |
| :--- | :--- | :--- |
| $52-56$ | Gov | White |
| $47-51$ | Private | White |
| $47-51$ | Self-emp | White |
| $52-56$ | Without-pay | White |

Cube4_3

| ag_level1 | wc_level1 | ra_level1 |
| :--- | :--- | :--- |
| $37-41$ | Gov | White |
| $37-41$ | Private | White |
| $42-46$ | Self-emp | White |
| $42-46$ | Without-pay | White |

Cube4_7

| ag_level1 | wc_level2 | ra_level1 |
| :--- | :--- | :--- |
| $52-56$ | With-Pay | White |
| $52-56$ | With-Pay | Colored |
| $47-51$ | With-Pay | White |
| $52-56$ | Without-pay | White |

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## Scenarios of User Study

- Each variant cube: most similar to the reference cube according to a distance function
- 14 scenarios organized as:
- cubes with arithmetic type values ( 5 scenarios)
$\square$ cubes with categorical type values ( 2 scenarios)
- cubes with mixed type values ( 7 scenarios)


## Notation of distance functions

| Family | Abbr. | Distance function name |
| :---: | :---: | :---: |
| Local | $\delta_{\mathrm{M}}$ | Manhattan |
| Aggregation | $\delta_{\text {Low, } \mathrm{C}}$ | With respect to a lower level of hierarchy <br> $f_{\text {aggr }}=$ count |
|  | $\delta_{\text {Low,m }}$ | With respect to a lower level of hierarchy <br> $f_{\text {aggr }}=$ max |
|  | $\delta_{\text {LCA,P }}$ | Lowest common ancestor through $f_{\text {path }}$ |
|  | $\delta_{\text {LCA,D }}$ | Lowest common ancestor through $f_{\text {depth }}$ |
| Percentage | $\delta_{\%}$ | Applying percentage function |
| Highway | $\delta_{\text {Anc }}$ | With respect to an ancestor $x_{\mathrm{y}}$ |
|  | $\delta_{\text {Desc }}$ | With respect to a descendant $y_{\mathrm{x}}$ |
|  | $\delta_{\mathrm{H}, \text { Desc }}$ | Highway, selecting the representative from a |
|  |  |  |

- Top three most preferred distance functions

|  | Users_all | Users_cs | Users_non |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{\delta}_{\text {LCA,PP}}$ | $40.47 \%$ | $38.57 \%$ | $44.28 \%$ |
| $\boldsymbol{\delta}_{\text {Anc }}$ | $18.09 \%$ | $20 \%$ | $14.28 \%$ |
| $\boldsymbol{\delta}_{\mathbf{H}, \text { Desc }}$ | $9.52 \%$ | $10.71 \%$ | $7.14 \%$ |

- Most preferred function by users w.r.t value type

| Value Type | Users_all | Users_cs | Users_non |
| :---: | :---: | :---: | :---: |
| Arithmetic | $\delta_{\text {Anc }}$ | $\delta_{\text {LCA,P }} \delta_{\mathrm{H}, \text { Desc }}, \delta_{\mathrm{Anc}}$ | $\delta_{\mathrm{LCA}, \mathrm{P}}$ |
| Categorical | $\delta_{\mathrm{LCA}, \mathrm{P}}$ | $\delta_{\mathrm{LCA}, \mathrm{P}}$ | $\delta_{\mathrm{LCA}, \mathrm{P}}$ |
| Arithmetic <br> \& Categorical | $\delta_{\mathrm{Anc}}$ | $\delta_{\mathrm{Anc}}$ | $\delta_{\mathrm{LCA}, \mathrm{P}} \delta_{\mathrm{Anc}}$ |

## winner distance function

## per scenario

- winner function: is the most frequent function per scenario for all 15 users
- The most frequent winner function was $\delta_{\text {LCA, } \mathrm{P}}$
- Percentages
- $35.71 \%$ for the Users_all group
- $35,71 \%$ for the Users_cs group
- $57.14 \%$ for the Users_non group


## Diversity and spread of user choices

- Two major findings
- (a) All functions were picked by some user
- (b) certain functions appeared as user choices for all users of a user group
- $\delta_{\mathrm{LCA}, \mathrm{P}}, \delta_{\mathrm{H}, \text { Desc }}$ and $\delta_{\text {Anc }}$ for Users_cs
- $\delta_{\text {LCA,P }}, \delta_{\text {Low, } \mathrm{m}}$ and $\delta_{\text {Anc }}$ for Users_non


## most preferred family of functions

|  | Local | Aggregation | Hierarchy Path | Percentage | Highway |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Users_cs | 1 | 9 | 69 | 9 | 52 |
| Users_non | 2 | 5 | 34 | 5 | 24 |
| Users_all | 3 | 14 | 103 | 14 | 76 |

## Selection stability of users

- $13^{\text {th }}$ and $14^{\text {th }}$ scenarios replicas of $3^{\text {rd }}$ and $10^{\text {th }}$ scenario
- 4 out of 5 Users_non users
- 6 out of 10 Users_cs users
- selected the same function for both of the two replicas scenarios
- The rest of the users selected the same function for only one replica


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## User study between 2 sets of points in

M/D space

- which distance function between two cubes do the users prefer?
- Closest Relative
- Hausdorff
- Between dimensions $\delta_{\text {LCA,P }}$
- Between cells weighted sum


## Scenarios of User Study

- 14 scenarios
- Each scenario contains 4 cubes $(A, B, C, D)$
- Cube A: reference cube
- B,C,D: variant cubes
- one most similar to $A$ according to the Closest relative
- one most similar to $A$ according to the Hausdorff
- remaining less similar to $A$ for both functions
- Users were asked to order the three cubes from the most similar to the less similar when compared to the cube $A$


## Sample scenario

A
ag_level1

| wc_level1 | AVG(hours_per_week) |  |
| :---: | :---: | :---: |
| $27-31$ | Gov | 41.636 |
| $27-31$ | Private | 42.2742 |
| $27-31$ | Self-emp | 46.3854 |
| $27-31$ | Without-pay | 65 |

B
ag_level1

| $37-41$ | wr_level1 | AVG(hours_per_week) |
| :---: | :---: | :---: |
| $62-66$ | Without- | 40.2509 |

C
ag_level1

| $22-26$ | Gc_level1 | AVG(hours_per_week) |
| :---: | :---: | :---: |
| $22-26$ | Private | 36.5979 |
| $22-26$ | Self-emp | 38.602 |
| $22-26$ | Without-pay | 43.6528 |


|  | $\mathbf{d}(\mathbf{A , B})$ | $\mathbf{d}(\mathbf{A}, \mathbf{C})$ | $\mathbf{d}(\mathbf{A}, \mathbf{D})$ |
| :--- | :--- | :--- | :--- |
| Closest <br> Relative | 0.34126 | 0.19812 | 0.10799 |
| Hausdorff | 0.38151 | 0.25170 | 0.30385 |

D
ag_level1

| wc_level1 | AVG(hours_per_week) |  |
| :---: | :---: | :---: |
| $27-31$ | Gov | 41.636 |
| $32-36$ | Private | 42.8008 |

## Scenario groups

- no_measures
$\square$ Cube distances computed ignoring measures
- not_equal
$\square$ Cube distances computed with different weights between $k$ dimensions and $l$ measures
- $w_{\mathrm{d}}=k / k+l, w_{\mathrm{m}}=l / k+l$
- equal
- Cube distances computed with equal weights between dimensions and measures
- $w_{\mathrm{d}}=w_{\mathrm{m}}$


## User Reliability \& Stability

- User Reliability
- $6^{\text {th }}$ scenario has cube $B$ identical to cube $A$
- 2 out of 39 users answered wrong
- 37 valid users
- User Stability
- $13^{\text {th }}$ and $14^{\text {th }}$ scenario were replicas of the $5^{\text {th }}$ and $9^{\text {th }}$ scenario
- User_ok: same ordering for one scenario
- User_half_ok: same first choice
- User_Stable : User_ok for both replicas
or User_ok and User_half_ok


## User Stability

|  | User_OK |  | User_Half_OK |  | User_Stable |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Frequency | Pct | Frequency | Pct | Frequency | Pct |
| $13^{\text {th }}$ <br> scenario | 28 | $75 \%$ | 5 | $13 \%$ | 24 | $65 \%$ |
| 14 <br> scenario | 19 | $51 \%$ | 8 | $21 \%$ | 24 | $65 \%$ |

## Most frequent distance function

- Most frequent function chosen as the first ordering in all scenarios

| Over all scenarios | Frequency | Percentage |
| :---: | :---: | :---: |
| Hausdorff | 154 | $\mathbf{3 8 \%}$ |
| Closest relative | $\mathbf{2 3 2}$ | $\mathbf{5 7 \%}$ |
| Most distant cube | 21 | $5 \%$ |

## Local scenario winner

- Local scenario winner function:
- function that was mostly selected as the first choice from the users in each scenario
- closest relative: 6 scenarios
- Hausdorff: 5 scenarios


## Group winner function

| Scenario Group | Scenario | Winning function | Winner function |
| :---: | :---: | :---: | :---: |
| no_measures | Scenario1 | Closest relative | Closest relative |
|  | Scenario2 | Closest relative |  |
|  | Scenario3 | Closest relative |  |
|  | Scenario4 | Hausdorff | Hausdorff |
| not_equal | Scenario5 | Hausdorff |  |
|  | Scenario7 | Closest relative |  |
|  | Scenario8 | Hausdorff |  |
|  | Scenario9 | Hausdorff |  |
|  | Scenario10 | Hausdorff |  |
|  | Scenario11 | Closest relative |  |
|  | Scenario12 | Closest relative |  |

## Conclusions

- Taxonomy of distances
- Distance between values of a dimension:
- Most preferred function according to the path of the lowest common ancestor
- Distance between sets of points in a m/d space
- Closest relative and Hausdorff
- Future work
- More user studies
- Combine texts


## Thank you for your attention!



User study Questionnaires \& Results can be found :
http://www.cs.uoi.gr/~ebaikou/publications/2011_ICDE/

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[^0]:    $\square$ the number of measures that have the same value for both cells

