# Computing and Handling Cardinal Direction Information

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**Abstract.** Qualitative spatial reasoning forms an important part of the commonsense reasoning required for building intelligent Geographical Information Systems (GIS). Previous research has come up with models to capture cardinal direction relations for typical GIS data. In this paper, we target the problem of efficiently computing the cardinal direction relations between regions that are composed of sets of polygons and present the first two algorithms for this task. The first of the proposed algorithms is purely qualitative and computes, in linear time, the cardinal direction relations between the input regions. The second has a quantitative aspect and computes, also in linear time, the cardinal direction relations with percentages between the input regions. The algorithms have been implemented and embedded in an actual system, CarDirect, that allows the user to annotate regions of interest in an image or a map, compute cardinal direction relations and retrieve combinations of interesting regions on the basis of a query.

## 1 Introduction

Recent developments in the fields of mobile and collaborative computing, require that intelligent Geographical Information Systems (GIS) should support real-time response to complex queries. Related research in the field of spatiotemporal data management and reasoning has provided several results towards this problem. Among these research topics, qualitative spatial reasoning has received a lot of attention with several kinds of useful spatial relations being studied so far, e.g., topological relations [2, 17], cardinal direction relations [6, 8, 11] and distance relations [3]. The uttermost aim in these lines of research is to define new categories of spatial operators as well as to build efficient algorithms for the automatic processing of queries that use such operators.

The present paper concentrates on *cardinal direction relations* [6, 8]. Cardinal direction relations are qualitative spatial relations characterizing the relative position of a region with respect to another (e.g., region $a$ is *north of* region $b$). Our starting point is the cardinal direction framework presented in [5, 6, 20, 21]. To express the cardinal direction relation between a region $a$ and with respect to region $b$, this model approximates only region $b$ (using its minimum bounding box – MBB) while it uses the exact shape of region $a$. This offers a more precise and expressive model than previous approaches that approximate both extended regions using points or MBB's [4, 8, 13]. Particularly, in this paper we will employ the cardinal direction model presented in [21] because it is formally defined and can be applied to a wide set of regions (like disconnected regions and regions with holes). Additionally, we also study the interesting extension of cardinal directions relations that adds a quantitative aspect using percentages [6].

The goal of this paper is to address the problem of efficiently computing the cardinal direction relations between regions that are composed of sets of polygons (stored as lists of their edges). To the best of our knowledge, this is the first effort handling the aforementioned problem for the cardinal direction relations that can be expressed in [6, 21]. On the contrary, for other models of directions such algorithms do exist. For instance, Peuquet and Ci-Xiang [15] capture cardinal direction on polygons using points and MBB's approximations and present linear algorithms that compute the relative direction. Moreover, we present an implemented system, CarDirect that encapsulates the cardinal direction relations computation functionality in order to answer interesting user queries. The scenario for CarDirect usage is based on a simple scheme, where the user identifies and annotates interesting areas in an image or a map (possibly with the use of special segmentation software) and requires to retrieve regions that satisfy (spatial and thematic) criteria.

The technical contributions of this paper can be summarized as follows:

1. We present an algorithm for the efficient computation of cardinal direction relations. The proposed algorithm calculates the purely qualitative cardinal direction relations between the input regions and can be executed in linear time with respect to the number of input polygon's edges.
2. We complement the previous result with an algorithm for the linear computation of cardinal direction relations in a quantitative fashion with percentages. The computation is performed through a novel technique for the computation of areas of polygons.
3. We discuss the implementation of a tool, CarDirect that encapsulates the above algorithms. Using CarDirect the user can specify, edit and annotate regions of interest in an image or a map and compute the cardinal direction relations between these regions using the aforementioned linear algorithms. The configuration of the image (formed by the annotated regions and the calculated relations) are persistently stored using a simple XML description. Finally, the user is allowed to query the stored XML configuration of the image and retrieve combinations of interesting regions.

The rest of the paper is organized as follows. Section 2 presents the cardinal direction relations model. In Section 3, we present two algorithms for the problem of computing cardinal direction relations. Proofs of correctness and more details about the two algorithms can be found in the extended version of this paper [19]. Section 4 presents the CarDirect tool. Finally, Section 5 offers conclusions and lists topics of future research.

## 2 A Formal Model for Cardinal Direction Information

Cardinal direction relations, for various types of regions, have been defined in [5, 6, 20, 21]. Goyal and Egenhofer [5] first presented a set of cardinal direction relations for connected regions. Skiadopoulos and Koubarakis [20] formally define the above cardinal direction relations, propose composition algorithms and prove that these algorithm are correct. Moreover, Skiadopoulos and Koubarakis [21] presented an extension that handles disconnected regions and region with holes, and study the consistency problem for a given set of cardinal direction constraints. In this paper, we will start with the cardinal direction relations for the composite regions presented in [21] and then we will present an extension with percentages in the style of [5, 6].

We consider the Euclidean space $\Re^2$. *Regions* are defined as non-empty and bounded sets of points in $\Re^2$. Let $a$ be a region. The *infimum* (greatest lower bound) [9] of the *projection* of region $a$ on the $x$-axis (resp. $y$-axis) is denoted by $inf_x(a)$ (resp. $inf_y(a)$). The *supremum* (least upper bound) of the *projection* of region $a$ on the $x$-axis (resp. $y$-axis) is denoted by $sup_x(a)$ (resp. $sup_y(a)$). The *minimum bounding box* of a region $a$, denoted by $mbb(a)$, is the rectangular region formed by the straight lines $x = inf_x(a)$, $x = sup_x(a)$, $y = inf_y(a)$ and $y = sup_y(a)$ (see Fig. 1a). Obviously, the projections on the $x$-axis (resp. $y$-axis) of a region and its minimum bounding box have the same infimums and supremums.

Throughout this paper we will consider the following types of regions [20, 21]:

– Regions that are homeomorphic to the *closed unit disk* ($\{(x, y) : x^2 + y^2 \leq 1\}$). The set of these regions will be denoted by $REG$. Regions in $REG$ are *closed*, *connected* and have *connected boundaries* (for definitions of closeness and connectness see [9]). Class $REG$ excludes disconnected regions, regions with holes, points, lines and regions with emanating lines. Notice that our results are not affected if we consider regions that are homeomorphic to the *open unit disk* (as in [14]).
– Regions in $REG$ cannot model the variety and complexity of geographic entities [1]. Thus, we consider class $REG^*$, an extension of class $REG$, that accommodates *disconnected regions* and *regions with holes*. The set of these regions will be denoted by $REG^*$. Set $REG^*$ is a natural extension of $REG$. The regions that we consider are very common, for example, countries are made up of separations (islands, exclaves, external territories) and holes (enclaves) [1].

In Fig. 1, regions $a$, $b$ and $c$ are in $REG$ (also in $REG^*$) and region $d = d_1 \cup \cdots \cup d_8$ is in $REG^*$. Notice that region $d$ is disconnected and has a hole.

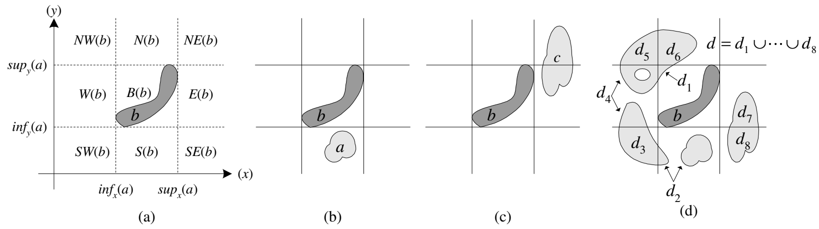


**Fig. 1.** Reference tiles and relations

Let us now consider two arbitrary regions $a$ and $b$ in $REG^*$. Let region $a$ be related to region $b$ through a cardinal direction relation (e.g., $a$ is north of $b$). Region $b$ will be called the *reference* region (i.e., the region which the relation refers to) while region $a$ will be called the *primary* region (i.e., the region for which the relation is introduced). The axes forming the minimum bounding box of the reference region $b$ divide the space into 9 areas which we call *tiles* (Fig. 1a). The peripheral tiles correspond to the eight cardinal direction relations *south*, *southwest*, *west*, *northwest*, *north*, *northeast*, *east* and *southeast*. These tiles will be denoted by $S(b)$, $SW(b)$, $W(b)$, $NW(b)$, $N(b)$, $NE(b)$, $E(b)$ and $SE(b)$ respectively. The central area corresponds to the region's minimum bounding box and is denoted by $B(b)$. By definition each one of these tiles includes the parts of the axes forming it. The union of all 9 tiles is $\Re^2$.

If a primary region $a$ is included (in the set-theoretic sense) in tile $S(b)$ of some reference region $b$ (Fig. 1b) then we say that *$a$ is south of $b$* and we write $a\ S\ b$. Similarly, we can define *southwest* ($SW$), *west* ($W$), *northwest* ($NW$), *north* ($N$), *northeast* ($NE$), *east* ($E$), *southeast* ($SE$) and *bounding box* ($B$) relations. If a primary region $a$ lies partly in the area $NE(b)$ and partly in the area $E(b)$ of some reference region $b$ (Fig. 1c) then we say that *$a$ is partly northeast and partly east of $b$* and we write $a\ NE{:}E\ b$.

The general definition of a cardinal direction relation in our framework is as follows.

**Definition 1.** *A* cardinal direction relation *is an expression* $R_1{:}\cdots{:}R_k$ *where (a)* $1 \leq k \leq 9$, *(b)* $R_1, \ldots, R_k \in \{B, S, SW, W, NW, N, NE, E, SE\}$ *and (c)* $R_i \neq R_j$ *for every* $i$, $j$ *such that* $1 \leq i, j \leq k$ *and* $i \neq j$. *A cardinal direction relation* $R_1{:}\cdots{:}R_k$ *is called* single-tile *if* $k = 1$; *otherwise it is called* multi-tile.

*Let $a$ and $b$ be two regions in $REG^*$. Single-tile cardinal direction relations are defined as follows:*

| | | |
|---|---|---|
| $a\ B\ b$ | *iff* | $inf_x(b) \leq inf_x(a)$, $sup_x(a) \leq sup_x(b)$, $inf_y(b) \leq inf_y(a)$ *and* $sup_y(a) \leq sup_y(b)$. |
| $a\ S\ b$ | *iff* | $sup_y(a) \leq inf_y(b)$, $inf_x(b) \leq inf_x(a)$ *and* $sup_x(a) \leq sup_x(b)$. |
| $a\ SW\ b$ | *iff* | $sup_x(a) \leq inf_x(b)$ *and* $sup_y(a) \leq inf_y(b)$. |
| $a\ W\ b$ | *iff* | $sup_x(a) \leq inf_x(b)$, $inf_y(b) \leq inf_y(a)$ *and* $sup_y(a) \leq sup_y(b)$. |
| $a\ NW\ b$ | *iff* | $sup_x(a) \leq inf_x(b)$ *and* $sup_y(b) \leq inf_y(a)$. |
| $a\ N\ b$ | *iff* | $sup_y(b) \leq inf_y(a)$, $inf_x(b) \leq inf_x(a)$ *and* $sup_x(a) \leq sup_x(b)$. |

$a \; NE \; b$ *iff* $sup_x(b) \le inf_x(a)$ *and* $sup_y(b) \le inf_y(a)$.

$a \; E \; b$ *iff* $sup_x(b) \le inf_x(a)$, $inf_y(b) \le inf_y(a)$ *and* $sup_y(a) \le sup_y(b)$.

$a \; SE \; b$ *iff* $sup_x(b) \le inf_x(a)$ *and* $sup_y(a) \le inf_y(b)$.

*In general, each multi-tile* ($2 \le k \le 8$) *relation is defined as follows:*

$$a \; R_1{:}\cdots{:}R_k \; b \text{ iff there exist regions } a_1, \ldots, a_k \in REG^* \text{ such that } a_1 \; R_1 \; b, \ldots, a_k \; R_k \; b \text{ and } a = a_1 \cup \cdots \cup a_k.$$

In Definition 1 notice that for every $i$, $j$ such that $1 \le i, j \le k$ and $i \ne j$, $a_i$ and $a_j$ have disjoint interiors but may share points in their boundaries.

*Example 1.* $S$, $NE{:}E$ and $B{:}S{:}SW{:}W{:}NW{:}N{:}E{:}SE$ are cardinal direction relations. The first relation is single-tile while the others are multi-tile. In Fig. 1, we have $a \; S \; b$, $c \; NE{:}E \; b$ and $d \; B{:}S{:}SW{:}W{:}NW{:}N{:}E{:}SE \; b$. For instance in Fig. 1d, we have $d \; B{:}S{:}SW{:}W{:}NW{:}N{:}SE{:}E \; b$ because there exist regions $d_1, \ldots, d_8$ in $REG^*$ such that $d = d_1 \cup \cdots \cup d_8$, $d_1 \; B \; b$, $d_2 \; S \; b$, $d_3 \; SW \; b$, $d_4 \; W \; b$, $d_5 \; NW \; b$, $d_6 \; N \; b$, $d_7 \; SE \; b$ and $d_8 \; E \; b$.

In order to avoid confusion, we will write the single-tile elements of a cardinal direction relation according to the following order: $B$, $S$, $SW$, $W$, $NW$, $N$, $NE$, $E$ and $SE$. Thus, we always write $B{:}S{:}W$ instead of $W{:}B{:}S$ or $S{:}B{:}W$. Moreover, for a relation such as $B{:}S{:}W$ we will often refer to $B$, $S$ and $W$ as its *tiles*.

The set of cardinal direction relations for regions in $REG^*$ is denoted by $\mathcal{D}^*$. Relations in $\mathcal{D}^*$ are jointly exhaustive and pairwise disjoint, and can be used to represent *definite information* about cardinal directions, e.g., $a \; N \; b$. Using the relations of $\mathcal{D}^*$ as our basis, we can define the *powerset* $2^{\mathcal{D}^*}$ of $\mathcal{D}^*$ which contains $2^{511}$ relations. Elements of $2^{\mathcal{D}^*}$ are called *disjunctive cardinal direction relations* and can be used to represent not only definite but also *indefinite information* about cardinal directions, e.g., $a \; \{N, W\} \; b$ denotes that region $a$ is north *or* west of region $b$.

Notice that the inverse of a cardinal direction relation $R$, denoted by $inv(R)$, is not always a cardinal direction relation but, in general, it is a disjunctive cardinal direction relation. For instance, if $a \; S \; b$ then it is possible that $b \; NE{:}N{:}NW \; a$ or $b \; NE{:}S \; a$ or $b \; N{:}NW \; a$ or $b \; N \; a$. Specifically, the relative position of two regions $a$ and $b$ is fully characterized by the pair $(R_1, R_2)$, where $R_1$ and $R_2$ are cardinal directions such that (a) $a \; R_1 \; b$, (b) $b \; R_2 \; a$, (c) $R_1$ is a disjunct of $inv(R_2)$ and (d) $R_2$ is a disjunct of $inv(R_1)$. An algorithm for computing the inverse relation is discussed in [21]. Moreover, algorithms that calculate the composition of two cardinal direction relations and the consistency of a set of cardinal direction constraints are discussed in [20–22].

Goyal and Egenhofer [5,6] use *direction relation matrices* to represent cardinal direction relations. Given a cardinal direction relation $R = R_1{:}\cdots{:}R_k$ the cardinal direction relation matrix that corresponds to $R$ is a 3×3 matrix defined as follows:

$$R = \begin{bmatrix} P_{NW} & P_N & P_{NE} \\ P_W & P_B & P_E \\ P_{SW} & P_S & P_{SE} \end{bmatrix} \quad \text{where} \quad P_{dir} = \begin{cases} \square \text{ if } dir \notin \{R_1, \ldots, R_k\} \\ \blacksquare \text{ if } dir \in \{R_1, \ldots, R_k\} \end{cases}.$$

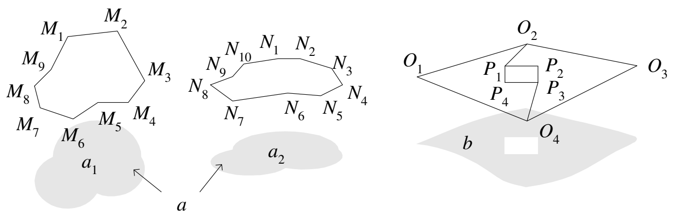


**Fig. 2.** Using polygons to represent regions

For instance, the direction relation matrices that correspond to relations $S$, $NE{:}E$ and $B{:}S{:}SW{:}W{:}NW{:}N{:}E{:}SE$ of Example 1 are as follows:

$$S = \begin{bmatrix} \square\square\square \\ \square\square\square \\ \square\blacksquare\square \end{bmatrix},\ NE{:}E = \begin{bmatrix} \square\blacksquare\blacksquare \\ \square\square\square \\ \square\square\square \end{bmatrix} \text{ and } B{:}S{:}SW{:}W{:}NW{:}N{:}E{:}SE = \begin{bmatrix} \blacksquare\blacksquare\square \\ \blacksquare\blacksquare\blacksquare \\ \blacksquare\blacksquare\blacksquare \end{bmatrix}.$$

At a finer level of granularity, the model of [5, 6] also offers the option to record how much of the a region falls into each tile. Such relations are called *cardinal direction relations with percentages* and can be represented with *cardinal direction matrices with percentages*. Let $a$ and $b$ be two regions in $REG^*$. The cardinal direction matrices with percentages can be defined as follows:

$$a \quad \frac{100\%}{area(a)} \cdot \begin{bmatrix} area(NW(b)\cap a) & area(N(b)\cap a) & area(NE(b)\cap a) \\ area(W(b)\cap a) & area(B(b)\cap a) & area(E(b)\cap a) \\ area(SW(b)\cap a) & area(S(b)\cap a) & area(SE(b)\cap a) \end{bmatrix} \quad b$$

where $area(r)$ denotes the area of region $r$.

Consider for example regions $c$ and $b$ in Fig. 1c; region $a$ is 50% northeast and 50% east of region $b$. This relation is captured with the following cardinal direction matrix with percentages.

$$c \begin{bmatrix} 0\% & 0\% & 50\% \\ 0\% & 0\% & 50\% \\ 0\% & 0\% & 0\% \end{bmatrix} b$$

In this paper, we will use simple assertions (e.g., $S$, $B{:}S{:}SW$) to capture cardinal direction relations [20, 21] and direction relations matrices to capture cardinal direction relations with percentages [5, 6].

## 3 Computing cardinal direction relations

Typically, in Geographical Information Systems and Spatial Databases, the connected regions in $REG$ are represented using single polygons, while the composite regions in $REG^*$ are represented using sets of polygons [18, 23]. In this paper, the edges of polygons are taken in a clockwise order. For instance, in Fig. 2 region $a_1 \in REG$ is represented using polygon $(M_1 M_2 \cdots M_9)$ and region $a = a_1 \cup a_2 \in REG^*$ is represented using polygons $(M_1 M_2 \cdots M_9)$ and

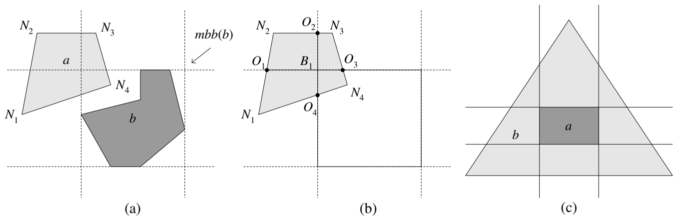


**Fig. 3.** Polygon clipping

$(N_1 N_2 \cdots N_{10})$. Notice than using sets of polygons, we can even represent regions with holes. For instance, in Fig. 2 region $b \in REG^*$ is represented using polygons $(O_2O_3O_4P_3P_2P_1)$ and $(O_1O_2P_1P_4P_3O_4)$.

Given the polygon representations of a primary region $a$ and a reference region $b$, the *computation of cardinal direction relations problem* lies in the calculation of the cardinal direction relation $R$, such that $a\ R\ b$ holds. Similarly, we can define the *computation of cardinal direction relations with percentages problem*.

Let us consider a primary region $a$ and a reference region $b$. According to Definition 1, in order to calculate the cardinal direction relation between region $a$ and $b$, we have to divide the primary region $a$ into segments such that each segment falls exactly into one tile of $b$. Furthermore, in order to calculate the cardinal direction relation with percentages we also have to measure the area of each segment. Segmenting polygons using *bounded boxes* is a well-studied topic of Computational Geometry called *polygon clipping* [7, 10]. A polygon clipping algorithm can be extended to handle *unbounded boxes* (such as the tiles of reference region $b$) as well. Since polygon clipping algorithms are very efficient (linear in the number of polygon edges), someone would be tempted to use them for the calculation of cardinal direction relations and cardinal direction relations with percentages. Let us briefly discuss the disadvantages of such an approach.

Let us consider regions $a$ and $b$ presented in Fig. 3a. Region $a$ is formed by a quadrangle (i.e., a total of 4 edges). To achieve the desired segmentation, polygon clipping algorithms introduce to $a$ new edges [7, 10]. After the clipping algorithms are performed (Fig. 3b), region $a$ is formed by 4 quadrangles (i.e., a total of 16 edges). The worst case that we can think (illustrated in Fig. 3c) starts with 3 edges (a triangle) and ends with 35 edges (2 triangles, 6 quadrangles and 1 pentagon). These new edges are only used for the calculation of cardinal direction relations and are discarded afterwards. Thus, it would be important to minimize their number. Moreover, in order to perform the clipping the edges of the primary region $a$ must be scanned 9 times (one time for every tile of the reference region $b$). In real GIS applications, we expect that the average number of edges is high. Thus, each scan of the edges of a polygon can be quite time consuming. Finally, polygon clipping algorithms sometimes require complex floating point operations which are costly.

In Sections 3.1 and 3.2, we consider the problem of calculating cardinal direction relations and cardinal direction relations with percentages respectively. We provide algorithms specifically tailored for this task, which avoid the drawbacks

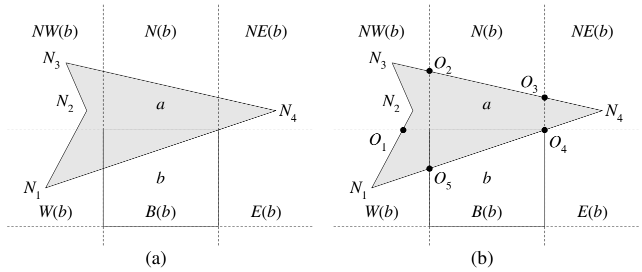


**Fig. 4.** Illustration of Examples 2 and 3

of polygon clipping methods. Our proposal does not segment polygons; instead it only divides some of the polygon edges. In Example 2, we show that such a division is necessary for the correct calculation. Interestingly, the resulting number of introduced edges is significantly smaller than the respective number of polygon clipping methods. Furthermore, the complexity of our algorithms is not only linear in the number of polygon edges but it can be performed with a single pass. Finally, our algorithms use simple arithmetic operations and comparisons.

### 3.1 Cardinal direction relations

We will start by considering the calculation of cardinal constraints relations problem. First, we need the following definition.

**Definition 2.** *Let $R_1, \ldots, R_k$ be basic cardinal direction relations. The* tile-union *of $R_1, \ldots, R_k$, denoted by tile-union$(R_1, \ldots, R_k)$, is a relation formed from the union of the tiles of $R_1, \ldots, R_k$.*

For instance, if $R_1 = S{:}SW$, $R_2 = S{:}E{:}SE$ and $R_3 = W$ then we have *tile-union*$(R_1, R_2) = S{:}SW{:}E{:}SE$ and *tile-union*$(R_1, R_2, R_3) = S{:}SW{:}W{:}E{:}SE$.

Let $S_a = \{p_1, \ldots, p_k\}$ and $S_b = \{q_1, \ldots, q_l\}$ be sets of polygons representing a primary region $a$ and a reference region $b$. To calculate the cardinal direction $R$ between the primary region $a$ and the reference region $b$, we first record the tiles of region $b$ where the points forming the edges of the polygons $p_1, \ldots, p_k$ fall in. Unfortunately, as the following example presents, this is not enough.

*Example 2.* Let us consider the region $a$ (formed by the single polygon $(N_1 N_2 N_3 N_4)$) and the region $b$ presented in Fig. 4a. Clearly points $N_1$, $N_2$, $N_3$ and $N_4$ lie in $W(b)$, $NW(b)$, $NW(b)$ and $NE(b)$ respectively, but the relation between $p$ and $b$ is $B{:}W{:}NW{:}N{:}NE$ and not $W{:}NW{:}NE$.

The problem of Example 2 arises because there exist edges of polygon $(N_1 N_2 N_3 N_4)$ that expand over three tiles of the reference region $b$. For instance, $N_3 N_4$ expands over tiles $NW(b)$, $N(b)$ and $NE(b)$. In order to handle such situations, we use the lines forming the minimum bounding box of the reference region $b$ to divide the edges of the polygons representing the primary region $a$ and create new edges such that (a) region $a$ does not change and (b) every new edge lies in exactly one tile. To this end, for every edge $AB$ of region $a$, we compute the set of intersection points $\mathcal{I}$ of $AB$ with the lines forming box $b$. We use the intersection points of $\mathcal{I}$ to divide $AB$ into a number of segments $\mathcal{S} = AO_1, \ldots, O_kB$. Each segment $AO_1, \ldots, O_kB$ lies in exactly one tile of $b$ and the union of all tiles is $AB$. Thus, we can safely replace edge $AB$ with $AO_1, \ldots O_kB$ without affecting region $a$. Finally, to compute the cardinal direction between regions $a$ and $b$ we only have to record the tile of $b$ where each new segment lies. Choosing a single point from each segment is sufficient for this purpose; we choose to pick the middle of the segment as a representative point. Thus, the tile where the middle point lies gives us the tile of the segment too. The above procedure is captured in Algorithm Compute-CDR (Fig. 5) and is illustrated in the following example.

*Example 3.* Let us continue with the regions of Example 2 (see also Fig. 4). Algorithm Compute-CDR considers every edge of region $a$ (polygon $(N_1N_2N_3N_4)$) in turn and performs the replacements presented in the following table.

| Edge | Replace with (new edges) | Edge | Replace with (new edges) |
|---|---|---|---|
| $N_1N_2$ | $N_1O_1,\ O_1N_2$ | $N_2N_3$ | $N_2N_3$ |
| $N_3N_4$ | $N_3O_2,\ O_2O_3,\ O_3N_4$ | $N_4N_1$ | $N_3O_4,\ O_4O_5,\ O_5N_4$ |

It easy to verify that every new edge lies in exactly one tile of $b$ (Fig. 4b). The middle points of the new edges lie in $B(b)$, $W(b)$, $NW(b)$, $N(b)$, $NE(b)$ and $E(b)$. Therefore, Algorithm Compute-CDR returns $B{:}W{:}NW{:}N{:}NE{:}E$, which precisely captures the cardinal direction relation between regions $a$ and $b$.

Notice that in Example 3, Algorithm Compute-CDR takes as input a quadrangle (4 edges) and returns 9 edges. This should be contrasted with the polygon clipping method that would have resulted in 19 edges (2 triangles, 2 quadrangles and 1 pentagon). Similarly, for the shapes in Fig. 3b-c, Algorithm Compute-CDR introduces 8 and 11 edges respectively while polygon clipping methods introduce 16 and 34 edges respectively.

The following theorem captures the correctness of Algorithm Compute-CDR and measures its complexity.

**Theorem 1.** *Algorithm* Compute-CDR *is correct, i.e., it returns the cardinal direction relation between two regions $a$ and $b$ in $REG^*$ that are represented using two sets of polygons $S_a$ and $S_b$ respectively. The running time of Algorithm* Compute-CDR *is $\mathcal{O}(k_a + k_b)$ where $k_a$ (respectively $k_b$) is the total number of edges of all polygons in $S_a$ (respectively $S_b$).*

Summarizing this section, we can use Algorithm Compute-CDR to compute the cardinal direction relation between two sets of polygons representing two regions $a$ and $b$ in $REG^*$. The following section considers the case of cardinal direction relations with percentages.

### 3.2 Cardinal direction relations with percentages

In order to compute cardinal direction relations with percentages, we have to calculate the area of the primary region that falls in each tile of the reference

**Algorithm** COMPUTE-CDR
Input: Two sets of polygons $S_a$ and $S_b$ representing regions $a$ and $b$ in $REG^*$.
Output: The cardinal direction relation $R$ such that $a\ R\ b$ holds.
Method:
Use $S_b$ to compute the minimum bounding box $mbb(b)$ of $b$.
Let $R$ be the empty relation.
For every polygon $p$ of $S_a$
    For every edge $AB$ of polygon $p$
        Let $\mathcal{I}$ be the set of intersection points of $AB$ with the lines forming box $b$.
        Let $\mathcal{S} = AO_1, \ldots O_k B$ be the segments that points in $\mathcal{I}$ divide $AB$.
        Replace $AB$ with $AO_1, \ldots O_k B$ in the representation of polygon $p$.
        Let $T_1, \ldots, T_k$ be the tiles of $b$ in which the middle points of
        edges $AO_1, \ldots O_k B$ lie.
        $R = \textit{tile-union}(\ R,\ T_1, \ldots, T_k\ )$
    EndFor
    If the center of $mbb(b)$ is in $p$ Then $R = \textit{tile-union}(\ R,\ B\ )$
EndFor
Return $R$

**Fig. 5.** Algorithm COMPUTE-CDR

region. A naive way for this task is to segment the polygons that form the primary region so that every polygon lies in exactly one tile of the reference region. Then, for each tile of the reference region we find the polygons of the primary region that lie inside it and compute their area. In this section, we will propose an alternative method that is based on Algorithm COMPUTE-CDR. This method simply computes the area between the edges of the polygons that represent the primary region and an appropriate reference line without segmenting these polygons.

We will now present a method to compute the area between a line and an edge. Then, we will see how we can extend this method to compute the area of a polygon. We will first need the following definition.

**Definition 3.** *Let $AB$ be an edge and $e$ be a line. We say that $e$* does not cross *$AB$ if and only if one of the following holds: (a) $AB$ and $e$ do not intersect, (b) $AB$ and $e$ intersect only at point $A$ or $B$, or (c) $AB$ completely lies on $e$.*

For example, in Fig. 6 lines $e_1$, $e_1$ and $e_3$ do not cross edge $AB$. Let us now calculate the area between an edge and a line.

**Definition 4.** *Let $A(x_A, y_A)$ and $B(x_B, y_B)$ be two points forming edge $AB$, $y = l$ and $x = m$ be two lines that do not cross $AB$. Let also $L_A$ and $L_B$ (respectively $M_A$ and $M_B$) be the projections of points $A$,$B$ to line $y = l$ (respectively $x = m$) – see also Fig. 7. We define expression $E_l(AB)$ and $E'_m(AB)$ as follows:*

$$E_l(AB) = \frac{(x_B - x_A)(y_A + y_B - 2l)}{2} \text{ and } E'_m(AB) = \frac{(y_B - y_A)(x_A + x_B - 2l)}{2}.$$

Expressions $E_l(AB)$ and $E'_m(AB)$ can be positive or negative depending on the direction of vector $\overrightarrow{AB}$. It is easy to verify that $E_l(AB) = -E_l(BA)$ and

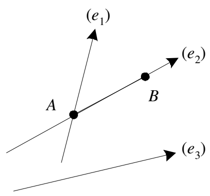


**Fig. 6.** Lines not crossing $AB$

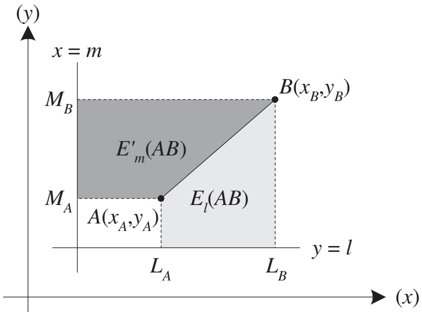


**Fig. 7.** Area between an edge and a line

$E'_m(AB) = -E'_m(BA)$ holds. The absolute value of $E_l(AB)$ equals to the area between edge $AB$ and line $y = l$, i.e., the area of polygon $(ABL_BL_A)$. In other words, the following formula holds.

$$area(\ (ABL_BL_A)\ ) = E_l(AB) = \frac{(x_B - x_A)(y_A + y_B - 2l)}{2}$$

Symmetrically, area between edge $AB$ and line $x = m$, i.e., the area of polygon $(ABM_BM_A)$, equals to the absolute value of $E'_m(AB)$.

$$area(\ (AM_AM_BB)\ ) = E'_m(AB) = \frac{(y_B - y_A)(x_A + x_B - 2l)}{2}$$

Expressions $E_l$ and $E'_m$ can be used to calculate the area of polygons. Let $p = (N_1 \cdots N_k)$ be a polygon, and $y = l$, $x = m$ be two lines that do not cross with any edge of polygon $p$. The area of polygon $p$, denoted by $area(p)$, can be calculated as follows:

$$area(p) = |\ E_l(N_1N_2) + \cdots + E_l(N_kN_1)\ | = |\ E'_m(N_1N_2) + \cdots + E'_m(N_kN_1)\ |.$$

Notice that Computational Geometry algorithms, in order to calculate the area of a polygon $p$ use a similar method that is based on a reference point (instead of a line) [12, 16]. This method is not appropriate for our case because it requires to segment the primary region using polygon clipping algorithms (see also the discussion at the beginning of Section 3). In the rest of this section, we will present a method that utilizes expressions $E_l$ and $E'_m$ and does not require polygon clipping.

*Example 4.* Let us consider polygon $p = (N_1N_2N_3N_4)$ and line $y = l$ presented in Fig. 8d. The area of polygon $p$ can be calculated using formula $area(p) = |\ E_l(N_1N_2) + E_l(N_2N_3) + E_l(N_3N_4) + E_l(N_4N_1)\ |$. All the intermediate expressions $E_l(N_1N_2)$, $E_l(N_1N_2)+E_l(N_2N_3)$, $E_l(N_1N_2)+E_l(N_2N_3)+E_l(N_3N_4)$, $E_l(N_1N_2) + E_l(N_2N_3) + E_l(N_3N_4) + E_l(N_4N_1)$ are presented as the gray areas of Fig. 8a-d respectively.

We will use expressions $E_l$ and $E_m$ to compute the percentage of the area of the primary region that falls in each tile of the reference region. Let us consider

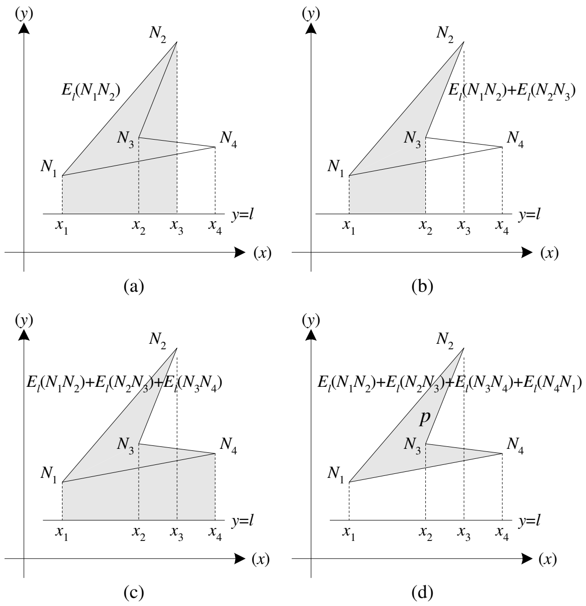


**Fig. 8.** Using expression $E_l$ to calculate the area of a polygon

region $a$ presented Fig. 9. Region $a$ is formed by polygons $(N_1N_2N_3N_4)$ and $(M_1M_2M_3)$. Similarly to Algorithm COMPUTE-CDR, to compute the cardinal direction relation with percentages of $a$ with $b$ we first use the $mbb(b)$ to divide the edges of region $a$. Let $x = m_1$, $x = m_2$, $y = l_1$ and $y = l_2$ be the lines forming $mbb(b)$. These lines divide the edges of polygons $(N_1N_2N_3N_4)$ and $(M_1M_2M_3)$ as shown in Fig. 9.

Let us now compute the area of $a$ that lies in the $NW$ tile of $b$ (i.e., $area(NW(b) \cap a)$). Notice that $area(NW(b) \cap a) = area((O_1N_2O_2B_1))$. To compute the area of polygon $(O_1N_2O_2B_1)$ it is convenient to use as a reference line $x = m_1$. Doing so, we do not have to compute edges $B_1O_1$ and $O_2B_1$ because $E'_{m_1}(B_1O_1) = 0$ and $E'_{m_1}(O_2B_1) = 0$ hold and thus the area we are looking for can be calculated with the following formula:

$$area(NW(b) \cap a) = area((O_1N_2O_2B_1)) = \mid E'_{m_1}(O_1N_2) + E'_{m_1}(N_2O_2) \mid$$

In other words, to compute the area of $a$ that lies in $NW(b)$ ($area(NW(b) \cap a)$) we calculate the area between the west line of $mbb(b)$ ($x = m_1$) and every edge of $a$ that lies in $NW(b)$, i.e., the following formula holds:

$$area(NW(b) \cap a) = \mid \sum_{AB \in NW(b)} E'_{m_1}(AB) \mid.$$

Similarly, to calculate the area of $a$ that lies in the $W(b)$ and $SW(b)$ we can use the expressions:

$$area(W(b) \cap a) = \mid \sum_{AB \in W(b)} E'_{m_1}(AB) \mid, \; area(SW(b) \cap a) = \mid \sum_{AB \in SW(b)} E'_{m_1}(AB) \mid$$

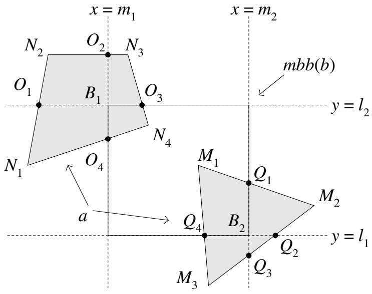


**Fig. 9.** Computing cardinal direction relations with percentages

For instance, in Fig. 9 we have $area(W(b) \cap a) = |E'_{m_1}(N_1O_1) + E'_{m_1}(O_1B_1)|$ and $area(SW(b) \cap a) = 0$

To calculate the area of $a$ that lies in $NE(b)$, $E(b)$, $SE(b)$, $S(b)$ and $N(b)$ we simply have to change the line of reference that we use. In the first three cases, we use the east line of $mbb(b)$ (i.e., $x = m_2$ in Fig. 9), in the fourth case, we use the south line of $mbb(b)$ ($y = l_1$) and in the last case, we use the north line of $mbb(b)$ ($y = l_2$). In all cases, we use the edges of $a$ that fall in the tile of $b$ that we are interested in. Thus, have:

$$\begin{array}{ll} area(T(b) \cap a) = \mid \sum_{AB \in T(b)} E'_{m_2}(AB) \mid & \text{if} \quad T \in \{NE, E, SE\} \\ area(S(b) \cap a) = \mid \sum_{AB \in S(b)} E_{l_1}(AB) \mid & \\ area(N(b) \cap a) = \mid \sum_{AB \in N(b)} E_{l_2}(AB) \mid & \end{array} .$$

For instance, in Fig. 9 we have $area(N(b) \cap a) = \mid E_{l_2}(O_2N_3) + E'_{m_1}(N_3O_3) \mid$ and $area(W(b) \cap a) = \mid E'_{m_2}(Q_1M_2) + E'_{m_2}(M_2Q_2) \mid$.

Let us now consider the area of $a$ that lies in $B(b)$. None of the lines of $mbb(b)$ can help us compute $area(B(b) \cap a)$ without segmenting the polygons that represent region $a$. For instance, in Fig. 9 using line $y = l_1$ we have:

$$\begin{array}{l} area(B(b) \cap a) = \mid E_{l_1}(Q_4M_1) + E_{l_1}(M_1Q_4) + E_{l_1}(O_4N_4) + E_{l_1}(N_4O_4) + \\ \qquad E_{l_1}(O_4B_1) + E_{l_1}(B_1O_3) \mid . \end{array}$$

Edge $B_1O_3$ is not an edge of any of the polygons representing $a$. To handle such situations, we employ the following method. We use the south line of $mbb(b)$ ($y = l_1$) as the reference line and calculate the areas between $y = l_1$ and all edges that lie *both* in $N(b)$ and $B(b)$. This area will be denoted by $area((B+N)(b) \cap a)$ and is practically the area of $a$ that lies on $N(b)$ and $B(b)$, i.e., $area((B+N)(b) \cap a) = area(N(b) \cap a) + area(B(b) \cap a)$. Since $area(N(b) \cap a)$ has been previously computed, we just have to subtract it from $area((B + N)(b) \cap a)$ in order to derive $area(B(b) \cap a)$. For instance, in Fig. 9 we have:

$$\begin{array}{l} area((B + N)(b) \cap a) = \mid \sum_{AB \in B(b) \cup N(b)} E_{l_1}(AB) \mid = \\ \quad \mid E_{l_1}(O_2N_3) + E_{l_1}(O_3N_4) + E_{l_1}(N_4O_4) + E_{l_1}(Q_4M_1) + E_{l_1}(M_1Q_4) \mid = \\ \quad area(\ (O_2N_3O_3N_4O_4) + (M_1Q_1B_2Q_4)\ ) \end{array}$$

**Algorithm** COMPUTE-CDR%

Input: Two sets of polygons $S_a$ and $S_b$ representing regions $a$ and $b$ in $REG^*$ resp.

Output: The cardinal direction relation with percentages $R$, such that $a\ R\ b$ holds.

Method:

Use $S_b$ to compute the minimum bounding box $mbb(b)$ of $b$.

Divide all the edges in $S_a$ so that every new edge lies in exactly one tile of $b$.

Let $y = l_1$, $y = l_2$, $x = m_1$ and $x = m_2$ be the lines forming $mbb(b)$.

$a_{B+N} = a_S = a_{SW} = a_W = a_{NW} = a_N = a_{NE} = a_E = a_{SE} = 0$

<u>For</u> every edge $AB$ of $S_a$

  Let $t$ be the tile of $b$ that $AB$ falls in.

  <u>Case</u> $t$  // Expressions $E'$ and $E$ are defined in Definition 4

    $NW$, $W$, $SW$: $a_t = a_t + E'_{m_1}(AB)$

    $NE$, $E$, $SE$: $a_t = a_t + E'_{m_1}(AB)$

    $S$: $a_t = a_t + E_{l_1}(AB)$

    $N$: $a_t = a_t + E_{l_2}(AB)$

  <u>EndCase</u>

  <u>If</u> $(t = N)$ or $(t = B)$ <u>Then</u> $a_{B+N} = a_{B+N} + E_{l_1}(AB)$

<u>EndFor</u>

$a_B = |a_{B+N}| - |a_N|$

$totalArea = |a_B| + |a_S| + |a_{SW}| + |a_W| + |a_{NW}| + |a_N| + |a_{NE}| + |a_E| + |a_{SE}|$

<u>Return</u> $\frac{100\%}{totalArea} \cdot \begin{bmatrix} |a_{NW}| & |a_N| & |a_{NE}| \\ |a_S| & |a_B| & |a_E| \\ |a_{SW}| & |a_S| & |a_{SE}| \end{bmatrix}$

**Fig. 10.** Algorithm COMPUTE-CDR%

and

$$area(N(b) \cap a) = |\sum_{AB \in N(b)} E_{l_2}(AB)| = area((O_2 N_3 O_3 N_4 O_4)).$$

Therefore, $area(B(b) \cap a) = area((B + N)(b) \cap a) - area(N(b) \cap a)$ holds.

The above described method is summarized in Algorithm COMPUTE-CDR% presented in Fig. 10. The following theorem captures the correctness of Algorithm COMPUTE-CDR% and measures its complexity.

**Theorem 2.** *Algorithm* COMPUTE-CDR% *is correct, i.e., it returns the cardinal direction relation with percentages between two regions $a$ and $b$ in $REG^*$ that are represented using two sets of polygons $S_a$ and $S_b$ respectively. The running time of Algorithm* COMPUTE-CDR% *is $\mathcal{O}(k_a + k_b)$ where $k_a$ (respectively $k_b$) is the total number of edges of all polygons in $S_a$ (respectively $S_b$).*

In the following section, we will present an actual system, CARDIRECT, that incorporates and implements Algorithms COMPUTE-CDR and COMPUTE-CDR%.

## 4 A tool for the manipulation of cardinal direction information

In this section, we will present a tool that implements the aforementioned reasoning tasks for the computation of cardinal direction relationships among regions.

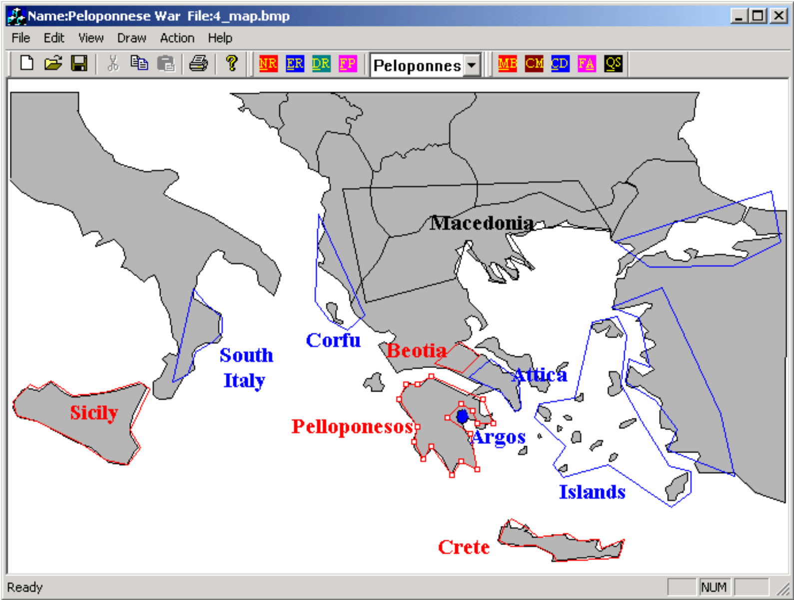


**Fig. 11.** Using CarDirect to annotate images

The tool, CarDirect, has been implemented in C++ over the Microsoft Visual Studio toolkit. Using CarDirect, the user can define regions of interest over some underlying image (e.g., a map), compute their relationships (with and without percentages) and pose queries. The tool implements an XML interface, through which the user can import and export the configuration he constructs (i.e., the underlying image and the sets of polygons that form the regions); the XML description of the configuration is also employed for querying purposes.

The XML description of the exported scenarios is quite simple: A configuration (`Image`) is defined upon an image file (e.g., a map) and comprises a set of regions and a set of relations among them. Each region comprises a set of polygons of the same color and each polygon comprises a set of edges (defined by $x$- and $y$-coordinates). The direction relations among the different regions are all stored in the XML description of the configuration. The DTD for CarDirect configurations is as follows.

```
<?xml version="1.0" encoding="UTF-8"?>
<!ELEMENT Image (Region+, Relation*)>
<!ATTLIST Image name CDATA #IMPLIED file CDATA #IMPLIED>
<!ELEMENT Region (Polygon*)>
<!ATTLIST Region id ID #REQUIRED name CDATA #IMPLIED color CDATA #IMPLIED>
<!ELEMENT Polygon (Edge, Edge, Edge, Edge*)>
<!ATTLIST Polygon id CDATA #REQUIRED>
<!ELEMENT Edge EMPTY>
<!ATTLIST Edge x CDATA #REQUIRED y CDATA #REQUIRED>
<!ELEMENT Relation EMPTY>
<!ATTLIST Relation type CDATA #REQUIRED
          primary IDREF #REQUIRED reference IDREF #REQUIRED>
```

Observe Fig. 11. In this configuration, the user has opened a map of Ancient Greece at the time of the Peloponnesian war as the underlying image. Then, the

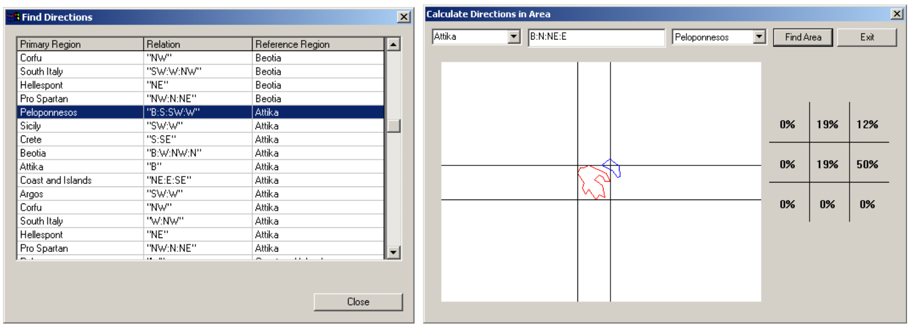


**Fig. 12.** Using CarDirect to extract cardinal direction relations

user defined three sets of regions: the "Athenean Alliance" in blue, comprising of Attica, the Islands, the regions in the East, Corfu and South Italy; (b) the "Spartan Alliance" in red, comprising of Peloponnesos, Beotia, Crete and Sicely; and (c) the "Pro-Spartan" in black, comprising of Macedonia.

Moreover, using CarDirect, the user can compute the cardinal direction relations and the cardinal direction relations with percentages between the identified regions. In Fig. 12, we have calculated the relations between the regions of Fig. 11. For instance, Peloponnesos is $B{:}S{:}SW{:}W$ of Attica (left side of Fig. 12) while Attica is

$$\begin{bmatrix} 0\% & 19\% & 12\% \\ 0\% & 19\% & 50\% \\ 0\% & 0\% & 0\% \end{bmatrix}$$

of Peloponnesos (right-hand side of Fig. 12).

The query language that we employ is based on the following simple model. Let $A = \{a_1, \ldots, a_n\}$ be a set of regions in $REG^*$ over a configuration. Let $C$ be a finite set of thematic attributes for the regions of $REG^*$ (e.g., the color of each region) and $f$ a function, $f : REG^* \rightarrow dom(C)$, practically relating each of the regions with a value over the domain of $C$ (e.g., the fact that the Spartan Alliance is colored red).

A *query condition* over variables $x_1, \ldots, x_k$ is a conjunction the following types of formulae $x_i = a$, $f(x_i) = c$, $x_i \ R \ x_j$ where $1 \leq i, j \leq n$, $a \in A$ is a region of the configuration, $c \in dom(C)$ is a value of a thematic attribute and $R \in 2^{\mathcal{D}^*}$ is a (possibly disjunctive) cardinal direction relation. A *query* $q$ over variables $x_1, \ldots, x_n$ is a formula of the form

$$q = \{(x_1, \ldots, x_n) \mid \phi(x_1, \ldots, x_n)\}$$

where $\phi(x_1, \ldots, x_n)$ is a query condition.

Intuitively, the query returns a set of regions in the configuration of an image that satisfy the query condition, which can take the form of: (a) a cardinal direction constraint between the query variables (e.g., $x_1 \ B{:}SE{:}S \ x_2$); (b) a restriction in the thematic attributes of a variable (e.g., $color(x_1) = blue$) and (c) direct reference of a particular region (e.g., $x_1 = Attica$).

For instance, for the configuration of Fig. 11 we can pose the following query: "Find all regions of the Athenean Alliance which are surrounded by a region in the Spartan Alliance". This query can be expressed as follows:

$$q = \{(a, b) \mid color(a) = red,\ color(b) = blue,\ a\ S{:}SW{:}W{:}NW{:}N{:}NE{:}E{:}SE\ b\}$$

## 5 Conclusions and future work

In this paper, we have addressed the problem of efficiently computing the cardinal direction relations between regions that are composed of sets of polygons (a) by presenting two linear algorithms for this task, and (b) by explaining their incorporation into an actual system. These algorithms take as inputs two sets of polygons representing two regions respectively. The first of the proposed algorithms is purely qualitative and computes the cardinal direction relations between the input regions. The second has a quantitative aspect and computes the cardinal direction relations with percentages between the input regions. To the best of our knowledge, these are the first algorithms that address the aforementioned problem. The algorithms have been implemented and embedded in an actual system, CarDirect, which allows the user to specify, edit and annotate regions of interest in an image. Then, CarDirect automatically computes the cardinal direction relations between these regions. The configuration of the image and the introduced regions is persistently stored using a simple XML description. The user is allowed to query the stored XML description of the image and retrieve combinations of interesting regions on the basis of the query.

Although this part of our research addresses the problem of relation computation to a sufficient extent, there are still open issues for future research. First, we would like to evaluate experimentally our algorithm against polygon clipping methods. A second interesting topic is the possibility of combining topological [2] and distance relations [3]. Another issue is the possibility of combining the underlying model with extra thematic information and the enrichment of the employed query language on the basis of this combination. Finally, a long term goal would be the integration of CarDirect with image segmentation software, which would provide a complete environment for the management of image configurations.

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