1.C

Nuclear Physics A334 (1980) 217–228; © *North-Holland Publishing Co., Amsterdam*

VARIATIONAL CALCULATIONS OF v_8 MODELS OF NUCLEAR MATTER †

I. E. LAGARIS and V. R. PANDHARIPANDE

Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801

Received 15 August 1979

Abstract: We report variational calculations of v_8 models of nuclear matter which contain central, spin, isospin, tensor and spin-orbit potentials. These semi-realistic models can explain the nucleon-nucleon scattering in $^1S_0$, $^3S_1$–$^3D_1$, $^1P_1$ and $^3P_2$–$^3F_2$ states up to ~ 300 MeV. The variational wave function has two-body central, spin, isospin, tensor and spin-orbit correlations. The terms in the cluster expansion of the energy expectation value, that do not contain the spin-orbit correlations are summed by chain summation techniques developed for the v_6 models. Of the terms containing spin-orbit correlations, the two-body and three-body-separable ones are calculated, and the magnitude of the rest is estimated. Results for three phase-equivalent v_8 models, which differ significantly in the strength of tensor and spin-orbit potentials, are reported. They suggest that simple v_8 models may not be able to simultaneously explain the binding energy and density of nuclear matter.

1. Introduction

Non-relativistic nuclear Hamiltonians containing only two-body forces can be expressed in the form:

$$H = \sum_i -\frac{\hbar^2}{2m}\nabla_i^2 + \sum_{i<j}\sum_p v^p(r_{ij})O_{ij}^p, \tag{1.1}$$

where $v^p(r_{ij})$ are functions of $|r_i - r_j|$, and O_{ij}^p are operators that operate on the spin, isospin and position variables of particles i and j. In principle one should take as many operators as are required to explain the NN scattering at non-relativistic energies. Simpler models of nuclear matter are obtained by considering a few of the most important operators in (1.1). The so-called "v_6 models" have $p = 1, 6$ and the $O_{ij}^{p=1,6}$ are 1, $\sigma_i \cdot \sigma_j$, $\tau_i \cdot \tau_j$, $(\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j)$, S_{ij} and $S_{ij}(\tau_i \cdot \tau_j)$, S_{ij} being the tensor operator. Many authors have studied v_6 models based on different NN potentials. Recent results have been reviewed by Pandharipande and Wiringa [1], and particularly for the v_6 model based on the Reid [2] potential, an agreement seems to be emerging.

In this paper we study the so-called "v_8 models" which contain the spin-orbit potentials associated with the operators:

$$O_{ij}^{7,8} = (L \cdot S)_{ij}, \qquad (L \cdot S)_{ij}(\tau_i \cdot \tau_j), \tag{1.2}$$

† Work supported by NSF PHY 78-26582.

in addition to the six contained in the v_6 models. Very crude calculations of v_8 models containing many approximations were reported earlier [3]).

A v_8 Hamiltonian can explain the elastic scattering in the $^1S_0$, $^3S_1$–$^3D_1$, $^1P_1$ and $^3P_2$–$^3F_2$ states. In this sense these are the simplest of the "realistic" models. They are, however, unable to explain phase shifts in D and higher waves. A Hamiltonian containing the eight v_8 potentials, two quadratic spin-orbit potentials and four momentum-dependent terms like $(\nabla^2 v^m(r) + v^m(r)\nabla^2)$ can explain all scattering data up to 330 MeV [ref. [4])].

The variational method developed by Wiringa and Pandharipande (WP) [ref. [3])] is used to calculate the $E(\rho)$ of the v_8 models. The pair-correlation operator is assumed to be:

$$F_{ij}(d, \beta_p) = \sum_{p=1,8} \beta_p f^p_{LO}(d, r_{ij}) O^p_{ij}, \tag{1.3}$$

β_p and d are the variational parameters, and the functions $f^p_{LO}(d, r_{ij})$ are obtained by minimizing the two-body cluster contribution to the energy under the healing constraint

$$f^p_{LO}(r_{ij} \geqq d) = \delta_{p1}. \tag{1.4}$$

The variational wave function is assumed to be

$$\Psi = [\mathcal{S} \prod_{i<j} F_{ij}] \Phi(\rho), \tag{1.5}$$

where $\mathcal{S}$ is a symmetrizing operator and $\Phi(\rho)$ is the Fermi gas wave function. We will refer to $\beta_p f^p_{LO}(d, r_{ij})$ by f^p_{ij} for brevity.

WP expand $E(\rho, d, \beta_p)$,

$$E(\rho, d, \beta_p) = \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle}, \tag{1.6}$$

in a diagrammatic series, and we will continue to use the diagrammatic notation given in fig. 1 of ref. [3]). In v_6 models $E(\rho, d, \beta_p)$ can be calculated, presumably quite accurately [5]), by summing the important diagrams of this expansion by chain summation methods. In v_8 models the contribution of all diagrams that do not contain either spin-orbit correlations ($f^{7,8}_{ij}$) or potential ($v^{7,8}_{mn}$) links can be calculated with the chain summation techniques developed for the v_6 models. If this contribution is called $E_6(\rho, d, \beta_p)$ we have:

$$E(\rho, d, \beta_p) = E_6(\rho, d, \beta_p) + E_{LS}(\rho, d, \beta_p), \tag{1.7}$$

where $E_{LS}(\rho, d, \beta_p)$ is the contribution of diagrams having $\boldsymbol{L} \cdot \boldsymbol{S}$ correlation and/or potential links. This paper primarily deals with the calculation of E_{LS}, that of $f^p_{LO}(d, r_{ij})$ is given in ref. [3]).

The contribution of two-body clusters to E_{LS} is calculated in sect. 2, and it is

generally much smaller than the two-body contribution in E_6. The three-body separable contributions to E_{LS} are calculated in sect. 3, while the many-body chain diagrams, and the results, are discussed in sects. 4 and 5 respectively.

2. Two-body cluster contribution

The two-body W-direct, W-exchange and W_F diagrams are shown in fig. 1. The contribution of two-body W-direct diagrams, per nucleon, is given by:

$$\frac{1}{2A\Omega}\sum_{m,n}\sum_{i,j,k}\int e^{-i(k_m\cdot r_m+k_n\cdot r_n)}C(f^i_{mn}O^i_{mn}H^j_{mn}O^j_{mn}f^k_{mn}O^k_{mn})e^{i(k_m\cdot r_m+k_n\cdot r_n)}d^3r_{mn}. \quad (2.1)$$

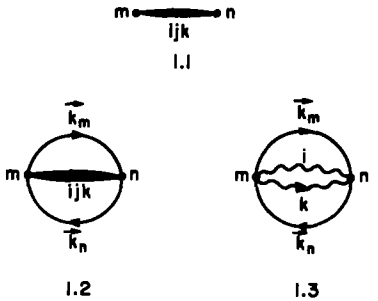

Fig. 1. Diagrams 1.1, 1.2 and 1.3 respectively give the two-body W-direct, W-exchange and W_F contributions to E_{LS}.

Here A is the number of nucleons, Ω the normalization volume, and

$$H^j_{mn} = v^j_{mn} - \delta_{j1}\frac{\hbar^2}{m}\nabla^2_{mn}. \quad (2.2)$$

By convention the ∇^2 in H^c operates only on the F_{mn}, the kinetic energy terms having $\nabla_m F_{mn}\cdot\nabla_m\Phi$ are included in W_F and U_F, while those having $\nabla_m F_{mn}\cdot\nabla_m F_{ml}$ form the U. A sum over all occupied momentum, spin and isospin states m, n is implied in (2.1).

The C-part of a product is defined as the part independent of any σ or τ operators, and methods to calculate it are given in ref. [1]). We can commute out the ∇^2 in H_c and obtain:

$$C(f^i_{mn}O^i_{mn}H^j_{mn}O^j_{mn}f^k_{mn}O^k_{mn}) = (f^iH^jf^k)C(O^i_{mn}O^j_{mn}O^k_{mn})$$
$$-\delta_{j1}\frac{\hbar^2}{m}\{2f^i_{mn}C(O^i_{mn}(\nabla f^k_{mn})\cdot(\nabla O^k_{mn}))+f^i_{mn}f^k_{mn}C(O^i_{mn}\nabla^2O^k_{mn})\}. \quad (2.3)$$

We note that:

$$\nabla^2(L\cdot S)_{mn} = 0,$$
$$(\nabla f^k_{mn})\cdot(\nabla(L\cdot S)_{mn}) = \frac{f'^k}{r_{mn}}(L\cdot S)_{mn}. \quad (2.4)$$

The operators O^i_{mn} generally have two factors, $O^{i_\sigma}_{mn}$ which can be 1, $\sigma_m \cdot \sigma_n$, S_{mn} and $(L \cdot S)_{mn}$, and $O^{i_\tau}_{mn}$ which can be 1 or $\tau_m \cdot \tau_n$. We will use the abbreviations c, σ, t, b and τ to denote these operators. Obviously the C-part of a product can be factorized into a C-part of ΠO^{i_σ} operators, and a C-part of ΠO^{i_τ} operators:

$$C(O^i_{mn} O^j_{mn} O^k_{mn}) = C(O^{i_\sigma}_{mn} O^{j_\sigma}_{mn} O^{k_\sigma}_{mn}) C(O^{i_\tau}_{mn} O^{j_\tau}_{mn} O^{k_\tau}_{mn}). \tag{2.5}$$

The $(C(\Pi\, \tau_m \cdot \tau_n)$ is trivial to calculate [1]) and we will not discuss it. It is convenient to use the following generalizations of tensor and $L \cdot S$ operators to calculate the $C(\Pi\, O^{i_\sigma}_{mn})$. Let A, $B \ldots$ be the vector operators r, ∇ and L, the operators $\beta(A)$ are defined as

$$\beta_{mn}(A) = \tfrac{1}{2}(\sigma_m + \sigma_n) \cdot A, \tag{2.6}$$

and $\beta_{mn}(L) = L \cdot S$. The operators $\alpha(A, B)$ are defined as:

$$\alpha_{mn}(A, B) = \tfrac{3}{2}(\sigma_m \cdot A \sigma_n \cdot B + \sigma_n \cdot A \sigma_m \cdot B) - A \cdot B \sigma_m \cdot \sigma_n, \tag{2.7}$$

and $\alpha_{mn}(r, r) = r^2 S_{mn}$. Even though the operators r, ∇ and L do not commute, it may be verified that $\alpha(A, B) = \alpha(B, A)$. Some of the useful products of these operators are given below:

$$\sigma_m \cdot \sigma_n \beta_{mn}(A) = \beta_{mn}(A) \sigma_m \cdot \sigma_n = \beta_{mn}(A), \tag{2.8}$$

$$\sigma_m \cdot \sigma_n \alpha_{mn}(A, B) = \alpha_{mn}(A, B) \sigma_m \cdot \sigma_n = \alpha_{mn}(A, B), \tag{2.9}$$

$$(L \cdot S)^2_{mn} = \tfrac{1}{2} L^2_{mn} + \tfrac{1}{6} \sigma_m \cdot \sigma_n L^2_{mn} - \tfrac{1}{2}(L \cdot S)_{mn} + \tfrac{1}{6} \alpha_{mn}(L, L), \tag{2.10}$$

$$(L \cdot S)_{mn} S_{mn} = -(L \cdot S)_{mn} - 3 S_{mn} + \alpha_{mn}(r_{mn}, \nabla_{mn}) - S_{mn}(r_{mn} \cdot \nabla_{mn}), \tag{2.11}$$

$$S_{mn}(L \cdot S)_{mn} = -(L \cdot S)_{mn} - \alpha_{mn}(r_{mn}, \nabla_{mn}) + S_{mn}(r_{mn} \cdot \nabla_{mn}). \tag{2.12}$$

Since the operators $\beta_{mn}(A)$ have two terms, one linear in σ_m and other in σ_n, while $\alpha_{mn}(A, B)$ is linear in σ_m and σ_n, only products like $\beta_{mn}(A)\, \beta_{mn}(B)$ and $\alpha_{mn}(A, B) \alpha_{mn}(C, D)$ can have a C-part.

TABLE 1

Summary of contributions of two-body W-diagrams to E_{LS} of Reid v_8 ($k_F = 1.7$ fm^{-1}, $d = 2.5\, r_0$) and BJ-II v_8 ($k_F = 1.4$ fm^{-1}, $d = 2.4\, r_0$) models

$i_\sigma j_\sigma k_\sigma$	$C(O^{i_\sigma} O^{j_\sigma} O^{k_\sigma})$	Reid v_8		BJ-II	
		direct	exch.	direct	exch.
tbt	-18	3.43	-0.00	2.37	-0.45
cbb	$\frac{1}{2} L^2$	-5.22	-3.72	-5.68	-4.51
σbb	$\frac{1}{2} L^2$	-0.05	-0.06	0.20	0.15
tbb	$-\frac{1}{2} L^2$	-0.15	-0.13	-0.14	-0.13
bcb	$\frac{1}{2} L^2$	1.40	1.02	1.37	1.07
$b\sigma b$	$\frac{1}{2} L^2$	0.13	0.10	0.22	0.18
btb	$-\frac{1}{2} L^2$	-0.08	-0.06	-0.05	-0.04
bbb	$-\frac{1}{4} L^2$	0.29	$+0.23$	0.43	0.36

Most of the two-body W-direct diagrams are included in E_6, those that contribute to E_{LS} must have one or more of i_σ, j_σ and $k_\sigma = b$. Their contributions to the E_{LS} of Reid v_8 and BJ-II v_8 (based on Bethe-Johnson II potentials [6,3]) are summarized in table 1. The equilibrium values of β_p in the Reid v_8 (BJ-II v_8) models are found to be:

$$\beta_\sigma = \beta_\tau = \beta_{\sigma\tau} = 0.83\,(0.90),$$
$$\beta_t = \beta_{t\tau} = 1.1\,(1.1),$$
$$\beta_b = \beta_{b\tau} = 0.65\,(0.7). \tag{2.13}$$

The β_p do not have appreciable dependence on k_F in the density range investigated. The Reid v_8 results are at $k_F = 1.7$ fm^{-1}, $d = 2.5\ r_0$, and BJ-II v_8 results are at $k_F = 1.4$ fm^{-1}, $d = 2.4\ r_0$. The equilibrium occurs at $k_F = 1.7$ and 1.38 fm^{-1} in the present calculations of the Reid v_8 and BJ-II v_8 models.

Only the spin parts $O^{i\sigma}O^{j\sigma}O^{k\sigma}$ are listed in the table, and all terms that differ in isospin operators are grouped together. Thus the contribution listed as tbt is in fact a sum of the eight terms in the product $(t+t\tau)(b+b\tau)(t+t\tau)$. The terms ijk and kji have identical contributions, and when $i \neq k$ the listed contribution is the sum of ijk and kji terms.

The C-parts of $O^{i\sigma}_{mn}O^{j\sigma}_{mn}O^{k\sigma}_{mn}$ are also given in table 1. We note that:

$$\frac{1}{A\Omega}\sum_{m,n} e^{-i(\boldsymbol{k}_m\cdot\boldsymbol{r}_m+\boldsymbol{k}_n\cdot\boldsymbol{r}_n)}e^{i(\boldsymbol{k}_m\cdot\boldsymbol{r}_m+\boldsymbol{k}_n\cdot\boldsymbol{r}_n)} = \rho, \tag{2.14}$$

$$\frac{1}{A\Omega}\sum_{m,n} e^{-i(\boldsymbol{k}_m\cdot\boldsymbol{r}_m+\boldsymbol{k}_n\cdot\boldsymbol{r}_n)}L^2 e^{i(\boldsymbol{k}_m\cdot\boldsymbol{r}_m+\boldsymbol{k}_n\cdot\boldsymbol{r}_n)} = \tfrac{1}{5}k_F^2 r^2\rho, \tag{2.15}$$

which makes the calculation rather simple.

The contribution of the two-body W-exchange diagram is given by

$$-\frac{1}{2}\frac{1}{A\Omega}\sum_{m,n}\sum_{i,j,k}\sum_{n'=1,4}\tfrac{1}{4}\int e^{-i(\boldsymbol{k}_m\cdot\boldsymbol{r}_n+\boldsymbol{k}_n\cdot\boldsymbol{r}_m)}$$
$$\times C(O^{n'}_{mn}f^i_{mn}O^i_{mn}H^j_{mn}O^j_{mn}f^k_{mn}O^k_{mn})e^{i(\boldsymbol{k}_m\cdot\boldsymbol{r}_m+\boldsymbol{k}_n\cdot\boldsymbol{r}_n)}d^3r_{mn}, \tag{2.16}$$

where $\frac{1}{4}\sum_{n'=1,4}O^{n'}_{mn}$ is simply the spin-isospin exchange operator. Thanks to eqs. (2.8) and (2.9) the C-parts in (2.16) are very simple to calculate; they are either numbers or a number times the L^2 operator, and

$$\frac{1}{A\Omega}\sum_{m,n} e^{-i(\boldsymbol{k}_m\cdot\boldsymbol{r}_n+\boldsymbol{k}_n\cdot\boldsymbol{r}_m)}e^{i(\boldsymbol{k}_m\cdot\boldsymbol{r}_m+\boldsymbol{k}_n\cdot\boldsymbol{r}_n)} = \rho l^2(k_F r), \tag{2.17}$$

$$\frac{1}{A\Omega}\sum_{m,n} e^{-i(\boldsymbol{k}_m\cdot\boldsymbol{r}_n+\boldsymbol{k}_n\cdot\boldsymbol{r}_m)}L^2 e^{i(\boldsymbol{k}_m\cdot\boldsymbol{r}_m+\boldsymbol{k}_n\cdot\boldsymbol{r}_n)} = \rho r l(k_F r)l'(k_F r), \tag{2.18}$$

where $l(k_F r)$ is the familiar Slater function:

$$l(x) = \frac{3}{x^3}(\sin x - x\cos x). \tag{2.19}$$

The two-body W-exchange contributions to E_{LS} are also listed in table 1, and the total contribution of two-body W diagrams to E_{LS} is given in table 2.

TABLE 2

Summary of contributions to E_6 and E_{LS} of v_8 models (in MeV)

	Reid-v_8, $k_F = 1.7$ fm^{-1}		BJ-II-v_8, $k_F = 1.4$ fm^{-1}	
	E_6	E_{LS}	E_6	E_{LS}
T_F	35.95	0	24.38	0
W-2 body	−58.01	−2.88	−35.96	−4.65
W_F-2 body	−5.43	−0.23	−7.04	−0.17
W_S	14.27	−0.53	14.82	−0.46
W_0-MB	−5.38	0.23*	−2.39	−0.03*
$W_C + W_{CS}$	4.88	0.32*	2.43	0.14*
W_F-MB	−1.25	NC	−0.28	NC
U_F	1.34	NC	0.66	NC
U	−0.45	NC	−0.41	NC
total	−14.09	−3.09	−3.79	−5.17

NC denotes terms that have not been calculated, while a * denotes crude estimates.

The only two-body W_F terms (fig. 1.3) that contribute to E_{LS} have $i, k = b$ or $b\tau$. Their contribution is given by:

$$\frac{\hbar^2}{8m}\rho\int[(f^b+3f^{b\tau})f'^b+3(f^b-f^{b\tau})f'^{b\tau}][rll''+rl'^2-ll']\mathrm{d}^3r, \tag{2.20}$$

and is listed in table 2.

The two-body contributions to E_{LS} are much smaller than those in E_6 (table 2). This may appear surprising in view of the large magnitudes of v^b and f^b (figs. 5 and 6 of ref. [3])]. But the important E_{LS} contributions have a C-part of $\sim \frac{1}{2}L^2$, which gives a factor of $\sim \frac{1}{10}k_F^2 r^2$. At $r < 1$ fm, where the f^b and v^b are appreciable, it cuts down all $L\cdot S$ contributions. However, at very high densities ($k_F \gtrsim 5$ fm^{-1}) the E_{LS} contributions may become comparable to those of E_6.

3. Three-body separable diagrams

The separable diagrams give the dominant many-body cluster contribution (MBCC) to E_6 (W_S of table 2), and since the $(L\cdot S)_{mn}$ operator does not commute with any part of F_{m1} we may expect them to give the leading MBCC to E_{LS}. The three-body separable diagrams are shown in fig. 2. Since $[(L\cdot S)_{mn}, f^c_{m1}] \neq 0$ even the $p, q = c$ terms of diagrams 2.1–3 contribute to the E_{LS}.

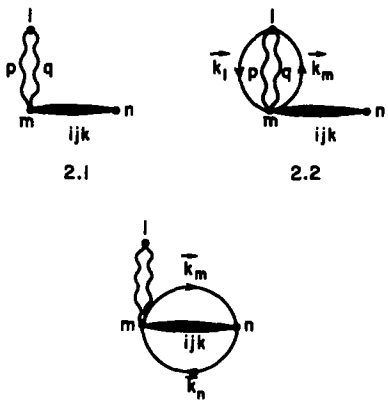

Fig. 2. Diagrams 2.1–3 give the three-body separable contributions to E_{LS}.

The general expression for the direct three-body W_S is:

$$\frac{1}{A\Omega^2}\sum_{m,n,1}\sum_{i,j,k,p,q}\int \Phi_3^*(m,n,1)C(\{f_{mn}^i O_{mn}^i, f_{m1}^p O_{m1}^p\}H_{mn}^j O_{mn}^j\{f_{mn}^k O_{mn}^k, f_{m1}^q O_{m1}^q\}$$
$$-(f_{mn}^i O_{mn}^i H_{mn}^j O_{mn}^j f_{mn}^k O_{mn}^k)(f_{m1}^p O_{m1}^p f_{m1}^q O_{m1}^q))\Phi_3(m,n,1)\mathrm{d}^3 r_{mn}\mathrm{d}^3 r_{m1}, \qquad (3.1)$$

where $\{,\}$ denote anticommutators, and we must not let the O_{mn} in the second, so-called "separated", term of 3.1 operate on the $f_{m1}O_{m1}$ [ref. [1])]. The $\Phi(m,n,1)$, $\Phi(n,m,1)$ etc. are given by:

$$\Phi_3(m,n,1) = \mathrm{e}^{i(k_m\cdot r_m + k_n\cdot r_n + k_1\cdot r_1)},$$
$$\Phi_3(n,m,1) = \mathrm{e}^{i(k_m\cdot r_n + k_n\cdot r_m + k_1\cdot r_1)}. \qquad (3.2)$$

The exchange diagrams 2.2, 3 respectively have a $\Phi_3^*(1,n,m)$ or $\Phi_3^*(n,m,1)$ instead of the $\Phi_3^*(m,n,1)$, and the corresponding spin-isospin exchange operator. On calculating the C-part and summing of k_m, k_n and k_1 the various terms of the three-body W_S contribution can be expressed in the convenient form of the product of an integral over r_{mn} with an integral over r_{n1}.

Most of the terms are included in E_6, those that contribute to E_{LS} are classified as follows: The so-called BI terms have one or more $(L\cdot S)_{mn}$ operators, but no $(L\cdot S)_{m1}$ operators. Their contribution is divided into two parts BIF and BIK. The BIF includes all terms in which one or more $(L\cdot S)_{mn}$ operate on the F_{m1}, while those in which all $(L\cdot S)_{mn}$ operate on F_{mn} or the Φ's form the BIK. The terms which have $(L\cdot S)_{m1}$, but no $(L\cdot S)_{mn}$ operators contribute to BPK and BPF. The BPF contains contributions from $(L\cdot S)_{m1}$ operating on F_{mn}, and the rest of BP is called BPK. Only the terms in which $p, q = 7, 8$ contribute to BP, the terms having only one $(L\cdot S)_{m1}$ operator have zero C-part. Terms having both $(L\cdot S)_{mn}$ and $(L\cdot S)_{m1}$ operators are included in BB.

The calculated values of BIF, BIK, BPF and BPK are given in table 3. The BIF is large and attractive. It takes into account the increase in the average value L^2_{mn} due to correlations of m and n with other particles. The $ijk = cbb$ and bbc terms give the dominant contribution to BIF, and so BIF is approximately linear in β_b, the magnitude of $L \cdot S$ correlations. The BPF and BPK are respulsive and quadratic in β_b. These terms are responsible for bringing down the equilibrium β_b to ~ 0.7; if we include only two-body cluster contributions the equilibrium value of β_b is ~ 1.

TABLE 3

Summary of the contributions in MeV of three-body separable diagrams to E_{LS} of v_8 models

	Reid v_8	BJ-II v_8
k_F	1.7	1.4
BIF	−3.17	−3.89
BIK	0.19	0.38
BPF	1.25	1.83
BPK	1.20	1.22
total	−0.53	−0.46

The order of magnitude of BB diagrams will be given by the product of the two-body cluster contribution to E_{LS} and $\int \frac{1}{5} k_F^2 r^2 f^{b2} d^3r$. BB should be very small, and is neglected. The many-body separable contributions to E_{LS} are also neglected; they should be a small fraction (typically a quarter to a third [1])) of the calculated three-body-separable contribution to E_{LS}.

4. Many-body chain diagrams

The chain diagrams represent the influence of the medium on the distribution function of the interacting pair. In v_6 models the magnitude of the contributions of chain diagrams is typically a tenth of the two-body energy. Accordingly we may expect the contribution of chain diagrams to E_{LS} to be of the order of 0.5 MeV and thus not too important.

The terms of chain diagrams having $L \cdot S$ links can be grouped into two parts. The "K-part" includes those in which the L_{ij} operate on the F_{ij} or Φ. This part is calculated, but the "F-part" in which one or more L_{ij} operate on F_{ik} is neglected. Since the two-body and BIF diagrams have similar contributions we may expect the neglected F-part to be comparable to the calculated K-part. In this sense the present calculation of the chain diagrams in E_{LS} is just an order of magnitude estimate.

The E_{LS} diagrams with central chains (W_0-MB) are illustrated in figs. 3.1–4. The K-part of diagrams of type 3.1 and 3.2 having one or more G^c_{dd} chains are calculated by inserting an $\exp(G^c_{dd}) - 1$ in the integrands of the two-body diagrams. Similarly the terms of diagrams 3.3 and 3.4 containing only k_n are obtained by respectively inserting a G^c_{de} and G^c_{cc} in the two-body integrals. The terms having k_m give derivatives

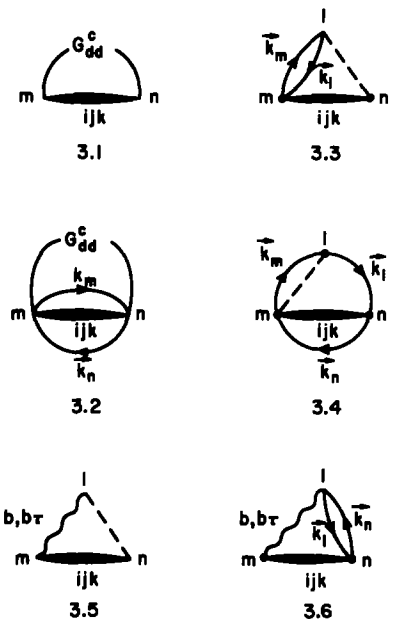

Fig. 3. Diagrams 3.1–4 give to W_0 – MB contribution to E_{LS}, whereas 2.5–6 are examples of W_c diagrams having $G^{b,b\tau}$.

of l_{m1}, and they have to be calculated explicitly as three-body integrals. The K-part of many W_0-MB diagrams is summed by dressing diagrams 3.3 and 3.4 by G^c_{dd} chains. The total estimated W_0-MB contribution to E_{LS} is quite small and it is given in table 2.

The contribution of W-diagrams having operator chains is called W_c. Of these we first consider those that have $(\boldsymbol{L}\cdot\boldsymbol{S})_{mn}$ operators and a $G^{p=2,6}_{mn}$ chain which does not contain $\boldsymbol{L}\cdot\boldsymbol{S}$ correlations. We may generate the leading terms of such diagrams by replacing an $f^{p=2,6}$ in the two-body integrals by f^cG^p. The f^cG^p are much smaller than f^p in general, and so the W_c is typically $\sim 10\,\%$ of the two-body W. A glance at table 1 then suggests that from this class of diagrams, we need to consider only those in which an $f^cG^{t\tau}$ replaces an $f^{t\tau}$ in the tbt terms. However, since the $G^{t\tau}$ are very small, and they change sign [1]), the contribution of these diagrams would be negligible.

Diagrams having $\boldsymbol{L}\cdot\boldsymbol{S}$ links in the chains can also contribute to W_c. Of these we estimate only 3.5 and 3.6 which can be thought of as the leading three-body terms of $G^{p=7,8}$. From table 1 we see that the terms having $G^{p=7,8}$ and $i_\sigma j_\sigma k_\sigma = bcc, cbc$ and ccb may be important, and so only these are calculated. The contribution of the K-part of diagram 3.5 with $p = b$ and $ijk = cbc$ is, for example, given by:

$$\frac{k_F^2}{40}\rho^2\int(v^bf^{c^2})_{mn}(2f^cf^b)_{m1}(f^{c^2}-1)_{n1}r_{mn}r_{m1}\cos\theta_m\mathrm{d}^3r_{mn}\mathrm{d}^3r_{n1}. \tag{4.1}$$

The diagrams 3.5 and 3.6 can also be dressed by G^c_{dd} chains, the contribution of their K-parts is given in table 2 as the order of magnitude of the W_c of E_{LS}. It is quite small.

W_F is generally the largest of the terms having $\nabla_m F_{mn} \cdot \nabla_m \Phi$ and $\nabla_m F_{mn} \cdot \nabla_m F_{m1}$ contributions. Since the two-body W_F contribution to E_{LS} is itself very small, we expect the many-body contributions W_F-MB, U_F and U to E_{LS}, to be negligible. These have not been calculated at all.

5. Results

The minimum energy $E(v_6)$ for the Reid v_6 model at $k_F = 1.7$ fm^{-1} is found to be -17.2 MeV by the WP variational calculation with the wave function Ψ_6 having correlation operators F_6:

$$F_6 = \sum_{p=1,6} f_6^p O^p. \tag{5.1}$$

The energy of the v_8 model obtained by treating the $L \cdot S$ potentials in first order perturbation theory, is given by:

$$E(v_8)_{\text{pert.}} = E(v_6) + \sum_{p=7,8} \frac{\langle \Psi_6 | \sum_{m<n} v^p(r_{mn}) O^p_{mn} | \Psi_6 \rangle}{\langle \Psi_6 | \Psi_6 \rangle}. \tag{5.2}$$

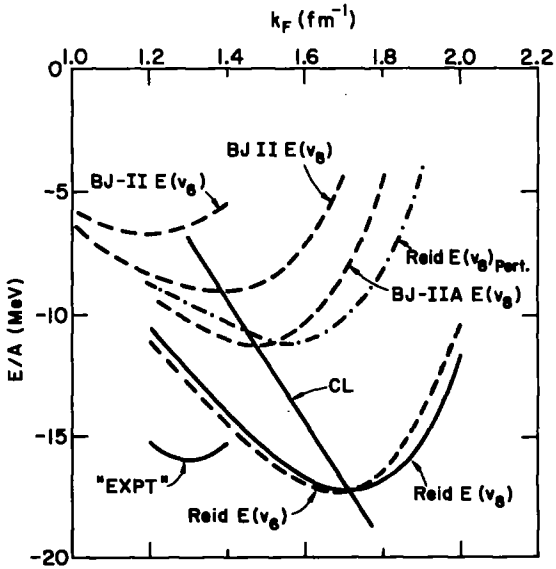

Fig. 4. The calculated energies of Reid, BJ-II and BJ-IIA models. The full curves give $E(v_8)$, dashed curves give $E(v_6)$ from ref. [3]), and the dot-dash curve gives $E(v_8)_{\text{pert.}}$ for the Reid model. The curve labelled "EXPT" assumes equilibrium $k_F = 1.3$ fm^{-1}, $E_0 = -16$ MeV and a compressibility of 250 MeV.

It is much simpler to calculate than $E(v_8)$. The E_{LS} diagrams having only v^b or v^{br} lines contribute to the second term of (5.2), and of these the two-body *tbt* diagram is dominant. The quantity $E(v_8)_{\text{pert.}}$ is the expectation value of the v_8 Hamiltonian with the optimum wave function Ψ_6 for the v_6 model. It is thus an upper bound to $E(v_8)$; but it is much to high (fig. 4). For example, $E(v_8)_{\text{pert.}}$ for the Reid model at $k_F = 1.7$ is -10.2 MeV.

The operator F_8 in the Ψ_8 of v_8 models have (i) $\boldsymbol{L}\cdot\boldsymbol{S}$ correlations absent in F_6, and (ii) spin, isospin and tensor correlations $f_8^{p=1,6}$ that are slightly different from those in F_6. We first discuss the effect of the difference between $f_8^{p=1,6}$ and $f_6^{p=1,6}$. Let $\hat{\Psi}_6$ denote the wave function

$$\hat{\Psi}_6 = \{\mathcal{S}\prod_{i<j}[\sum_{p=1,6} f_8^p(r_{ij})O_{ij}^p]\}\Phi. \tag{5.3}$$

It is simply the Ψ_8 without its spin-orbit correlations. The quantity E_6 is the expectation value of the v_6 Hamiltonian with $\hat{\Psi}_6$. It is much higher (-14.1 MeV for Reid models at $k_F = 1.7$) than $E(v_6)$. However, the expectation value of the v_8 Hamiltonian with $\hat{\Psi}_6$ is slightly lower than $E(v_8)_{\text{pert.}}$ (-10.7 MeV in Reid at $k_F = 1.7$).

The $\boldsymbol{L}\cdot\boldsymbol{S}$ correlations in Ψ_8 lower the energy by several MeV. In Reid v_8 at $k_F = 1.7$ ($k_F = 2.0$) the energy goes down from $-10.7\,(+1.5)$ MeV to $-17.2\,(-10.5)$ on adding $\boldsymbol{L}\cdot\boldsymbol{S}$ correlations. The effect of spin-orbit correlations is nevertheless much smaller than the energy gain due to tensor correlations (~ 40 MeV at $k_F = 1.7$). In the Reid models $E(v_6)$ and $E(v_8)$ (fig. 4) are almost identical due to accidental cancellations. The difference between $E(v_8)_{\text{pert.}}$ and $E(v_8)$ gives a better indication of the effect of $\boldsymbol{L}\cdot\boldsymbol{S}$ correlations. The $\boldsymbol{L}\cdot\boldsymbol{S}$ force in the BJ-II model is approximately three times larger than that in the Reid case, and it has a more evident influence on $E(\rho)$.

The BJ-IIA v_8 model uses the $^3S_1$-$^3D_1$ potential "5.595" in the $TS = 01$ states where BJ-II uses potential "6.55". BJ label $^3S_1$-$^3D_1$ potentials by the D-state percentage they predict in the deuteron. The interaction in $TS = 00$, 10 and 11 states in BJ-IIA models is identical to that in BJ-II. Thus BJ-IIA and Reid v_8 models respectively have weaker tensor and $\boldsymbol{L}\cdot\boldsymbol{S}$ potentials than BJ-II v_8 models have. The equilibrium points of these three v_8 models (fig. 4) lie on an almost straight "Coester line" that misses the empirical equilibrium point. The energies given by the v_8 models at low densities are perhaps more distressing. At $k_F = 1.2\,\text{fm}^{-1}$ these phase-equivalent v_8 models give rather similar energies between 8–11 MeV, whereas the empirical $E(\rho)$ obtained with a compressibility of 250 MeV [ref. [1])] suggests that $E(k_F = 1.2)$ should be ~ 15 MeV. This undesirable behavior of the $E(\rho)$ at small densities may be responsible for the fact that calculations with the Reid potential generally underbind light nuclei such as ^{16}O and ^{40}Ca by 2–3 MeV per nucleon [7]), and overbind nuclear matter.

Earlier estimates by WP, of the effect of $\boldsymbol{L}\cdot\boldsymbol{S}$ potentials on the $E(\rho)$ were much too large, by a factor of ~ 2, primarily because they neglected the repulsive BPK and BPF terms which limit the size of f^b and f^{br}. All the two-body E_{LS}, and all

the important three-body separable E_{LS} diagrams have been calculated exactly in the present work. Crude estimates of the neglected E_{LS} diagrams suggest that these should be less than 1 MeV which is probably the accuracy of the E_6 calculation. If we trust these error estimates of the present many-body calculation then the need for a three-body force that gives a few MeV more binding at small densities, and possibly becomes repulsive at high densities is indicated. It is well known that two-body forces alone cannot provide all the binding energy of the triton [8]).

In the density range ($k_F < 2.2$ fm^{-1}) investigated the v_8 models do not exhibit instability towards a collapse as predicted by Calogero and Simonov [9]). However, it is possible that at very high densities some of the attractive terms in E_{LS}, which grow as k_F^5, may become dominant and produce a collapse.

References

1) V. R. Pandharipande and R. B. Wiringa, Rev. Mod. Phys. **51** (1979) 821
2) R. V. Reid, Ann. of Phys. **50** (1968) 411
3) R. B. Wiringa and V. R. Pandharipande, Nucl. Phys. **A317** (1979) 1
4) M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Côte, P. Pirès and R. de Tourreil, preprint (1979)
5) R. B. Wiringa, preprint (1979)
6) H. A. Bethe and M. B. Johnson, Nucl. Phys. **A230** (1974) 1
7) H. Kümmel, K. H. Lührmann and J. G. Zabolitzky, Phys. Reports **36C** (1978) 1
8) R. A. Brandenberg, Y. E. Kim and A. Tubis, Phys. Rev. **C12** (1975) 1368;
A. Laverne and C. Grignouse, Phys. Rev. Lett. **29** (1972) 436
9) F. Calogero and Yu. A. Simonov, Phys. Rev. Lett. **25** (1970) 881