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VARIATIONAL CALCULATIONS OF REALISTIC MODELS OF NUCLEAR MATTER*

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Abstract: We report variational calculations of nuclear matter with a semi-realistic Reid v_{12} model, and a realistic v_{14} model of the two-nucleon interaction operator. The v_{14} model fits the available nucleon–nucleon scattering data up to 425 MeV lab energy, and has relatively weak L^2 and $(\boldsymbol{L}\cdot\boldsymbol{S})^2$ interactions in addition to the standard central, tensor and $(\boldsymbol{L}\cdot\boldsymbol{S})$. The L^2 and $(\boldsymbol{L}\cdot\boldsymbol{S})^2$ interactions are treated semiperturbatively; their contribution reduces the overbinding of nuclear matter. However, the equilibrium $k_F = 1.7$ fm^{-1} and $E_0 = -17.5$ MeV obtained with the v_{14} model are both higher than their empirical values $k_F = 1.33$ fm^{-1} and $E_0 = -16$ MeV. We assume that the difference between the calculated and empirical $E(\rho)$ is entirely due to three-nucleon interactions (TNI). The TNI contributions are phenomenologically added to the nuclear matter energy, and their parameters are adjusted to obtain the correct equilibrium energy, density and compressibility. The required TNI contributions appear to be of reasonable magnitude.

1. Introduction

Most of the detailed variational calculations of nuclear matter use "Bethe's homework models" for the nuclear hamiltonian. A v_8 model [1,2]) is obtained by assuming that the interaction in all isospin–spin $T, S = 1.0; 1, 1; 0, 0$ and $0, 1$ states is given by that in the $^1S_0$, ($^3P_2$-$^3F_2$), $^1P_1$ and ($^3S_1$-$^3D_1$) channels. The potentials used by various authors such as Reid [3]), Bethe–Johnson [4]) etc., to fit the scattering data in $^1S_0$, $^3S_1$-$^3D_1$, $^1P_1$ and $^3P_2$-$^3F_2$ channels are used to construct a two-nucleon interaction operator:

$$v_{8,ij} = \sum_{p=1,8} v^p(r_{ij})\, O^p_{ij}, \tag{1.1}$$

where $v^p(r_{ij})$ are functions of the interparticle distance r_{ij}, and O^p_{ij} are operators:

$$O^{p=1,8}_{ij} = 1, \sigma_i\cdot\sigma_j, \tau_i\cdot\tau_j, (\sigma_i\cdot\sigma_j)(\tau_i\cdot\tau_j), S_{ij}, S_{ij}(\tau_i\cdot\tau_j), (\boldsymbol{L}\cdot\boldsymbol{S})_{ij}, (\boldsymbol{L}\cdot\boldsymbol{S})_{ij}(\tau_i\cdot\tau_j)\,. \tag{1.2}$$

Here S_{ij} is the tensor operator, and $\boldsymbol{L}\cdot\boldsymbol{S}$ the spin-orbit. The v_6 homework models are obtained by neglecting the two $\boldsymbol{L}\cdot\boldsymbol{S}$ interactions in v_8 models. It is well known that the v_8 models fail to explain the D-state phase shifts. Thus the v_6 and v_8 models are not completely realistic, but they are very good pedagogical models in the sense that they contain the bulk of the nucleon-nucleon interaction.

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Recently we [5]) obtained a realistic v_{14} model of the two-nucleon interaction operator:

$$v_{14,ij} = \sum_{p=1,14} v^p(r_{ij}) O^p_{ij}, \tag{1.3}$$

that fits the phase shifts in all S, P, D and F waves. The six additional operators are:

$$O^{p=9,14}_{ij} = L^2, L^2(\sigma_i \cdot \sigma_j), L^2(\tau_i \cdot \tau_j), L^2(\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j), (\boldsymbol{L} \cdot \boldsymbol{S})^2, (\boldsymbol{L} \cdot \boldsymbol{S})^2(\tau_i \cdot \tau_j)\,. \tag{1.4}$$

The potentials $v^{p=9,14}(r)$ associated with these operators are relatively weak, but necessary to fit the D and F wave phases.

In the present work we report variational calculations of nuclear matter with this and a Reid v_{12} interaction operator [5]) that does not have the $(\boldsymbol{L} \cdot \boldsymbol{S})^2$ terms. The variational wave function is taken to be:

$$\Psi_v = \left(\mathcal{S} \prod_{i<j} \mathcal{F}_{ij}\right) \Phi, \tag{1.5}$$

where $\mathcal{S}$ is a symmetrizing operator, $\mathcal{F}_{ij}$ is a pair correlation operator and Φ the Fermi gas wave function. The calculations use Fermi hypernetted chain and single operator chain summation methods developed earlier [1,2]).

The $\mathcal{F}_{ij}$ is taken to be:

$$\mathcal{F}_{ij} = \sum_{p=1,8} f^p(r_{ij}) O^p_{ij}\,. \tag{1.6}$$

We neglect L^2 and $(\boldsymbol{L} \cdot \boldsymbol{S})^2$ correlations that the potentials $v^{p=9,14}$ may induce in matter, however the effect of $v^{p=9,14}$ on the $f^{p=1,8}$ is taken into account. This semi-perturbative treatment of the $v^{p=9,14}$ may be reasonable since their contributions to the nuclear matter energy are relatively small. Two-body Euler–Lagrange equations used to calculate the $f^p(r_{ij})$ are reported in sect. 2. These equations, based on recent work on the v_6 models [6]), allow different ranges for the tensor and other correlations, and use "quenched" interactions.

The results of several nuclear matter calculations with Reid models are given in sect. 3. First we show that the $f^p(r_{ij})$ calculated with the new Euler-Lagrange equations give nuclear matter energies that are 3–4 MeV lower than those obtained previously [2]) for the Reid v_8 model. Next we report the energies obtained with the Reid v_{12} model. This model has Reid's potentials in $^1S_0$, $^3S_1$-$^3D_1$, $^1P_1$, $^3P_2$-$^3F_2$ and $^1D_2$ channels. It is not repulsive enough in $^3P_0$ and $^3P_1$ channels. It does not give the $^3D_2$ and $^3D_3$ phases individually, but approximately reproduces the weighed average of ^3D_J phases:

$$\delta(^3D_{av}) = \tfrac{1}{15}[3\delta(^3D_1) + 5\delta(^3D_2) + 7\delta(^3D_3)]\,. \tag{1.7}$$

It is rather bad in the $^1F_3$ state. We obtain equilibrium $k_F = 1.65$ fm^{-1} and energy $E_0 = -17.2$ MeV with the Reid v_{12} model.

Recently Day [7]) has used a realistic Reid model for Brueckner-Bethe calculations of nuclear-matter. This model has different potentials supplied by Reid in each *JS* state having total angular momentum $J \leq 6$. The interaction in states having $J > 6$ is approximated by the Reid v_6 model. It corresponds to an extremely complicated two-nucleon interaction operator which is very inconvenient for variational calculations. However, the Reid v_{12} model should be fairly close to this Reid–Day model. Day obtains equilibrium $k_F = 1.53$ fm^{-1} and energy $E_0 = -17.2$ MeV which are reasonably close to our results for the Reid v_{12} model. The results obtained with the realistic v_{14} model are also given in sect. 3. It gives equilibrium $k_F = 1.7$ fm^{-1} and $E_0 = 17.5$ MeV.

In sect. 4 we assume that the entire difference between the calculated equilibrium k_F and E_0 and the empirical values deduced from liquid drop model fits, is due to the three-nucleon interactions. This assumption may not be completely valid, because our nuclear matter calculations are certainly not exact. Nevertheless we add, in a phenomenological way, the contributions of three-nucleon interactions to obtain the correct energy, density and compressibility of nuclear matter. The main object of this work is to define a nuclear hamiltonian that explains the two-nucleon scattering data and the nuclear matter properties.

The equation of state of hot and cold nucleon matter, and the structure of neutron stars has been studied with this hamiltonian [8]). It should also be possible to study the effect of the postulated three-nucleon interactions on the binding energy and structure of light nuclei such as ^{3}H, ^{4}He, ^{6}He etc.

2. Nuclear matter calculations with v_{14} interaction

Variational calculations of nuclear matter generally begin by defining a set of correlation functions $f^p(r, d^p, \alpha^p \ldots)$ where d^p, $\alpha^p \ldots$ are variational parameters. The energy expectation value $E(k_F, d^p, \alpha^p \ldots)$ is calculated and minimized with respect to variations in d^p, $\alpha^p \ldots$. This approximate procedure is used because the variational problem $\delta E = 0$ with respect to arbitrary variations in f^p is too complex.

In practice the $f^p(r, d^p, \alpha^p \ldots)$ are obtained by minimizing the two-body cluster contribution of an interaction $(\bar{v} - \lambda)$. The interaction $\bar{v}$ is related to the bare interaction:

$$\bar{v}_{ij} = \sum_{p=1,14} \alpha^p v^p(r_{ij}) O^p_{ij}, \tag{2.1}$$

the variational parameters α^p are meant to simulate the quenching of the spin-isospin interactions between particles 1 and 2 due to flipping of the spin and/or isospin of particle 1 or 2 by other particles in matter [6]). The $\lambda^p(r)$ are primarily meant to simulate the screening effects in matter. They are determined by the healing distances d^p of $f^p(r_{ij})$; the d^p are treated as variational parameters.

For the sake of brevity and clarity we use letters c, σ, τ, t, b and q to represent operators 1, $\sigma_i \cdot \sigma_j$, $\tau_i \cdot \tau_j$, S_{ij}, $(\boldsymbol{L} \cdot \boldsymbol{S})_{ij}$ and L^2_{ij}. So v^c, $v^{\sigma\tau}$, v^t, $v^{q\tau}$, v^{bb}, respectively,

denote v^1, v^4, v^5, v^{11}, v^{13} etc. In general minimizing the two-body cluster energy with the interaction $\bar{v}-\lambda$ gives eight coupled equations for the functions $f^{p=1,8}$. If, however, we assume that the healing distances satisfy the relations:

$$d^{\mathrm{c}} = d^{\sigma} = d^{\tau} = d^{\sigma\tau} = d_{\mathrm{c}}\,, \tag{2.2}$$

$$d^{\mathrm{t}} = d^{\mathrm{t}\tau} = d_{\mathrm{t}}\,, \tag{2.3}$$

$$d^{\mathrm{b}} = d^{\mathrm{b}\tau} = d_{\mathrm{b}}\,, \tag{2.4}$$

the equations decouple into four sets, one for each pair of spin and isospin: $T, S = 1, 0;\ 1, 1;\ 0, 0$ and $0, 1$. At least in v_6 models (2.2) is found to be a reasonable assumption [6]).

It is convenient to define T, S channel functions:

$$x^{\mathrm{c}}_{T,S} = x^{\mathrm{c}} + (4S-3)x^{\sigma} + (4T-3)x^{\tau} + (4S-3)(4T-3)x^{\sigma\tau}\,, \tag{2.5}$$

$$x^{\mathrm{q}}_{T,S} = x^{\mathrm{q}} + (4S-3)x^{\mathrm{q}\sigma} + (4T-3)x^{\mathrm{q}\tau} + (4S-3)(4T-3)x^{\mathrm{q}\sigma\tau}\,, \tag{2.6}$$

$$x^{\mathrm{y}}_{T,S} = x^{\mathrm{y}} + (4T-3)x^{\mathrm{y}\tau}\,, \quad \mathrm{y} = \mathrm{t},\ \mathrm{b}\ \text{and}\ \mathrm{bb}\,. \tag{2.7}$$

In the above equations x can be v, $\bar{v}$ or f. The $x^{\mathrm{y}}_{T,S}$ are relevant only in the $S=1$ channels. The "uncorrelated ϕ" are defined as:

$$\phi^{\mathrm{c}}_{T,S}(r) = \{1-(-1)^{T+S}l^2(k_{\mathrm{F}}r)\}^{1/2}\,, \tag{2.8}$$

$$\phi^{\mathrm{q}}_{T,S}(r) = \{\tfrac{1}{5}k_{\mathrm{F}}^2r^2 - (-1)^{T+S}rl(k_{\mathrm{F}}r)l'(k_{\mathrm{F}}r)\}^{1/2}\,, \tag{2.9}$$

$$\phi^{\mathrm{qq}}_{T,S}(r) = \{\tfrac{12}{175}k_{\mathrm{F}}^4r^4 + \tfrac{2}{5}k_{\mathrm{F}}^2r^2 - (-1)^{T+S}\tfrac{1}{2}(r\cdot\nabla)^2 l^2(k_{\mathrm{F}}r)\}^{1/2}\,, \tag{2.10}$$

$$l(x) = 3(\sin(x) - x\cos(x))/x^3\,. \tag{2.11}$$

They give the expectation values of operators 1, L^2 and L^4 with Fermi gas wave functions:

$$\frac{1}{A}\left\langle \Phi\Big|\sum_{i<j} x^{p}_{T,S}(r_{ij})O^{p}_{ij}\Big|\Phi\right\rangle = \tfrac{1}{32}\rho\,(2S+1)(2T+1)\int \mathrm{d}^3r\, x^{p}_{T,S}(r)(\phi^{p}_{T,S}(r))^2\,. \tag{2.12}$$

Here A is the number of nucleons, and $p = \mathrm{c}$, q or qq. Note that the present $\phi^{\mathrm{c}}_{T,S}$ corresponds to both $\phi_{T,S}$ and $\phi_{\mathrm{t},T}$ of refs. [1,2]), while the $\phi^{\mathrm{q}}_{T,S}$ corresponds to $\phi_{\mathrm{b},T}$. The present notation is more transparent.

It is simple to calculate the two-body cluster energy C_2 as a sum of four terms, each coming from a T, S channel:

$$C_2 = \sum_{T,S} C_{2,T,S}\,, \tag{2.13}$$

The $C_{2,T,S}$ are functions of $\bar{v}^{p}_{T,S}$ and $f^{p'}_{T,S}$; $p = \mathrm{c}$ and q, and $p' = \mathrm{c}$ in $S=0$ states, while $p = \mathrm{c}$, t, b, q and bb, and $p' = \mathrm{c}$, t and b in $S=1$ states. Minimizing the $C_{2,T,S}$ gives us the Euler–Lagrange (EL) equation for the $f^{\mathrm{c}}_{T,0}$ in singlet states:

$$\left\{-\frac{\hbar^2}{m}[\nabla^2 + 2(\phi^{\mathrm{c}}_{T,0})^{-1}\nabla\phi^{\mathrm{c}}_{T,0}\cdot\nabla) + \bar{v}^{\mathrm{c}}_{T,0} - \lambda^{\mathrm{c}}_{T,0} + (\phi^{\mathrm{c}}_{T,0})^{-2}(\phi^{\mathrm{q}}_{T,0})^2\bar{v}^{\mathrm{q}}_{T,0}\right\} f^{\mathrm{c}}_{T,0} = 0\,, \tag{2.14}$$

and a set of coupled EL equations for the $f^{c}_{T,1}$, $f^{t}_{T,1}$ and $f^{b}_{T,1}$ in triplet states:

$$\left\{-\frac{\hbar^2}{m}[\nabla^2+2(\phi^{c}_{T,1})^{-1}\nabla\phi^{c}_{T,1}\cdot\nabla]+\bar{v}^{c}_{T,1}-\lambda^{c}_{T,1}+(\phi^{c}_{T,1})^{-2}(\phi^{q}_{T,1})^2(\bar{v}^{q}_{T,1}+\tfrac{2}{3}\bar{v}^{bb}_{T,1})\right\}f^{c}_{T,1}$$
$$+\{8(\bar{v}^{t}_{T,1}-\lambda^{t}_{T,1})-\tfrac{2}{3}(\phi^{c}_{T,1})^{-2}(\phi^{q}_{T,1})^2\bar{v}^{bb}_{T,1}\}f^{t}_{T,1}$$
$$+\tfrac{2}{3}(\bar{v}^{b}_{T,1}-\lambda^{b}_{T,1}-\tfrac{1}{2}\bar{v}^{bb}_{T,1})(\phi^{c}_{T,1})^{-2}(\phi^{q}_{T,1})^2 f^{b}_{T,1}=0\,, \tag{2.15}$$

$$\left\{-\frac{\hbar^2}{m}[\nabla^2+2(\phi^{c}_{T,1})^{-1}\nabla\phi^{c}_{T,1}-6/r^2]+\bar{v}^{c}_{T,1}-\lambda^{c}_{T,1}-2(\bar{v}^{t}_{T,1}-\lambda^{t}_{T,1})-3(\bar{v}^{b}_{T,1}-\lambda^{b}_{T,1})\right.$$
$$\left.+6\bar{v}^{q}_{T,1}+9\bar{v}^{bb}_{T,1}+(\bar{v}^{q}_{T,1}+\tfrac{5}{6}\bar{v}^{bb}_{T,1})(\phi^{c}_{T,1})^{-2}(\phi^{q}_{T,1})^2\right\}f^{t}_{T,1}$$
$$+\{\bar{v}^{t}_{T,1}-\lambda^{t}_{T,1}-\tfrac{1}{12}(\phi^{c}_{T,1})^{-2}(\phi^{q}_{T,1})^2\bar{v}^{bb}_{T,1}\}f^{c}_{T,1}$$
$$-\tfrac{1}{12}\{\bar{v}^{b}_{T,1}-\lambda^{b}_{T,1}-2\bar{v}^{bb}_{T,1}\}(\phi^{c}_{T,1})^{-2}(\phi^{q}_{T,1})^2 f^{b}_{T,1}=0\,, \tag{2.16}$$

$$\left\{-\frac{\hbar^2}{m}[\nabla^2+2(\phi^{q}_{T,1})^{-1}\nabla\phi^{q}_{T,1}\cdot\nabla]+\bar{v}^{c}_{T,1}-\lambda^{c}_{T,1}-\bar{v}^{t}_{T,1}+\lambda^{t}_{T,1}-\tfrac{1}{2}(\bar{v}^{b}_{T,1}-\lambda^{b}_{T,1})\right.$$
$$\left.+(\phi^{q}_{T,1})^{-2}(\phi^{qq}_{T,1})^2(\bar{v}^{q}_{T,1}+\bar{v}^{bb}_{T,1})\right\}f^{b}_{T,1}+(\bar{v}^{b}_{T,1}-\lambda^{b}_{T,1}-\tfrac{1}{2}\bar{v}^{bb}_{T,1})f^{c}_{T,1}$$
$$-(\bar{v}^{b}_{T,1}-\lambda^{b}_{T,1}-2\bar{v}^{bb}_{T,1})f^{t}_{T,1}=0\,. \tag{2.17}$$

The $\lambda^{p}_{T,S}$ are chosen to satisfy the imposed constraints on the $f^{p}_{T,S}$:

$$f^{c}_{T,S}(r>d_c)=1\,, \tag{2.18}$$
$$f^{t}_{T,S}(r>d_t)=0\,, \tag{2.19}$$
$$f^{b}_{T,S}(r>d_b)=0\,. \tag{2.20}$$

For the sake of brevity we assume $d_t\geq d_b\geq d_c$, a condition satisfied by the equilibrium values, and obtain:

$$\lambda^{t}_{T,1}(r>d_t)=\bar{v}^{t}_{T,1}-\tfrac{1}{12}(\phi^{c}_{T,1})^{-2}(\phi^{q}_{T,1})^2\bar{v}^{bb}\,, \tag{2.21}$$
$$\lambda^{b}_{T,1}(r>d_b)=\bar{v}^{b}_{T,1}-\tfrac{1}{2}\bar{v}^{bb}(1-4f^{t})(1-f^{t})^{-1}\,, \tag{2.22}$$
$$\lambda^{c}_{T,1}(r>d_c)=\bar{v}^{c}_{T,1}+(\phi^{c}_{T,1})^{-2}(\phi^{q}_{T,1})^2(\bar{v}^{q}_{T,1}+\tfrac{2}{3}\bar{v}^{bb}_{T,1})+8(\bar{v}^{t}_{T,1}-\lambda^{t}_{T,1})f^{t}_{T,1}$$
$$-\tfrac{2}{3}(\phi^{c}_{T,1})^{-2}(\phi^{q}_{T,1})^2\{\bar{v}^{bb}_{T,1}f^{t}_{T,1}-(\bar{v}^{b}_{T,1}-\lambda^{b}_{T,1}-\tfrac{1}{2}\bar{v}^{bb}_{T,1})f^{b}_{T,1}\}\,, \tag{2.23}$$
$$\lambda^{c}_{T,0}(r>d_c)=\bar{v}^{c}_{T,0}+(\phi^{c}_{T,0})^{-2}(\phi^{q}_{T,0})^2\bar{v}^{q}_{T,0}\,. \tag{2.24}$$

The $\lambda^{p}_{T,S}(r<d_p)$ are constants chosen so as to make the gradients $f'^{p}_{T,S}=0$ at $r=d_p$.

The nuclear-matter $E(k_F, d^p, \alpha^p, \ldots)$ is calculated and minimized with respect to variations in $d^p, \alpha^p, \ldots$. The unspecified variational parameters could be the β^p used

in refs. [1,2]) or any other relevant choice. It is convenient to decompose E into three parts:

$$E = E_6 + E_{LS} + E_Q , \tag{2.25}$$

where E_6 is the sum of all diagrams that do not have $\boldsymbol{L}\cdot\boldsymbol{S}$ correlations, and $\boldsymbol{L}\cdot\boldsymbol{S}$ or v^{9-14} interactions. The E_6 is large, and hence calculated carefully as described in ref. [1]). E_{LS} is the sum of all diagrams that have at least one $\boldsymbol{L}\cdot\boldsymbol{S}$ line but no v^{9-14} interactions. The $\boldsymbol{L}\cdot\boldsymbol{S}$ lines may represent either interactions or correlations. The E_{LS} is calculated as discussed in ref. [2]).

The E_Q is the sum of all diagrams having v^{9-14} interaction lines. Unlike E_{LS} it is simply the expectation value:

$$E_Q = \frac{1}{A}\left\langle \Psi \middle| \sum_{i<j} \sum_{p=9,14} v^p(r_{ij}) O_{ij}^p \middle| \Psi \right\rangle \Big/ \langle \Psi | \Psi \rangle . \tag{2.26}$$

The contribution of two-body direct diagrams [fig. 1.1] to E_Q is given by:

$$\sum_{i=1,8} \sum_{j=9,14} \sum_{k=1,8} \tfrac{1}{2}\rho \int \mathrm{d}^3 r \langle C(f_{12}^i O_{12}^i v_{12}^j O_{12}^j f_{12}^k O_{12}^k) \rangle_{\mathrm{dir}} , \tag{2.27}$$

$$\langle \boldsymbol{X}_{12} \rangle_{\mathrm{dir}} = \left(\frac{3}{4\pi k_{\mathrm{F}}^3}\right)^2 \int \mathrm{d}^3 k_1 \int \mathrm{d}^3 k_2 \phi_{12}^* \boldsymbol{X}_{12} \phi_{12} , \tag{2.28}$$

$$\phi_{12} = \exp\left(i(\boldsymbol{k}_1 \cdot \boldsymbol{r}_1 + \boldsymbol{k}_2 \cdot \boldsymbol{r}_2)\right) . \tag{2.29}$$

The C-part of the product in eq. (2.27) can be written as a sum of three terms:

$$C(f_{12}^i O_{12}^i v_{12}^j O_{12}^j f_{12}^k O_{12}^k) = Z_0^d(r_{12}, ijk) + Z_2^d(r_{12}, ijk) L_{12}^2 + Z_4^d(r_{12}, ijk) L_{12}^4 . \tag{2.30}$$

It is relatively easy to calculate the Z_0^d, Z_2^d and Z_4^d. First, we recall that the C-part of a product of operators can be trivially factored into two parts, one each for spin and isospin operators. Secondly, products such as $O_{ij}^\sigma O_{ij}^{\mathrm{t}}$, $O_{ij}^\sigma O_{ij}^\sigma$ etc. can be reduced to sums of terms having operators 1, O^σ, O^{t} and O^{b}. Hence, relatively few new C-parts are needed to calculate the Z^d's, and these are given below,

$$m, n = \mathrm{c}, \sigma, \mathrm{t}, \mathrm{b}, \tag{2.31}$$

$$C(O_{12}^m L_{12}^2 O_{12}^n) = \delta_{mn}(36\delta_{n\mathrm{t}} + (1-\delta_{n\mathrm{b}}) A^n L_{12}^2 + \tfrac{1}{2}\delta_{n\mathrm{b}} L_{12}^4) , \tag{2.32}$$

$$C(O_{12}^m (\boldsymbol{L}\cdot\boldsymbol{S})_{12}^2 O_{12}^n) = 54\delta_{m\mathrm{t}}\delta_{n\mathrm{t}} + Q_{mn} L_{12}^2 + \tfrac{1}{2}\delta_{m\mathrm{b}}\delta_{n\mathrm{b}} L_{12}^4 , \tag{2.33}$$

$$Q_{mn} = \begin{array}{cccc} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & -\frac{1}{2} & 5 & 1 \\ -\frac{1}{4} & -\frac{1}{4} & 1 & 0 \end{array} \tag{2.34}$$

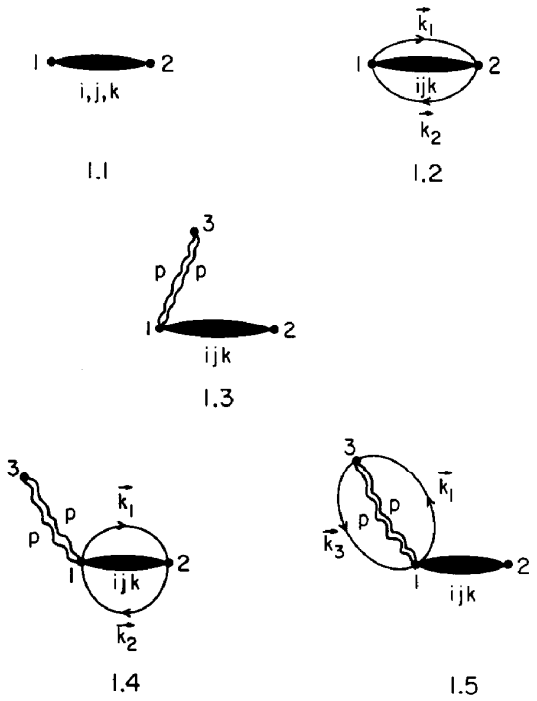

Fig. 1. The two- and three-body diagrams used to calculate W_Q. The diagrammatic notation is that given in ref. [1]), and the double wavy lines in diagrams 1.3–5 may be replaced by a dashed line representing $(f^c)^2-1$.

Here A^n is simply the $C((O^n)^2)$ as given in ref. [1]). The values of $\langle L^n\rangle_{\rm dir}$ are given in table 1.

The contribution of two-body exchange diagrams (fig. 1.2) is given by:

$$\sum_{n=1,4}\sum_{i=1,8}\sum_{j=9,14}\sum_{k=1,8} -\tfrac{1}{8}\rho\int \mathrm{d}^3r\langle C(O^n_{12}f^i_{12}O^i_{12}v^i_{12}O^j_{12}f^k_{12}O^k_{12})\rangle_{\rm ex}\,, \tag{2.35}$$

$$\langle X_{12}\rangle_{\rm ex}=\left(\frac{3}{4\pi k_{\rm F}^3}\right)^2\int \mathrm{d}^3k_1\int \mathrm{d}^3k_2\phi^*_{21}X_{12}\phi_{12}\,, \tag{2.36}$$

$$\phi_{21}=\exp\left(i(\boldsymbol{k}_1\cdot\boldsymbol{r}_2+\boldsymbol{k}_2\cdot\boldsymbol{r}_1)\right). \tag{2.37}$$

TABLE 1

$\langle L^n\rangle$

n	$\langle L^n\rangle_{\rm dir}$	$\langle L^n\rangle_{\rm ex}$
0	1	l^2
2	$\frac{1}{5}k_{\rm F}^2r^2$	rll'
4	$\frac{12}{175}k_{\rm F}^4r^4+\frac{2}{5}k_{\rm F}^2r^2$	$\frac{1}{2}(\boldsymbol{r}\cdot\nabla)^2l^2$

The C-part of the product in eq. (2.35) can also be expressed in the form (2.30); the Z_n^e (r_{12}, ijk) are calculated using eqs. (2.31)–(2.34), and $\langle L^n \rangle_{ex}$ are given in table 1.

Contributions of two-body clusters to W_Q in the v_{14} model of nuclear matter at $k_F = 1.6\,\mathrm{fm}^{-1}$ are given in table 2. They are not too large; the total two-body contribution at $k_F = 1.6\,\mathrm{fm}^{-1}$ is $\sim -54\,\mathrm{MeV}/A$. The main many-body diagram contributing to E_Q is expected to be the separable diagram 1.3 of fig. 1. We calculate only the main term of this diagram in which i and k = c. Its contribution is given by:

$$\tfrac{1}{4}\rho^2 \int (f_{12}^c)^2 (v_{12}^9 + \tfrac{1}{2} v_{12}^{13}) r_{12}^2 \, \mathrm{d}^3 r_{12} \times \tfrac{2}{3} \sum_{p=1,6} A^p \int f_{13}^p \left(-\nabla^2 f_{13}^p + (\delta_{p5} + \delta_{p6}) \frac{6}{r_{13}^2} f^p \right) \mathrm{d}^3 r_{13} \,. \tag{2.34}$$

We neglect the rather small contribution of terms having p = b, bτ to diagram 1.3. The contribution of fig. 1.3 calculated with these approximations is given in table 2.

TABLE 2

Contributions (in MeV/A) of diagrams shown in fig. 1 to v_{14} model of nuclear matter at $k_F = 1.6\,\mathrm{fm}^{-1}$

Diagram	Interaction	Contribution
1.1	j = 9–12	5.28
1.1	j = 13–14	−1.11
1.2	j = 9–12	−1.88
1.2	j = 13–14	1.26
1.3	j = 9 & 13	2.70
1.4	j = 9–12	0.26
1.5	j = 9	0.05

One can understand the magnitude of the contribution of fig. 1.3 as follows: The interaction energy of v^{9-14} interactions is roughly proportional to the average value of k^2; k is the momentum of the particle. It is thus proportional to the expectation value of the kinetic energy. The total kinetic energy in the liquid is made up of two terms, the Fermi gas kinetic energy T_F, and the kinetic energy due to correlations T_c. At $k_F = 1.6\,\mathrm{fm}^{-1}$ these terms are respectively 31.85 MeV/A and 25.54 MeV/A. The two-body diagrams get their contributions from T_F, and the $6(f^{t,t\tau})^2/r^2$ term of T_c, while the three-body diagrams get their contributions only from T_c.

The contributions of separable exchange diagrams 1.4 and 1.5 are calculated only for $v^{j=9,12}$. These are quite small (table 2). The contribution of chain diagrams to W_Q is neglected; it could be ~0.5 MeV in magnitude, and that is approximately the accuracy of our W_Q calculation. The estimated errors in the calculation of E_6 and E_{LS} are also ~0.5 MeV, and so a crude estimate of the accuracy of the energy expectation value is 1 MeV. On the other hand the variational wave function we use may not be general enough. First we minimize E by making limited variations in $f^p(r)$, second we do not include any f^{9-14} correlations that v^{9-14} may induce, and third we have no

three-body correlation operators in our Ψ_v. Improvements in Ψ_v could lower the energy by, and this is a possibly unreliable guess based on calculations done with oversimplified models [9]), about 1 MeV. So we feel that our results give reliable upper bounds, but the true energy may be up to 2 MeV below our results.

3. Results of Reid and v_{14} models

Most of the calculations use only three variational parameters d, d_t and α. We assume:

$$d^c = d^\sigma = d^\tau = d^{\sigma\tau} = d^b = d^{b\tau} = d\,, \tag{3.1}$$

$$d^t = d^{t\tau} = d_t\,. \tag{3.2}$$

At some test points we let d^b and $d^{b\tau}$ differ from d, but this did not produce an appreciable (>0.1 MeV) lowering of energy. Since v^c and v^q do not have spin and isospin operators we take:

$$\alpha^c = \alpha^q = 1\,, \tag{3.2}$$

$$\alpha^\sigma = \alpha^\tau = \alpha^{\sigma\tau} = \alpha^t = \alpha^{t\tau} = \alpha^b = \alpha^{b\tau} = \alpha^{q\sigma} = \alpha^{q\tau} = \alpha^{q\sigma\tau} = \alpha\,. \tag{3.3}$$

Again it was verified that taking α^t and $\alpha^{t\tau}$, or α^b and $\alpha^{b\tau}$ different from α does not have significant effect on $E(k_F)$, and the α^{bb} and $\alpha^{bb\tau}$ were simply set to unity in all calculations. The v^{bb} has a significant L^2 part which should not be quenched anyway.

At the equilibrium point of the v_{14} model a bigger set of variational parameters (d, d_t, α, β_σ, β_t, β_b) was used:

$$f^p(r, \alpha, d, d_t, \beta^p) = \beta^p f^p(r, \alpha, d, d_t)\,, \tag{3.4}$$

$$\beta^\sigma = \beta^\tau = \beta^{\sigma\tau} = \beta_\sigma\,, \tag{3.5}$$

$$\beta_t = \beta^{t\tau} = \beta_t\,, \tag{3.6}$$

$$\beta^b = \beta^{b\tau} = \beta_b\,; \tag{3.7}$$

fortunately the equilibrium values of β_p came out to be ~ 1, suggesting that it is adequate to minimize $E(d, d_t, \alpha)$.

The calculated $E(k_F)$ are shown in fig. 2. The Reid v_6 results are from refs. [6,10]), while the Reid-Day results are from ref. [7]). The Reid v_6 and Reid v_6–Day calculations use the same hamiltonian, the former is by the variational method, while the latter is with Brueckner–Bethe method. These two calculations are in essential agreement; the differences between their results are within their error estimates. The variational Reid v_8 energy is now much lower than that was previously reported [2]). This change is entirely due to the improvements in the calculation and parameterization of the $f^p(r)$.

The Reid v_{12} $E(k_F < 1.5\ \mathrm{fm}^{-1})$ is almost identical (within 0.2 MeV) of the v_{14} $E(k_F)$, though it saturates a little faster at $k_F > 1.6\ \mathrm{fm}^{-1}$. The breakdown of the $E(k_F)$

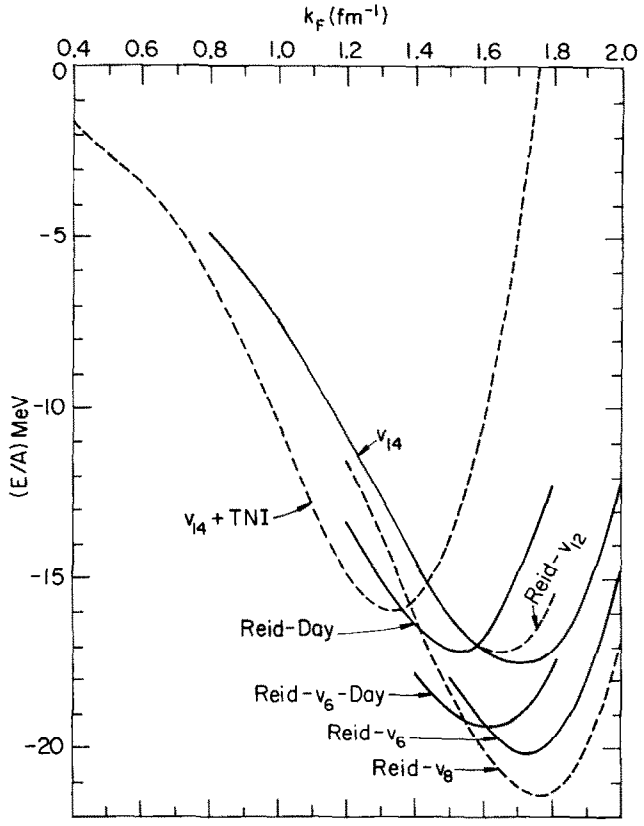

Fig. 2. The $E(k_F)$ of nuclear matter calculated with various models of nuclear hamiltonian. The curves bearing Day's name are obtained with Brueckner–Bethe method, while the others use variational method. At lower densities the Reid v_{12} and v_{14} models give similar $E(k_F)$.

is given in table 3 using the notation of ref. [1]). The various terms in E_6 of table 3 represent the following: (i) W_0 (MB) gives contribution of Fermi hypernetted chains to the two nucleon interaction energy W, (ii) W_c is the contribution of single operator chains to W, (iii) W_s and W_{cs} give contribution of separable diagrams to W, and (iv) W_F (MB), U_F and U give the many-body kinetic contributions via $\nabla_1\mathscr{F}_{12} \cdot \nabla_1\Phi$ and $\nabla_1\mathscr{F}_{12} \cdot \nabla_1\mathscr{F}_{13}$ terms.

The v_{14} interaction has a much weaker tensor force than the Reid models have. For example the deuteron D-state percentage is 5.2% and 6.5% respectively for these two models. It is thus rather remarkable that they give so close nuclear matter energies. The stronger tensor force generates larger repulsive W_s contributions, but it also gives more attractive W_c contributions (table 3). It was often remarked by Brown that the large repulsive dispersion corrections (which are buried in our W_s)

TABLE 3

Contributions to nuclear matter energy in MeV/A

Potential	v_{14}	v_{14}	v_{14}	Reid v_{12}
k_F	1.13	1.33	1.6	1.6
d(fm)	2.53	2.15	1.79	1.79
d_t(fm)	4.04	3.44	2.86	2.86
α	0.8	0.8	0.8	0.8
T_F	15.89	22.01	31.85	31.85
E_6: 2-body	−30.91	−40.06	−53.42	−51.67
E_6: W_0(MB)	−0.45	−3.14	−7.47	−7.85
E_6: W_c	2.33	3.12	3.30	−0.64
E_6: W_s	3.51	4.44	5.74	8.33
E_6: W_{cs}	−0.31	−0.30	−0.13	0.40
E_6: W_F(MB) + U_F	0.35	0.25	0.46	0.89
E_6: U	0.14	0.14	1.09	1.13
E_6: total	−9.56	−13.56	−18.58	−17.57
E_{LS}: 2-body	−1.24	−2.34	−4.75	−3.67
E_{LS}: MB	0.13	0.07	−0.40	−0.24
E_Q: 2-body	0.22	1.10	3.55	2.19
E_Q: MB	0.72	1.35	3.00	2.19
E: total	−9.71	−13.40	−17.19	−17.11

generated by tensor correlations will be significantly reduced by the attractive tensor ring diagrams (which contribute to our W_c). The Reid v_{12} variational results are in fair agreement with Day's Brueckner–Bethe results with the Reid-Day model. We may expect some differences between Reid-Day and Reid v_{12} models because they have different interactions in $^3P_0$, $^3P_1$, $^3D_2$, $^3D_3$ and higher waves, however, a fraction of the difference may be in the method used to calculate $E(k_F)$.

The v^{9-14} give a repulsive contribution which helps reduce the overbinding of nuclear matter significantly. It also seems to reduce the equilibrium k_F by ~0.1 fm^{-1}. But the calculated equilibrium k_F is much larger than the 1.33 fm^{-1} suggested by liquid drop fits. The curve labeled v_{14} + TNI (three-nucleon interactions) in fig. 2 has the correct equilibrium density ($k_F = 1.33$ fm^{-1}), energy (−16.0 MeV), and compressibility (240 MeV) [11]).

There does not seem to be a big difference between the Reid v_{12} and the v_{14} models. Earlier, larger differences were found in the $E(k_F)$ given by Reid v_8 and various BJ v_8 (Bethe–Johnson) models [2]). However, v_8 models based on BJ $^3P_2$-$^3F_2$ potentials give very repulsive phases in $^3P_0$ and $^3P_1$ states [5]), and so are almost unrealistic. The Reid v_{12} model is reasonably realistic, but still somewhat pedagogical, and its results should be taken accordingly. For example, there could be some effect of the stronger tensor force in the Reid v_{12} model, but it gets cancelled by that of the slightly weaker repulsion in $^3P_0$ and $^3P_1$ states, and does not show up in the comparison of the $E(k_F)$ of Reid v_{12} and v_{14} models.

4. Three-nucleon interaction contributions

Realistic two-nucleon interactions seem to overbind nuclear matter very significantly at $k_F > 1.5$ fm^{-1}, whereas at low k_F (<1.3 fm^{-1}) they give an underbinding. The overbinding at $k_F > 1.5$ is certainly beyond any possible errors in the many-body calculations, while the underbinding at $k_F < 1.3$ fm^{-1} is not that overwhelming. However, we know that the realistic two-nucleon interactions leave light nuclei like the triton [12]) and ^{4}He [ref. [13])] underbound, and this strongly suggests the need for more attraction at low densities. In this section we take a very phenomenological point of view, and add contributions of TNI to the v_{14} model to get the "correct" $E(k_F)$ around $k_F = 1.33$ fm^{-1}.

To begin with let us ignore the spin-isospin dependence of the TNI; we will put in the effect of its isospin dependence rather crudely. We hope that the TNI can be expanded as:

$$V_{123} = \sum_l \sum_{\text{cyc}} U_l u_l(r_{12}) u_l(r_{13}) P_l(\cos\theta_1)\,, \tag{4.1}$$

where U_l is a strength parameter, $u_l(r)$ are functions of interparticle distance, θ_i are the inner angles of the triangle $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, $\boldsymbol{r}_3$, and $\sum_{\text{cyc}}$ represents a sum of three terms obtained by cyclic rotation of indices 1, 2 and 3. At very high densities only the $l = 0$ component of V_{123} will contribute. It must be repulsive and so we refer to it as TNR. The $l \neq 0$ components will give negative contributions to $E(k_F)$ through correlations, but these will saturate as density increases, and we refer to them collectively as TNA.

If one takes very seriously the indication of a "hole" in the ^{3}He nucleus [14]), which two-nucleon interactions fail to give [15]), a three-nucleon P_2 interaction appears to be an attractive proposal to try out. A nucleon can be at the c.m. of the ^{3}He nucleus only when the three nucleons are in a line. The $l = 2$ term of V_{123} can be very repulsive in this configuration, and attractive when the nucleons are at the corners of an equilateral triangle leaving a hole in the middle.

The two-pion-exchange TNI corresponding to field theoretical diagram fig. 3.1 has been studied [16–18]), and it does contain terms of type (4.1) with $l = 2$ and $u_2(r) = T(r)$, where $T(r)$ is the radial shape of the one-pion-exchange tensor force:

$$T(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2}\right) \frac{e^{-\mu r}}{\mu r} (1 - \exp(-cr^2))^2\,, \tag{4.2}$$

$$\mu = 0.7 \text{ fm}^{-1}\,, \tag{4.3}$$

$$c = 2 \text{ fm}^{-2}\,. \tag{4.4}$$

The cut-off in (4.2) is that in the v_{14} model [5]); different authors generally use different cut-off's. The $l = 2$ terms of (4.1) may possibly dominate TNA.

As a model for the $l = 0$ TNR we consider separable diagrams involving isobar states [19]). Their field theory analogs are very schematically illustrated in figs. 3.2 and 3.3. This interaction requires the exchange of two pions between nucleons 1 and 2, and of another two pions between 1 and 3. We may thus take $u_0(r) = T^2(r)$.

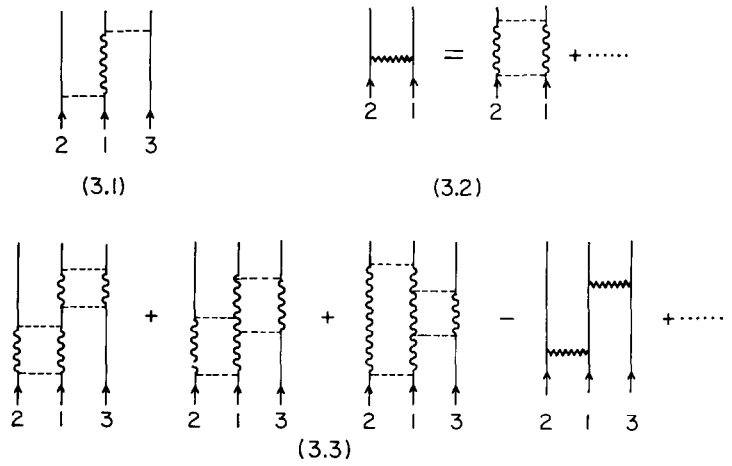

Fig. 3. Schematic field theoretical diagrams illustrating the TNI. The dashed and zig-zag lines represent one pion, and v_I interactions, while the full and wavy lines represent nucleons and isobars. All diagrams are not shown, but only a few are used for illustration. For example the r.h.s. diagram of 3.2 can have: (i) one of the isobar lines replaced by a nucleon line; or (ii) the two pions crossed; or (iii) interactions between the two pions, etc.

The v_{14} interaction model [5]) has three terms:

$$v_{14,ij} = v_\pi + v_I + v_S \tag{4.5}$$

which respectively correspond to the one-pion-exchange, two-pion-exchange isobar box (illustrated in fig. 3.2), and the short range core. The radial shape of v_I is taken to be $T^2(r)$ in accord with its assumed nature:

$$v_I = \sum_{p=1,14} I^p T^2(r_{ij}) O^p_{ij}, \tag{4.6}$$

and I^p are constants obtained by fitting the phase shifts. I^c is large and negative, and the rest $I^{p\neq c}$ are much smaller. The main effect of a TNR of type:

$$\sum_{\text{cyc}} U_0 T^2(r_{12}) T^2(r_{13}) \tag{4.7}$$

will be to weaken this attractive v_I, and we include it in the many-body calculation by making the two-nucleon density dependent:

$$v_{14} + \text{TNR} = v_\pi + v_I \exp(-\gamma_1 \rho) + v_S \tag{4.8}$$

We have chosen to use $\exp(-\gamma_1\rho)$, instead of $(1-\gamma_1\rho)$ the TNR gives, with the hope that it would take into account four- and more-body separable diagrams at very high densities. However, $\gamma_1\rho$ is very small (<0.03) in the region of nuclear densities, and whether one uses $\exp(-\gamma_1\rho)$ or $(1-\gamma_1\rho)$ hardly makes any difference.

The contribution of TNA will have the following characteristics. At low densities it will be proportional to ρ^2, and it will saturate as ρ increases. The interaction of fig.

3.1, which we use as a model for TNA, has zero C-part because it leaves free τ_2, τ_3, σ_2 and σ_3. It will contribute only when there are spin-isospin correlations between 2 and 3 that close the ring. Thus, its contribution will be proportional to the expectation value of $(\tau_1 \cdot \tau_2)^2$:

$$\langle(\tau_1 \cdot \tau_2)^2\rangle = 3 - 2\left(\frac{N-Z}{A}\right)^2, \tag{4.9}$$

where N and Z are the numbers of neutrons and protons. The C-parts of the spin operators do not depend upon N and Z, so we will ignore them. A reasonable parameterization of the TNA contribution is then given by:

$$\text{TNA} = \gamma_2 \rho^2 \exp(-\gamma_3 \rho)\left(3 - 2\left(\frac{N-Z}{A}\right)^2\right). \tag{4.10}$$

We calculate $E(k_F, v_{14}+\text{TNR})$ with the interaction (4.8) by the variational method, and add the TNA contribution (4.10) to obtain the nuclear matter energy. The parameters γ_{1-3} are varied to obtain equilibrium $k_F = 1.33$ fm^{-1}, $E_0 = -16$ MeV and $K = 240$ MeV. Their values are:

$$\gamma_1 = 0.15 \text{ fm}^3, \tag{4.11}$$

$$\gamma_2 = -700 \text{ fm}^6, \tag{4.12}$$

$$\gamma_3 = 13.6 \text{ fm}^3, \tag{4.13}$$

and the $E(k_F)$ obtained with these is shown by the curve labeled $v_{14}+\text{TNI}$ in fig. 2.

TABLE 4

"Interaction energies" in nuclear matter (MeV)

k_F(fm^{-1})	1.13	1.23	1.33	1.43	1.53
ρ(fm^{-3})	0.0975	0.1257	0.1589	0.1957	0.2419
T_F	15.89	18.82	22.01	25.44	29.12
v_{14}	−25.60	−30.36	−35.41	−40.57	−45.66
TNI	−3.89	−3.76	−2.59	−0.18	3.57
E	−13.60	−15.30	−15.99	−15.31	−12.97

Table 4 gives the contributions of various interactions to the nuclear matter energy. The interaction energies are defined as follows:

$$v_{14} - \text{interaction energy} = E(k_F, v_{14}) - T_F, \tag{4.14}$$

$$\text{TNI energy} = E(k_F, v_{14}+\text{TNR}) - E(k_F, v_{14}) + \text{TNA}. \tag{4.15}$$

They are not expectation values, because the wave function changes when the interaction is changed.

All the expectation values given in table 5 are calculated with the $v_{14}+\text{TNR}$ wave function. However, since $\gamma_1\rho$ is so very small there is little difference between the v_{14}

TABLE 5

"Expectation values" in nuclear matter (MeV)

$k_F(\mathrm{fm}^{-1})$	1.13	1.23	1.33	1.43	1.53
$\langle T\rangle$	27.54	32.79	38.72	45.33	52.62
$\langle v_\pi\rangle$	−20.69	−23.92	−27.26	−30.63	−33.94
$\langle v_I\rangle$	−94.31	−118.8	−148.9	−185.7	−230.0
$\langle v_S\rangle$	77.75	98.39	124.1	155.8	194.7
⟨TNR⟩	1.41	2.24	3.52	5.40	8.15
TNA	−5.30	−6.00	−6.11	−5.58	−4.58

and v_{14} + TNR wave functions at nuclear matter density. Note that the TNA given by eq. (4.10) is not an expectation value. There should be some kinetic energy of three-nucleon correlations, and a correction to $\langle v_{14}\rangle$ buried in it. Thus the expectation value of $\langle T\rangle$ could be a few MeV higher than listed in table 5; $2T_F$ is suggested as a reasonable estimate of $\langle T\rangle$ by the present work. The $\langle T\rangle$ is in fact measurable, and its measurement would be of great help in arriving at a correct theory of nuclear matter.

The TNA and TNR give much smaller contributions than the two-nucleon v_{14}. Microscopic calculations [18]) of TNA due to the two-pion-exchange three-nucleon interaction of fig. 3.1, give ∼−2 to −3 MeV at equilibrium density, against our −6 MeV. These calculations certainly do not rule out a larger TNA contribution. But it should be pointed out that we do not calculate $E(k_F, v_{14}+\mathrm{TNR})$ exactly. As mentioned in sect. 2 the variationally calculated $E(k_F, v_{14}+\mathrm{TNR})$ could be 0–2 MeV too high, and the phenomenological TNA may in part be correcting for the differences between the variational Ψ_v and the exact Ψ.

The sum of separable diagrams involving isobar states should give some idea of the TNR contribution in matter. It is difficult to calculate it microscopically, for the required isobar–nucleon interactions are experimentally hard to study. Of the allowed $\mathrm{NN}\rightleftarrows\mathrm{NN}$, $\mathrm{NN}\rightleftarrows\mathrm{N}\Delta$, $\mathrm{NN}\rightleftarrows\Delta\Delta$, $\mathrm{N}\Delta\rightleftarrows\mathrm{N}\Delta$, $\mathrm{N}\Delta\rightleftarrows\Delta\mathrm{N}$, $\mathrm{N}\Delta\rightleftarrows\Delta\Delta$, and $\Delta\Delta\rightleftarrows\Delta\Delta$ only the $\mathrm{NN}\to\mathrm{NN}$, $\mathrm{NN}\rightleftarrows\mathrm{N}\Delta$ and $\mathrm{NN}\rightleftarrows\Delta\Delta$ contribute significantly to the nucleon-nucleon scattering, but all are required to calculate TNR. Wiringa [19]) has attempted to calculate the sum of separable diagrams involving isobar states using the model of Niephaus, Gari and Sommer [20]) of transition potentials. Very preliminary results, which neglect $v^{p>6}$ $\mathrm{NN}\to\mathrm{NN}$ interactions, suggest that the sum of "isobar separable" diagrams is 1.42, 3.7 and 7.9 MeV at $k_F = 0.9$, 1.1 and 1.3 fm^{-1}, respectively. Our TNR contribution is smaller by approximately a factor of two than these estimates.

The expectation values $\langle v_I\rangle$ and $\langle v_S\rangle$ are both quite large, but their sum is much smaller. The $\langle v_I\rangle$ and $\langle v_S\rangle$ individually will be very dependent on the two-nucleon interaction model, and so their values should be regarded with some caution. In relativistic-mean-field theories, such as Walecka's [21]) the attraction $\langle v_I\rangle$ is attributed to a scalar field, and the kinetic energies are calculated with an effective mass

$m^* = m_N + \langle v_I \rangle$. The m^* is smaller than m_N, and so generates larger kinetic energies which help to saturate nuclear matter. Any such relativistic effect that may exist in nuclear matter is also buried in the phenomenological TNR.

The present approach to TNI contributions in matter is very phenomenological and not aesthetically satisfactory. A better approach will be to assume a nuclear hamiltonian:

$$H_{\text{nucl}} = -\frac{\hbar^2}{2m}\nabla_i^2 + \sum_{i<j}\sum_p v^p(r_{ij})O_{ij}^p + \sum_{i<j<k} V_{ijk}\,, \tag{4.16}$$

and determine the important components of V_{ijk} by fitting the binding energies and form factors of light nuclei such as ^{3}H and ^{4}He. Some of the very neutron rich isotopes of helium (^{6}He, ^{8}He, etc.) may provide information on the three-neutron interaction. Techniques to obtain reasonable wave functions and calculate energies of few-body systems with hamiltonians such as (4.16), are being developed. In the meanwhile, eqs. (4.8) and (4.10) define an effective hamiltonian that fits the two-nucleon scattering data and nuclear matter properties, and it seems to have reasonable TNI contributions. We hope that it may be of use in studies of the equation of state of hot and cold nucleon matter, and the structure of nuclei and neutron stars.

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