Digital Image Processing

Morphological Image Processing

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Images taken from: R. Gonzalez and R. Woods. Digital Image Processing, Prentice Hall, 2008

Morphological Image Processing and Analysis

In form and feature, face and limb,
I grew so like my brother,
That folks got taking me for him
And each for one another.

Henry Sambrooke Leigh,
Carols of Cockayne, The Twins

Contents

Mathematical morphology provides tools for the representation and description of image regions (e.g. boundary extraction, skeleton, convex hull).

It provides techniques for pre- and post-processing of an image (morphological thinning, pruning, filtering).

Its principles are based on set theory.

Applications to both binary and graylevel images.

Preliminaries

The four horizontal and vertical neighbours of a pixel p are called 4-*neighbours* of p and are denoted by $N_4(p)$.

The four diagonal neighbours of a pixel p are denoted by $N_D(p)$.

Together $N_4(p)$ and $N_D(p)$ are called the 8-*neighbours* of pixel p and are denoted by $N_8(p)$.

Preliminaries (cont.)

Adjacency of pixels

Let V be the set of intensity values used to define the adjacency (e.g. $V=\{1\}$ for binary images).

4-*adjacency*. Two pixels p and q with values in V are 4-adjacent if q is in $N_4(p)$.

8-*adjacency*. Two pixels p and q with values in V are 8-adjacent if q is in $N_8(p)$.

Preliminaries (cont.)

Adjacency of pixels

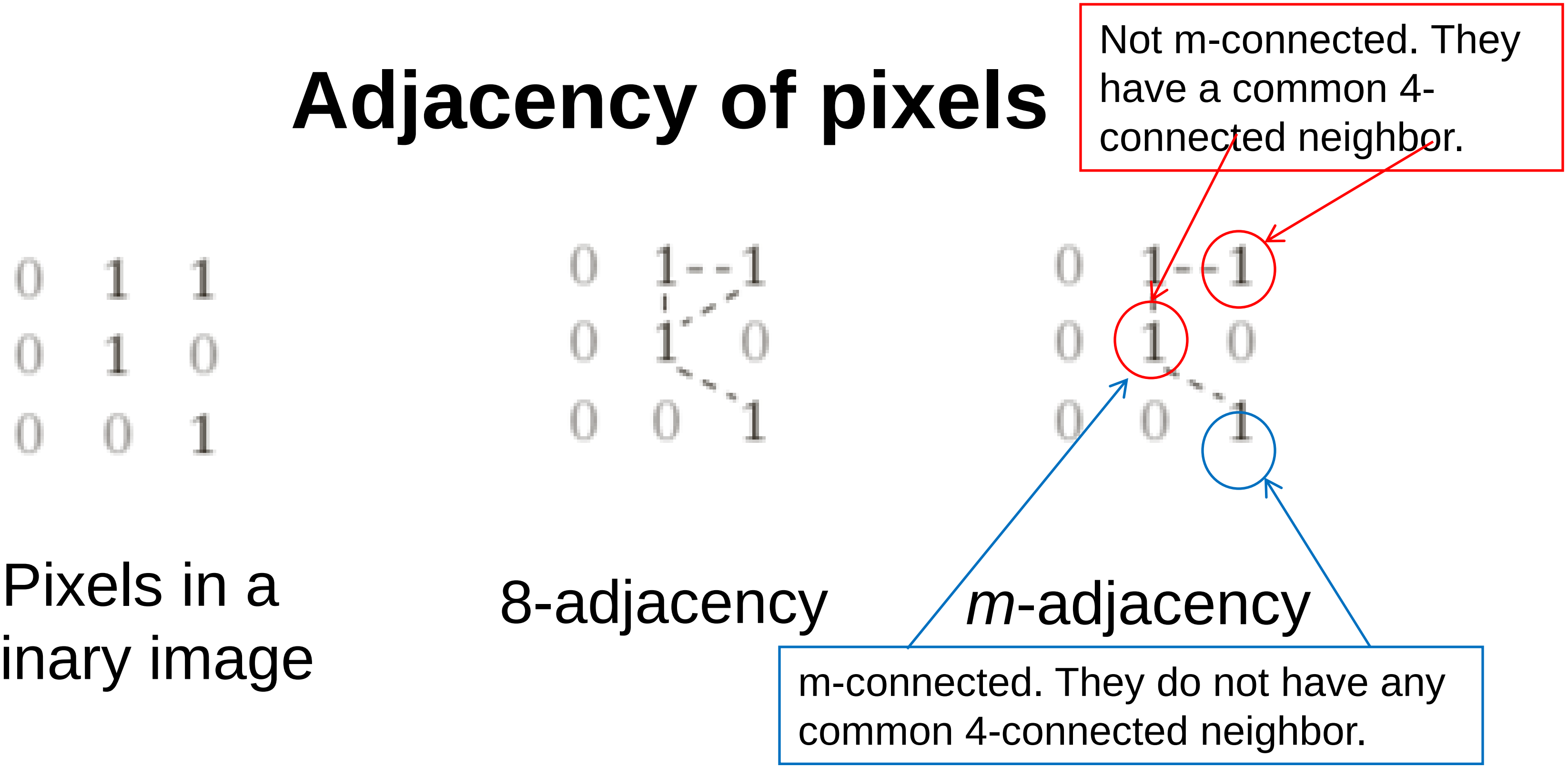
m-adjacency (mixed adjacency). Two pixels p and q with values in V are *m*-adjacent if

- q is in $N_4(p)$, or
- q is in $N_D(p)$ *and* the set $N_4(p) \cap N_4(q)$ has no pixels whose values are in V.

Mixed adjacency is a modification of the 8-*adjacency*. It is introduced to eliminate ambiguities of 8-*adjacency*.

Preliminaries (cont.)

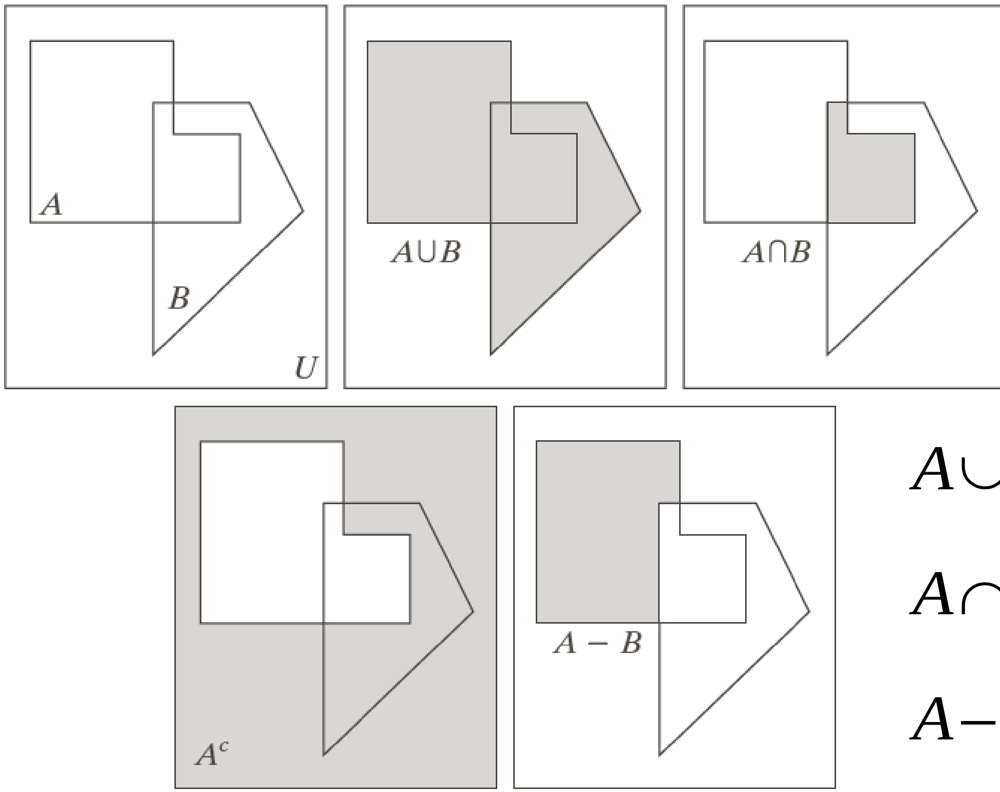
Adjacency of pixels



The role of *m*-adjacency is to define a single path between pixels. It is used in many image analysis and processing algorithms.

Preliminaries (cont.)

Basic set operations.



$$A \cup B = \{w \mid w \in A \text{ OR } w \in B\}$$

$$A \cap B = \{w \mid w \in A \text{ AND } w \in B\}$$

$$A - B = \{w \mid w \in A, \ w \notin B\} = A \cap B^c$$

$$A^c = \{w \mid w \notin A\}$$

Preliminaries (cont.)

The above operations assume that the images containing the sets are binary and involve only the pixel location.

Union and intersection are different when we define set operations involving intensity values:

$$A \cup B = \left\{ \max_z(a,b) \,|\, a \in A,\ b \in B \right\}$$

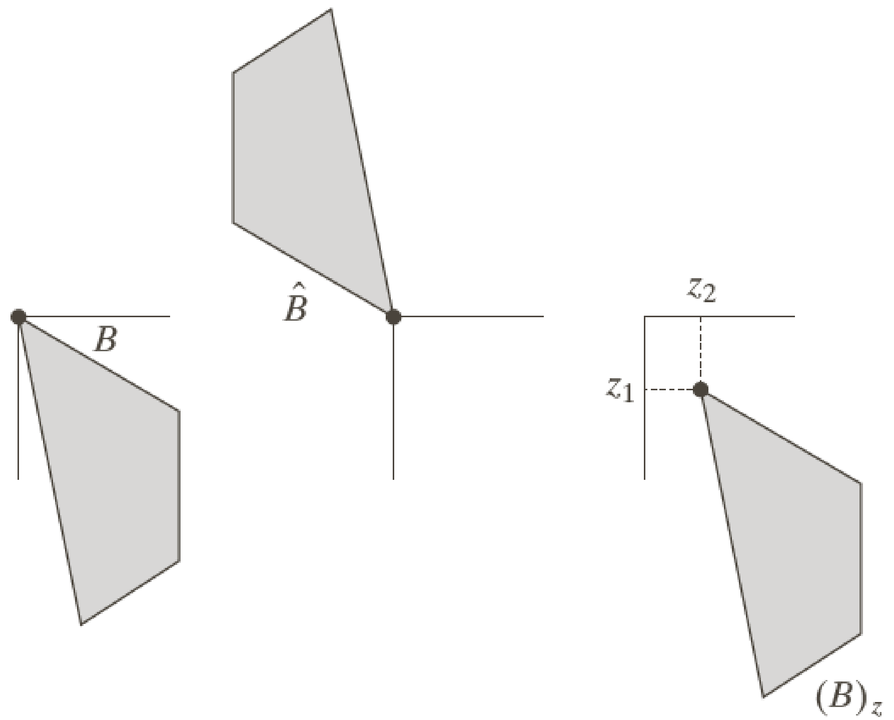
$$A \cap B = \left\{ \min_z(a,b) \,|\, a \in A,\ b \in B \right\}$$

The elements of the sets are gray values on the same location z.

Preliminaries (cont.)

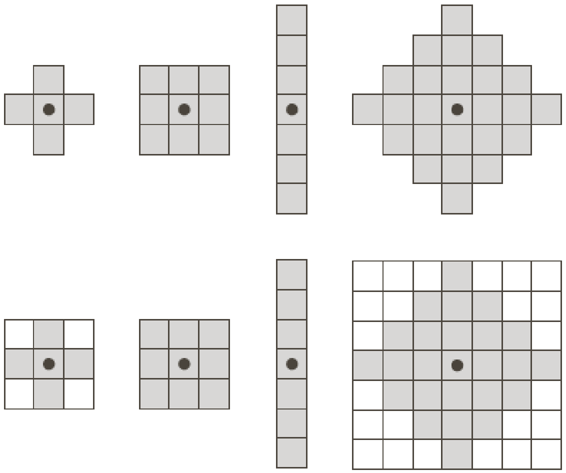
Set reflection: $\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$

Set translation by z: $(B)_z = \{c \mid c = b + z, \text{ for } b \in B\}$



Preliminaries (cont.)

- Set reflection and translation are employed to structuring elements (SE).
- SE Are small sets or subimages used to examine the image under study for properties of interest.
- The origin must be specified.
- Zeros are appended to SE to give them a rectangular form.



Note: gray colour represents a value of one and white colour a zero value.

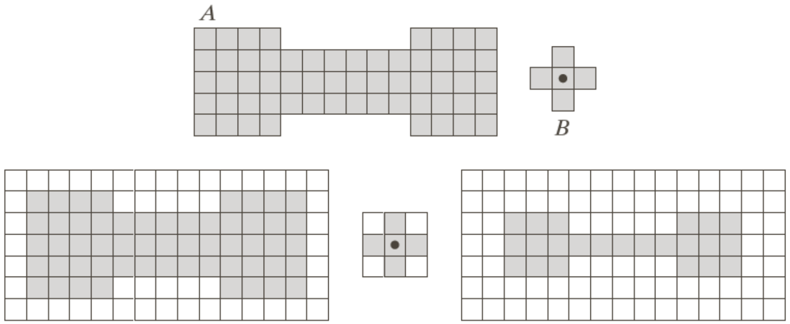
Preliminaries (cont.)

The origin of the SE B visits every pixel in an image A.

It performs an operation (generally non linear) between its elements and the pixels under it.

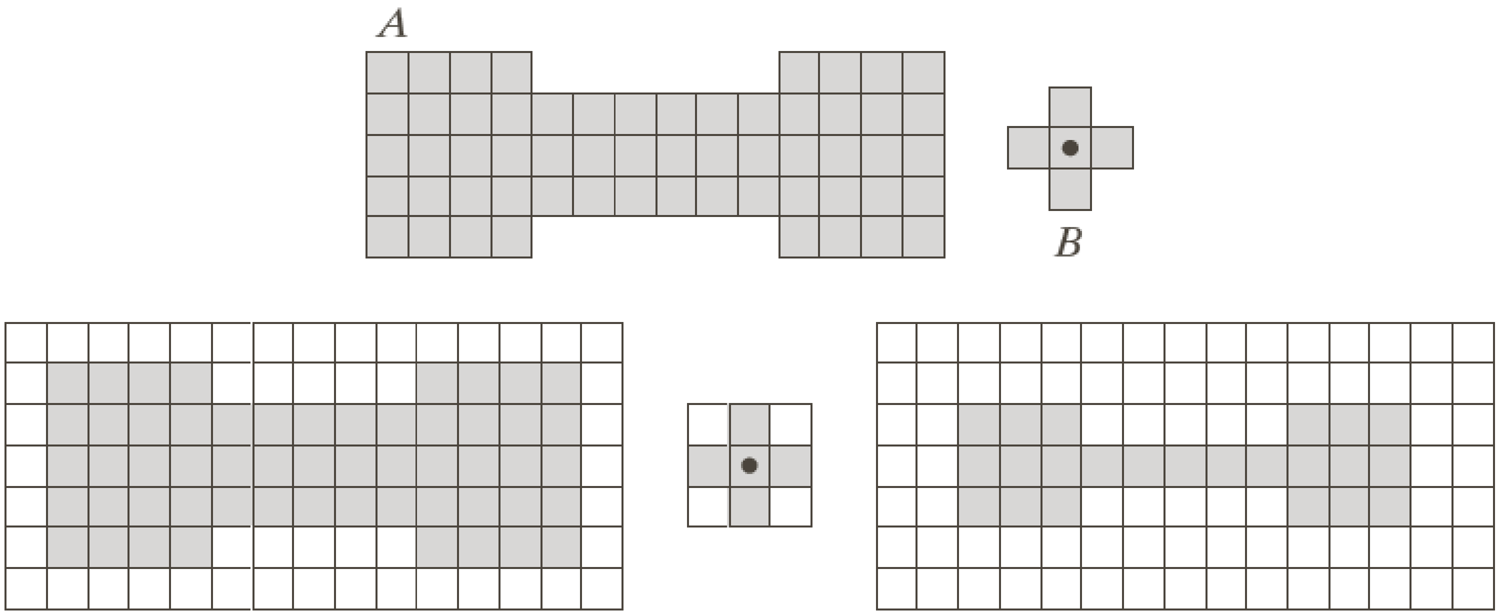
It is then decided if the pixel will belong to the resulting set or not based on the results of the operation.

Zero padding is necessary (like in convolution) to ensure that all of the elements of A are processed.



Preliminaries (cont.)

For example, it marks the pixel under its center as belonging to the result if B is completely contained in A ($A \in \mathbb{Z}^2$, $B \in \mathbb{Z}^2$).



Morphological Operations

- Some basic operations
  - Erosion.
  - Dilation.
  - Opening.
  - Closing.
- Applications
  - Morphological filtering.
  - The hit-or-miss transformation.

Erosion

The erosion of a set A by a SE B is defined as

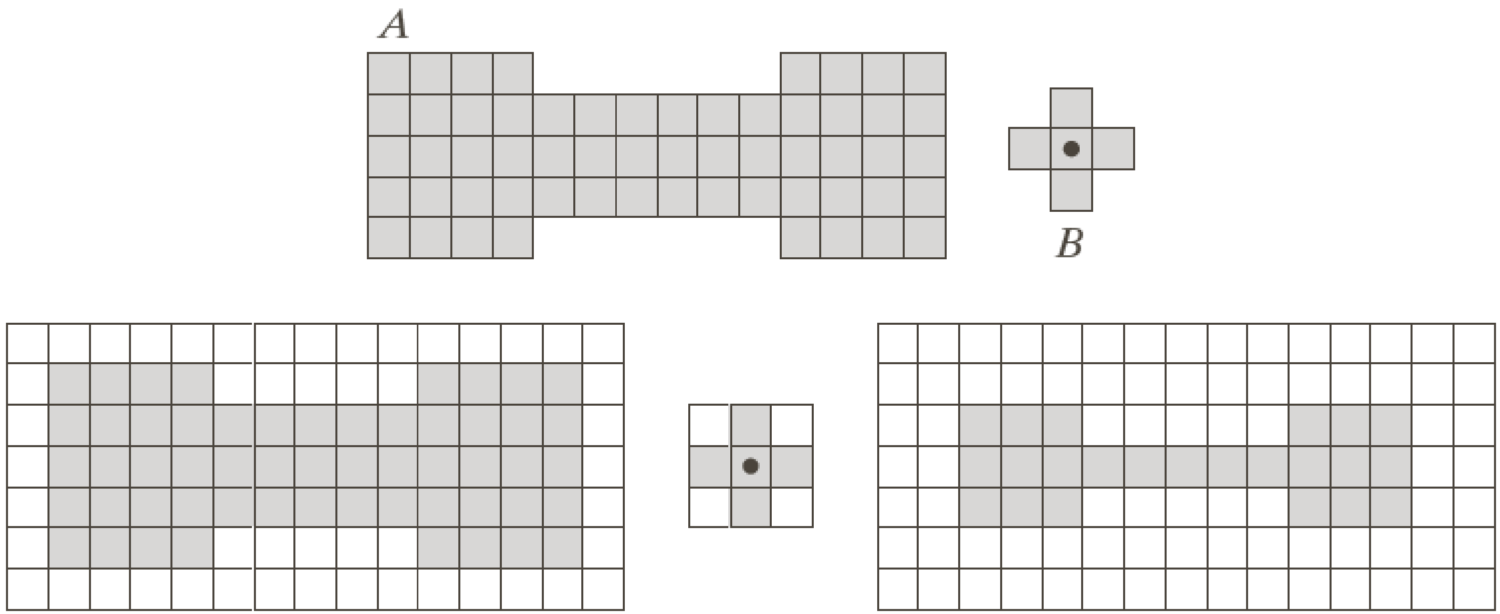
$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

The result is the set of all points z such that B translated by z is contained in A.

Equivalently:

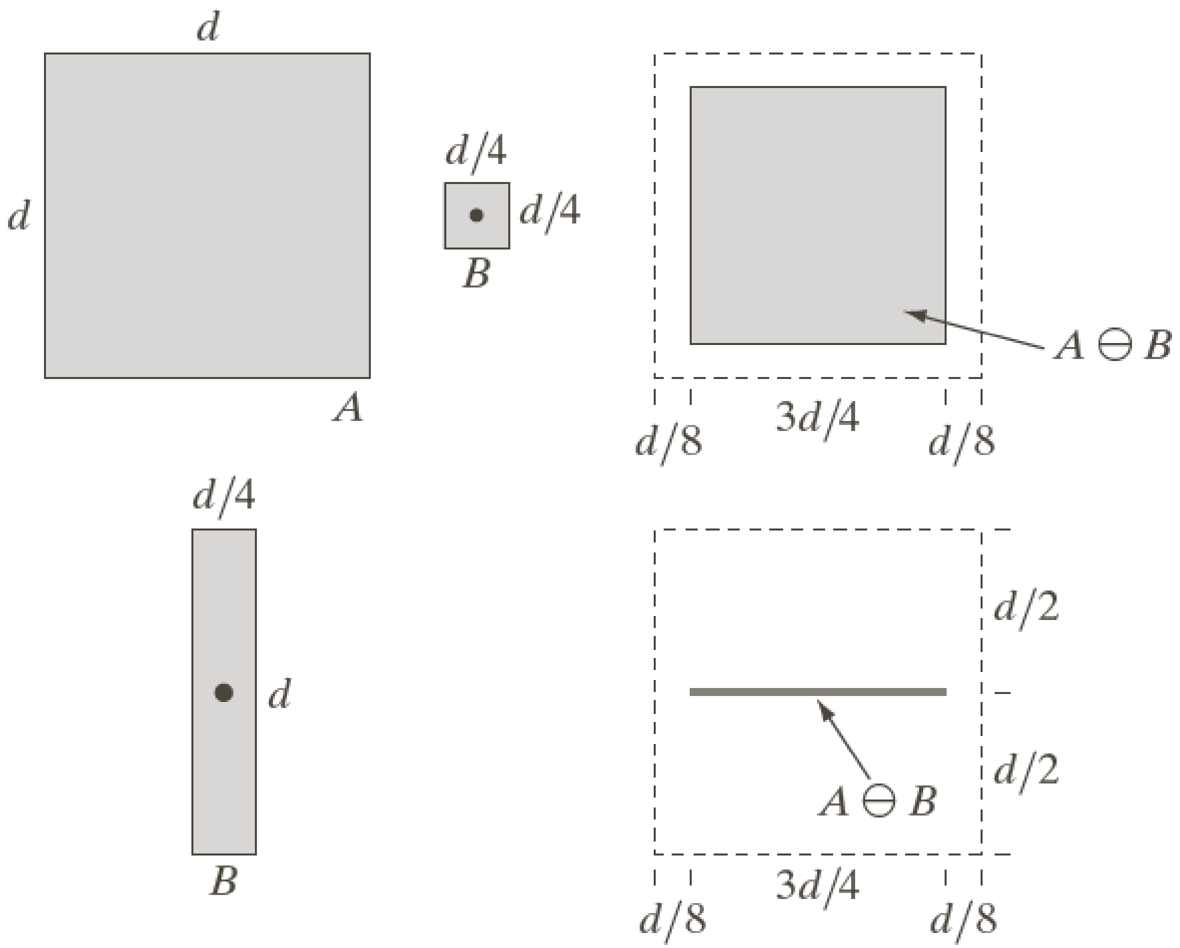
$$A \ominus B = \{z \mid (B)_z \cap A^c = \varnothing\}$$

Erosion (cont.)



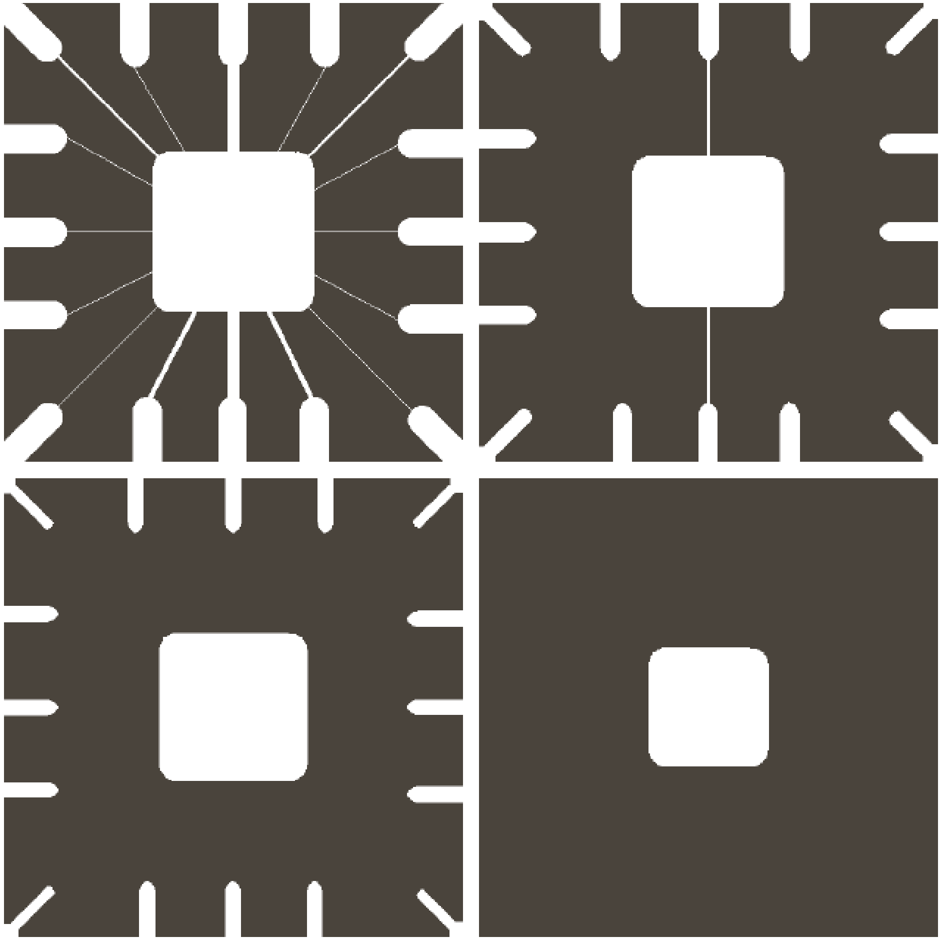
Erosion is a shrinking operation

Erosion (cont.)



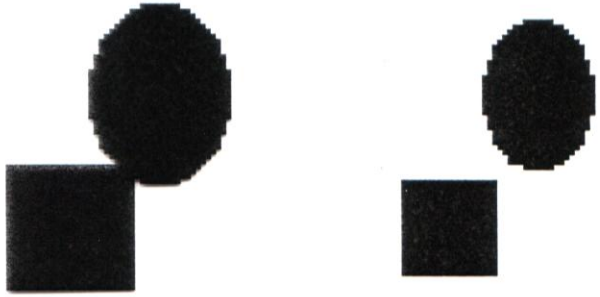
Erosion (cont.)

Erosion by a square SE of varying size

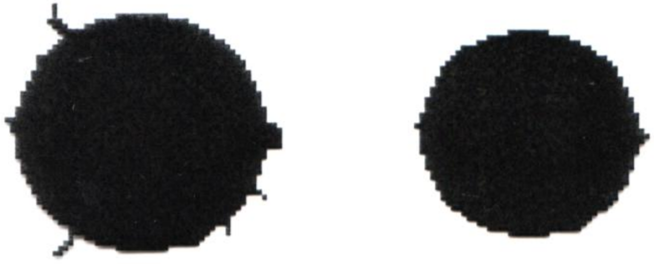


Erosion (cont.)

Erosion can split apart joined objects



Erosion can strip away extrusions



Watch out: Erosion shrinks objects

Dilation

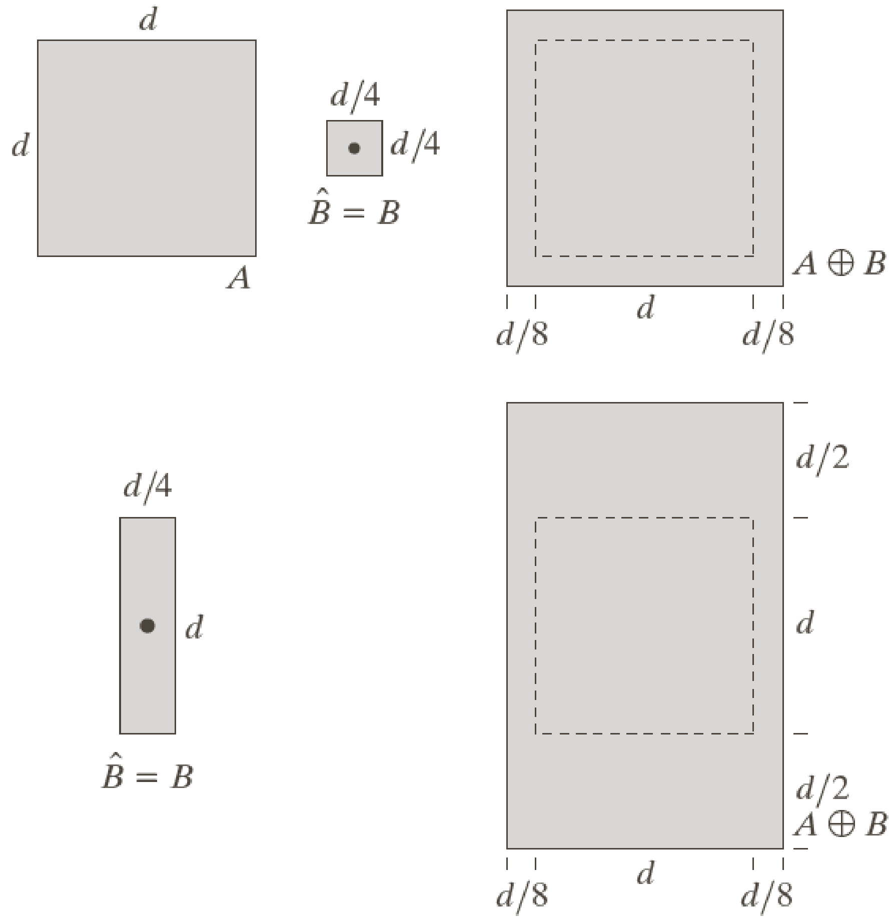
The dilation of a set A by a SE B is defined as

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$$

The result is the set of all points z such that the reflected B translated overlap with A at at least one element.

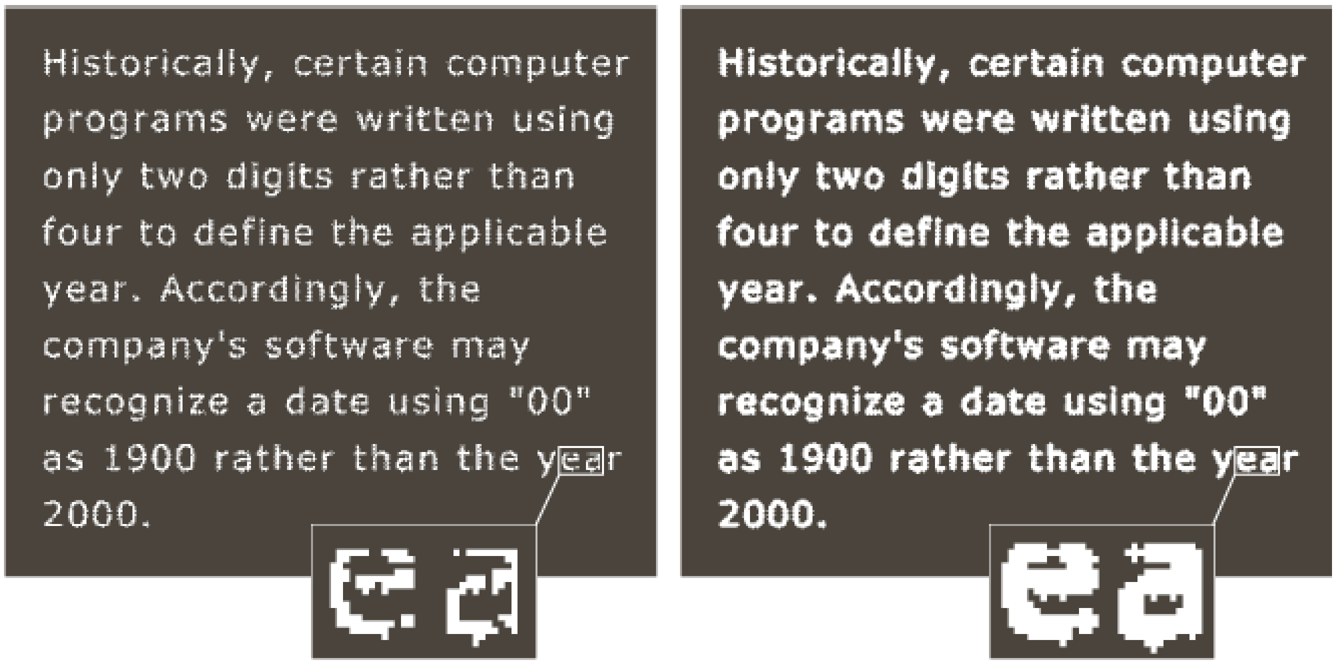
Equivalently: $A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$

Dilation (cont.)



Dilation is a thickening operation

Dilation (cont.)



0	1	0
1	1	1
0	1	0

Dilation bridges gaps.

Contrary to low pass filtering it produces a binary image.

Dilation (cont.)

Dilation can repair breaks

Dilation can repair intrusions

Watch out: Dilation enlarges objects

Duality

Erosion and dilation are dual operations with respect to set complementation and reflection:

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

Also,

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

The duality is useful when the SE is symmetric: The erosion of an image is the dilation of its background.

Compound Operations

More interesting morphological operations can be performed by combining erosions and dilations in order to reduce shrinking or thickening.

The most widely used of these *compound operations* are:

- Opening
- Closing

Opening

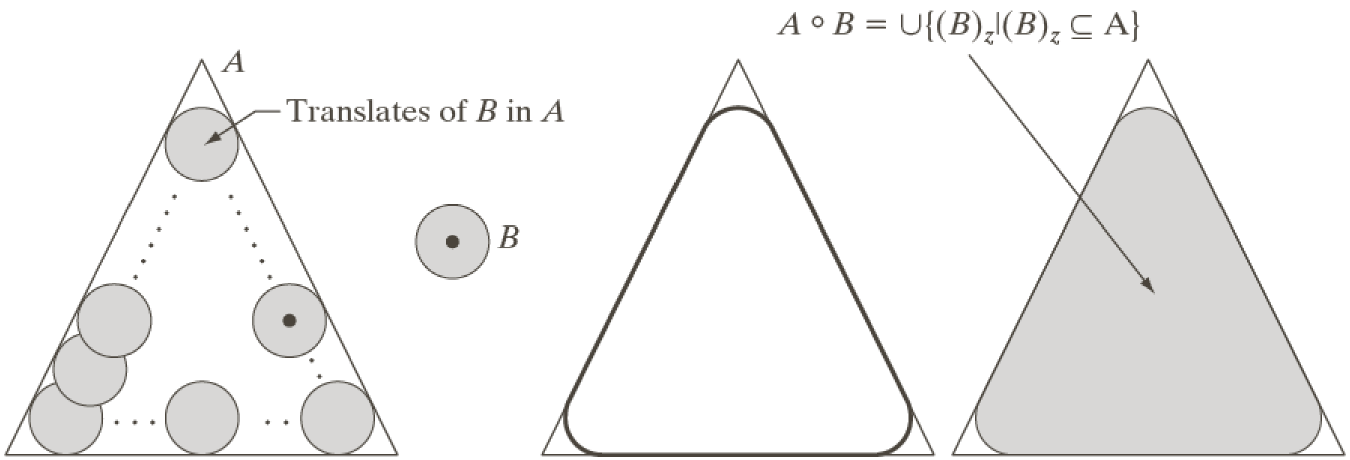
The opening of set A by structuring element B is defined as

$$A \circ B = (A \ominus B) \oplus B$$

which is an erosion of A by B followed by a dilation of the result by B.

Opening (cont.)

Geometric interpretation: The boundary of the opening is defined by points of the SE that reach the farthest into the boundary of A as B is "rolled" inside of this boundary.



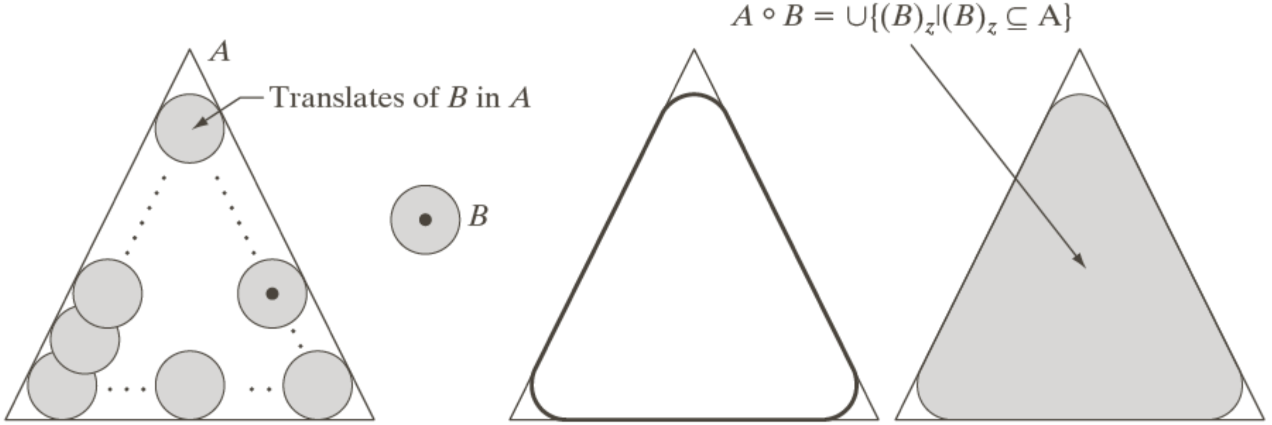
$$A \circ B = \bigcup \left\{ (B)_z \mid (B)_z \subseteq A \right\}$$

Opening (cont.)

Notice the difference with the simple erosion:

$$A \ominus B = \{z \mid (B)_z \subseteq A\} \qquad A \circ B = \bigcup \{(B)_z \mid (B)_z \subseteq A\}$$

If B translated by z lies inside A, then the result contains the whole set of points covered by the SE and not only its center as it is done in the erosion.



Closing

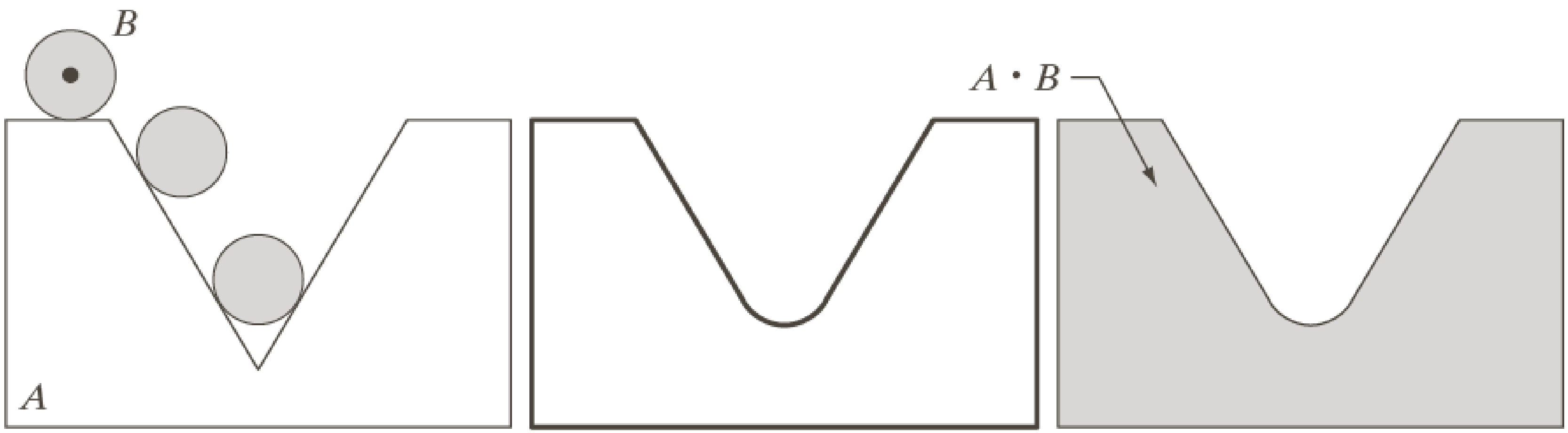
The closing of set A by structuring element B is defined as

$$A \bullet B = (A \oplus B) \ominus B$$

which is an dilation of A by B followed by an erosion of the result by B.

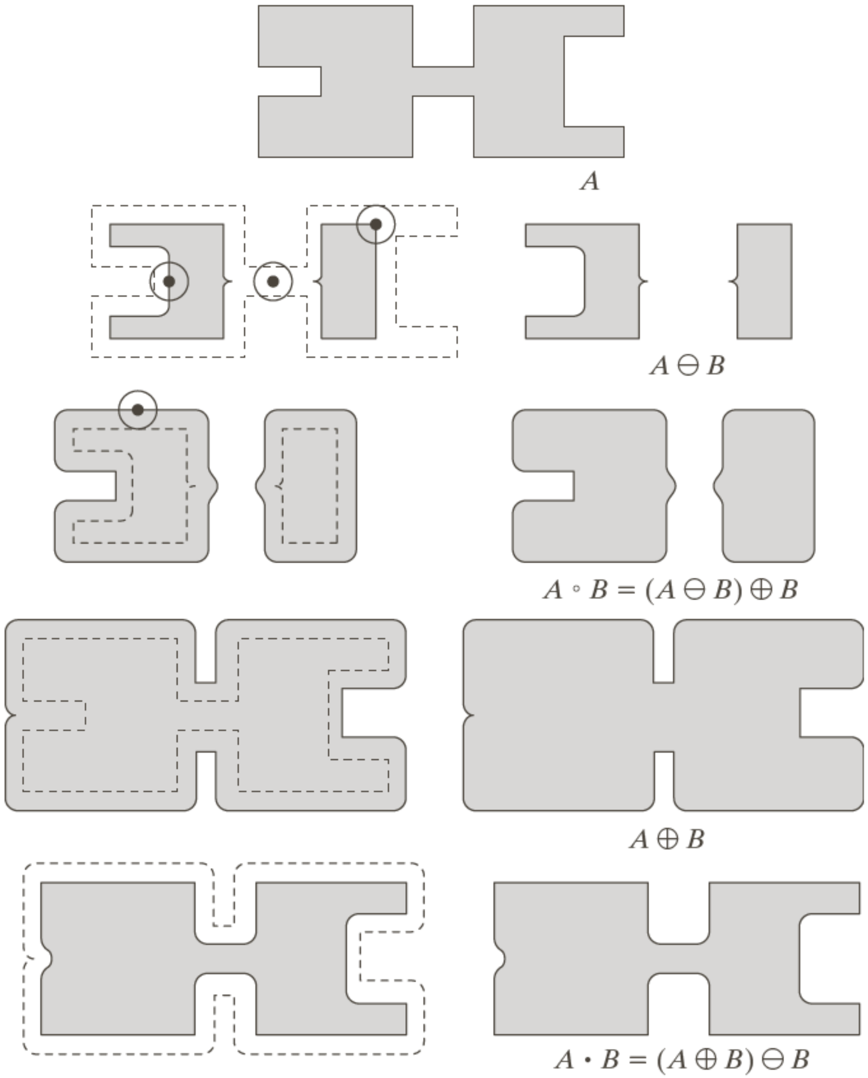
Closing (cont.)

It has a similar geometric interpretation except that *B* is rolled on the outside of the boundary:



$$A \bullet B = \{w \mid (B)_z \cap A \neq \varnothing, \text{for all translates of } (B)_z \text{ containing } w\}$$

Opening and Closing



Erosion: elements where the disk can not fit are eliminated.

Opening: outward corners are rounded.

Dilation: inward intrusions are reduced in depth.

Closing: inward corners are rounded.

Duality

Opening and closing are dual operations.

Erosion-Dilation duality

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

Opening-Closing duality

$$(A \bullet B)^c = A^c \circ \hat{B}$$

$$(A \circ B)^c = A^c \bullet \hat{B}$$

Properties of Opening and Closing

Opening:

$$A \circ B \subseteq A$$

$$C \subseteq D \Rightarrow C \circ B \subseteq D \circ B$$

$$(A \circ B) \circ B = A \circ B$$

Closing:

$$A \subseteq A \bullet B$$

$$C \subseteq D \Rightarrow C \bullet B \subseteq D \bullet B$$

$$(A \bullet B) \bullet B = A \bullet B$$

The last properties, in each case, indicate that multiple openings or closings have no effect after the first application of the operator

Morphological Filtering Example

B

1	1	1
1	1	1
1	1	1

The image contains noise:

•Light elements on dark background.

•Dark elements on the light components of the fingerprint.

Objective: Eliminate noise while distorting the image as little as possible.

We will apply an opening followed by closing.

Morphological Filtering Example (cont.)

A

$A \ominus B$

Background noise completely removed (noise components smaller than the SE).

The size of the dark noise elements in the fingerprint structure increased (inner dark structures).

Morphological Filtering Example (cont.)

$A \ominus B$

$(A \ominus B) \oplus B = A \circ B$

The dilation reduced the size of the inner noise or eliminated it completely.

However, new gaps were created by the opening between the fingerprint ridges.

Morphological Filtering Example (cont.)

$A \circ B$

$A \circ B \oplus B$

The dilation reduces the new gaps between the ridges but it also thickens the ridges.

Morphological Filtering Example (cont.)

$A \circ B \oplus B$

$[A \circ B \oplus B] \ominus B = (A \circ B) \bullet B$

The final erosion (resulting to a closing of the opened image) makes the ridges thinner.

Morphological Filtering Example (cont.)

A

$(A \circ B) \bullet B$

The final result is clean of noise but some ridges were not fully repaired.

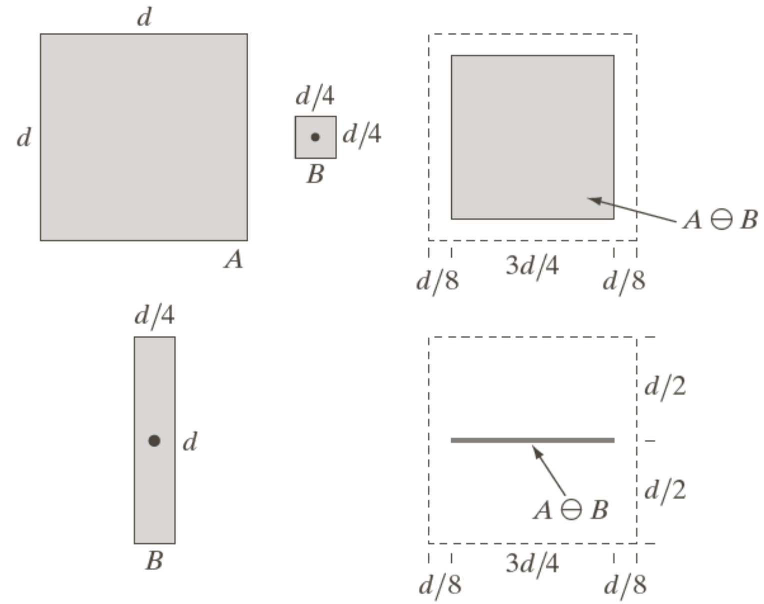
We should impose conditions for maintaining the connectivity (we will see a more advanced algorithm).

The Hit-or-Miss Transformation

Basic tool for shape detection.

Erosion of A by B: the set of all locations of the *origin* of B that B is completely contained in A.

Alternatively, it is the set of all locations that B found a match (hit) in A.



The Hit-or-Miss Transformation (cont.)

There are many possible locations for the shape we search (the SE!). If we are looking for disjoint (disconnected) shapes it is natural to assume a background for it.

Therefore, we seek to match B in A and simultaneously we seek to match the background of B in A^c.

Mathematically, the hit-or-miss transformation is:

$$A * B = (A \ominus B) \cap (A^c \ominus B_b)$$

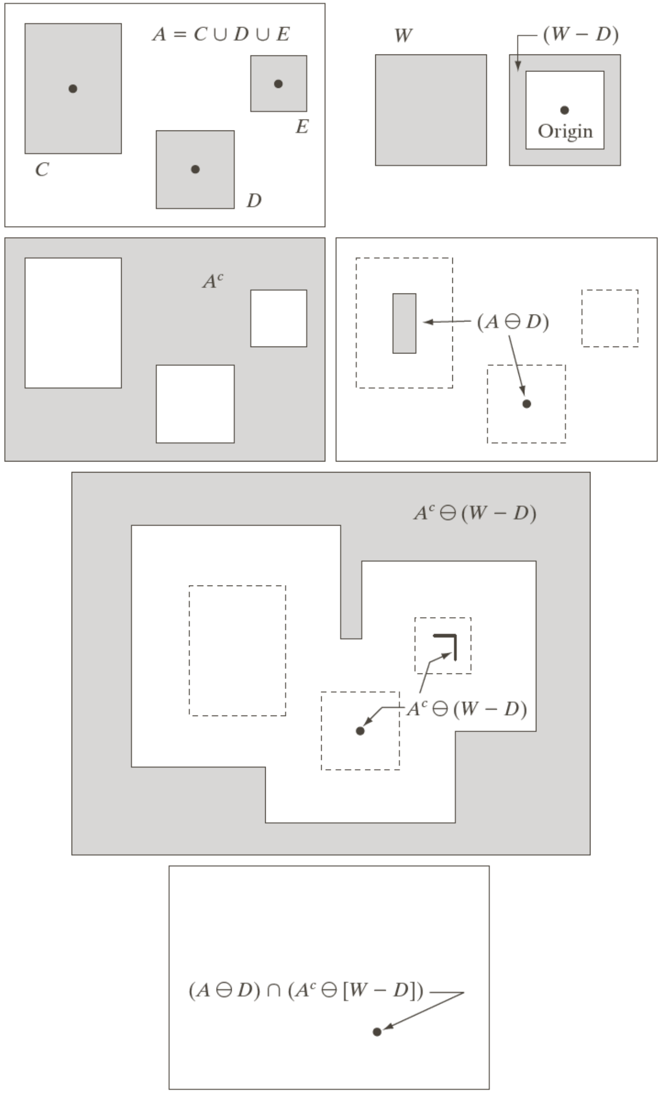
The Hit-or-Miss Transformation (cont.)

We seek to locate the shape D in the image A.

We define a thin background W for the shape.

We take the intersection of the two results

$$A * D = (A \ominus D) \cap \left[A^c \ominus (W - D) \right]$$



Morphological Algorithms

Using these morphological operations we may extract image components for shape representation:

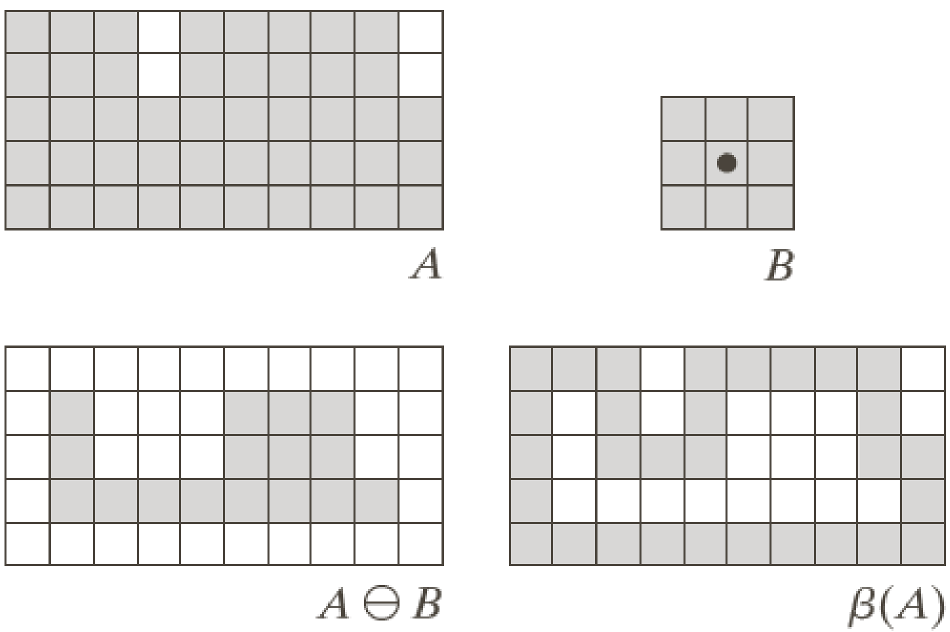
- Shape boundaries.
- Region filling.
- Connected components
- Convex hull.
- Shape thinning and thickening.
- Skeletons.

We may also accomplish a morphological image reconstruction.

Boundary Extraction

The boundary of a set A, denoted by $\beta(A)$, may be obtained by:

$$\beta(A) = A - (A \ominus B)$$



Boundary Extraction (cont.)

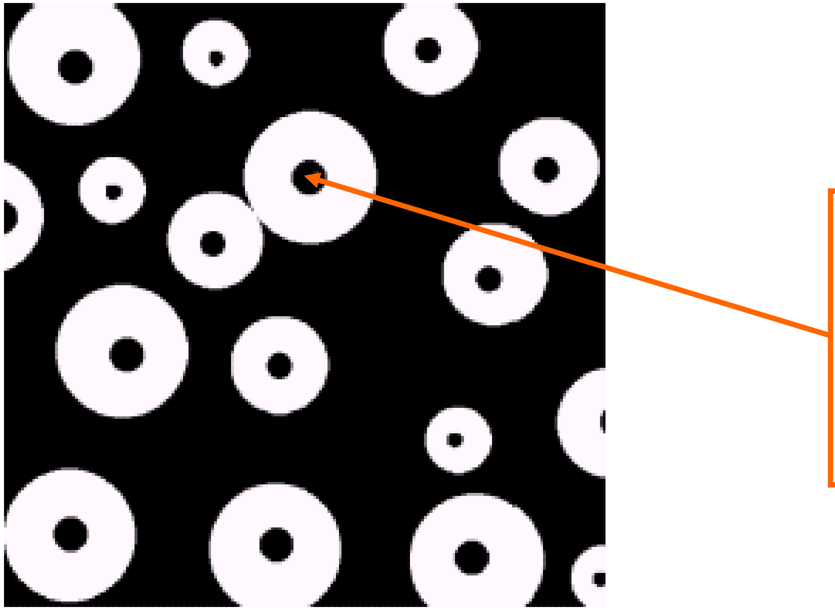
The boundary is one pixel thick due to the 3x3 SE. Other SE would result in thicker boundaries.

Original Image

Extracted Boundary

Region Filling

Given a pixel inside a boundary, region filling attempts to fill the area surrounded by that boundary with 1s.



Given a point inside here, can we fill the whole circle?

Region Filling (cont.)

Form a set X_0 with zeros everywhere except at the seed point of the region.

Then,

$$X_k = (X_{k-1} \oplus B) \cap A^c, \quad k = 1, 2, 3, \ldots$$

Where B is a 3x3 cross-shaped SE.

The algorithm terminates when $X_k = X_{k-1}$.

The set union of X_k and A contains all the filled holes and their boundaries.

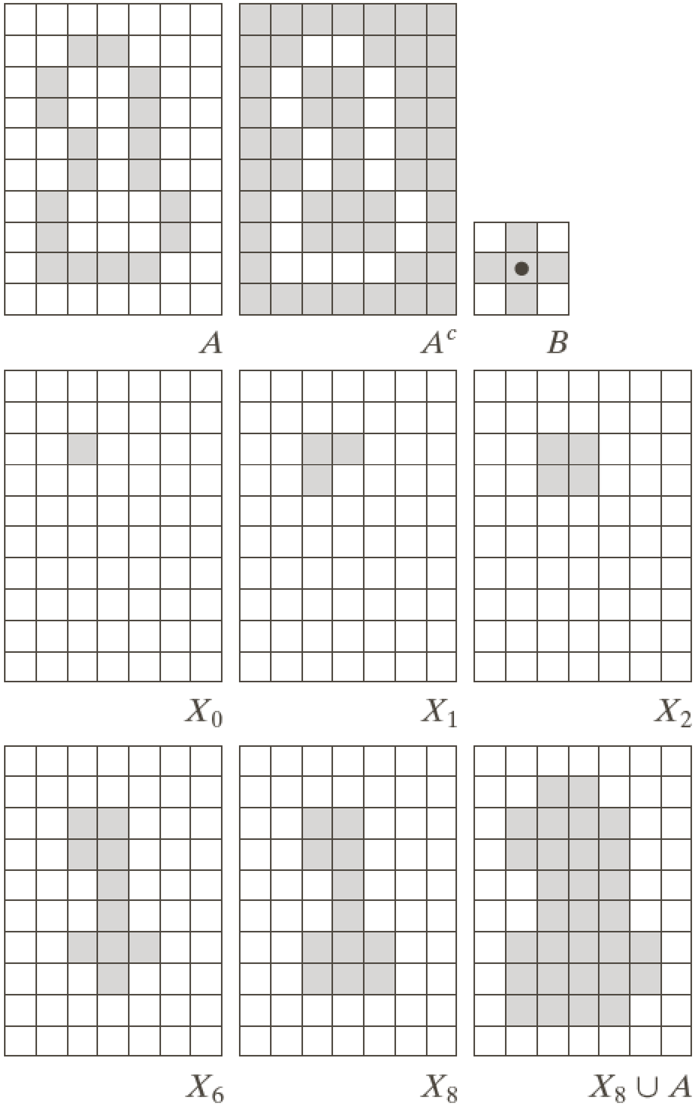
Region Filling (cont.)

This is a first example where the morphological operation (dilation) is conditioned.

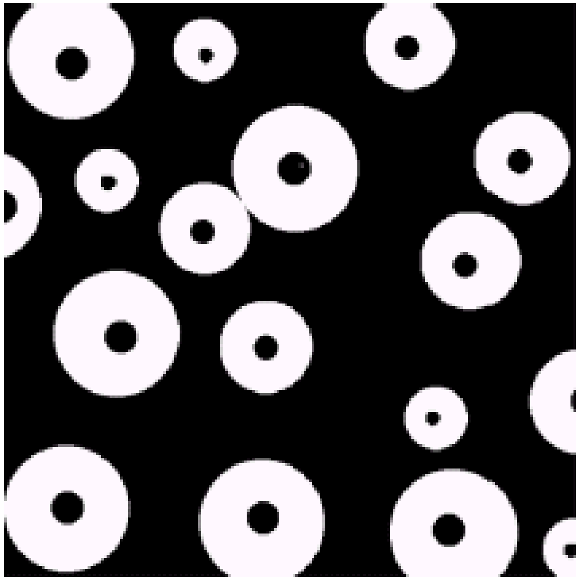
The intersection of the result with A^c limits the result inside the region of interest.

$$X_k = (X_{k-1} \oplus B) \cap A^c, \quad k = 1, 2, 3, \ldots$$

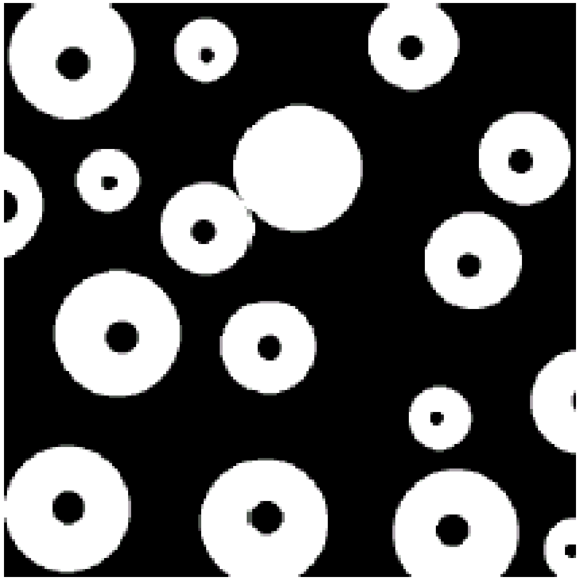
Region Filling (cont.)



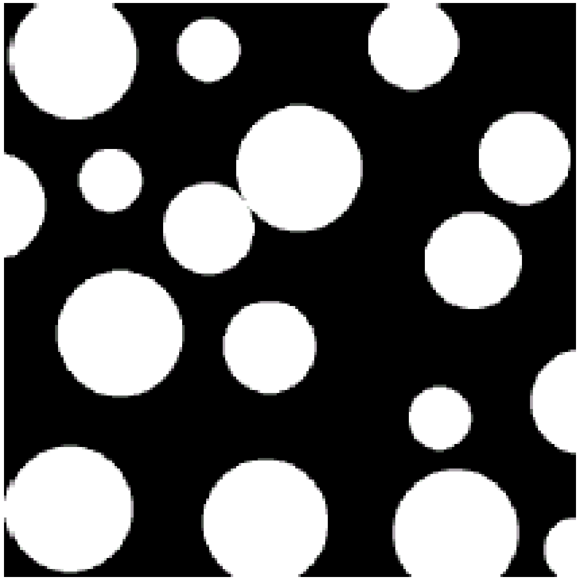
Region Filling (cont.)



Original Image



One Region Filled



All Regions Filled

Extraction of connected components

- Given a pixel on a connected component, find the rest of the pixels of that component.
- The algorithm may be applied to many connected components provided we know a pixel on each one of them.
- Disadvantage:
  - we have to provide a pixel on the connected component.
- There are more sophisticated algorithms that detect the number of components without manual interaction. The purpose here is to demonstrate the flexibility of mathematical morphology.

Extraction of connected components (cont.)

Form a set X_0 with zeros everywhere except at the seed point of the connected components.

Then,

$$X_k = (X_{k-1} \oplus B) \cap A, \quad k = 1, 2, 3, \ldots$$

Where B is a 3x3 square-shaped SE.

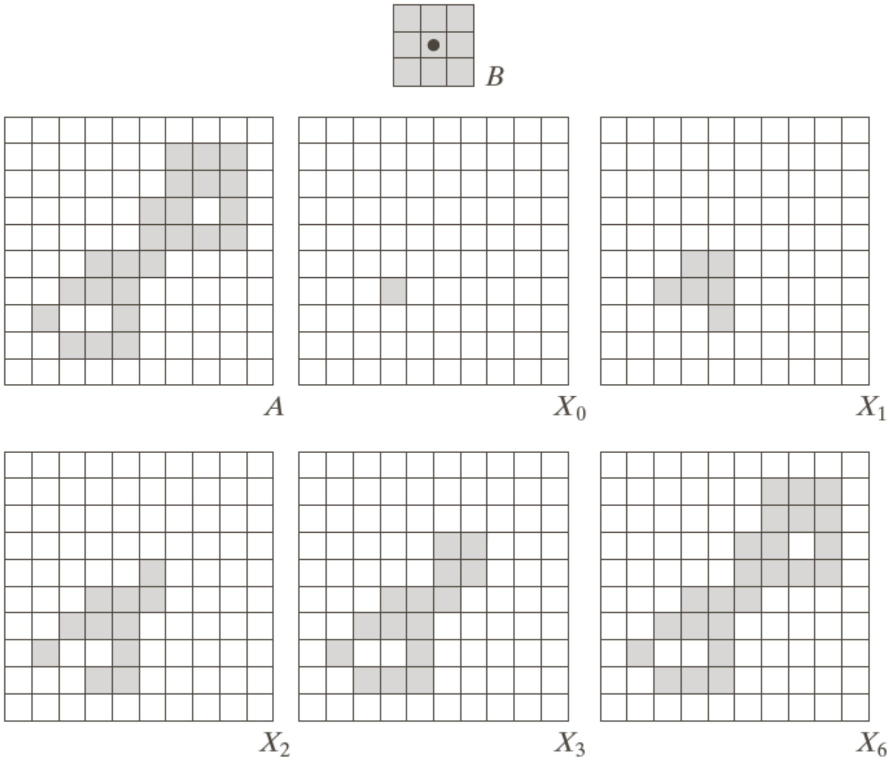
The algorithm terminates when $X_k = X_{k-1}$.

X_k contains all the connected components.

Extraction of connected components (cont.)

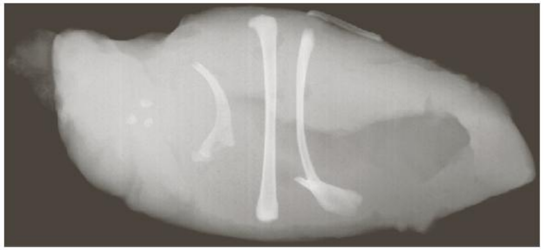
Note the similarity with region filling. The only difference is the use of A instead of A^c.

This is not surprising as we search for foreground objects.



Extraction of connected components (cont.)

Image of chicken filet containing bone fragments



Result of simple thresholding

Image erosion to retain only objects of significant size.

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

15 connected components detected with four of them being significant in size. This is an indication to remove the chicken filet from packaging.

Convex Hull

A set A is convex if the straight line segment joining any two points in A lies entirely within A.

The convex hull H of an arbitrary set S the smallest convex set containing S.

The difference H-S is called convex deficiency.

The convex hull and the convex deficiency are useful quantities to characterize shapes.

We present here a morphological algorithm to obtain the convex hull $C(A)$ of a shape A.

Convex Hull (cont.)

The procedure requires four SE B^i, i=1, 2, 3, 4, and implements the following equation:

$$X_k^i = (X_{k-1} * B^i) \cup A,\ i = 1,2,3,4 \text{ and } k = 1,2,3,...$$

$$\text{with } X_0^i = A$$

with i referring to the SE and k to iteration.

Then, letting

$$D^i = X_k^i$$

The convex hull of A is

$$C(A) = \bigcup_{i=1}^{4} D^i$$

Convex Hull (cont.)

$$X_k^i = (X_{k-1} * B^i) \cup A \quad i = 1, 2, 3, 4 \text{ and } k = 1, 2, 3, \ldots \text{ with } X_0^i = A$$

$$D^i = X_k^i, \; C(A) = \bigcup_{i=1}^{4} D^i$$

The method consists of iteratively applying the hit-or miss transform to A with B^1.

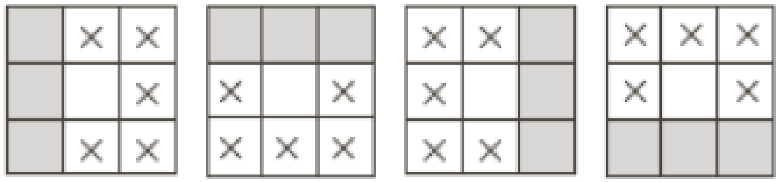
When no changes occur we perform the union with A and save the result to D^1.

The procedure is then continued with B^2 (applied to A) and so on.

The union of the results is the convex hull of A.

Note that a simple implementation of the hit or miss is applied (no background match is required).

Convex Hull (cont.)



B^1 B^2 B^3 B^4

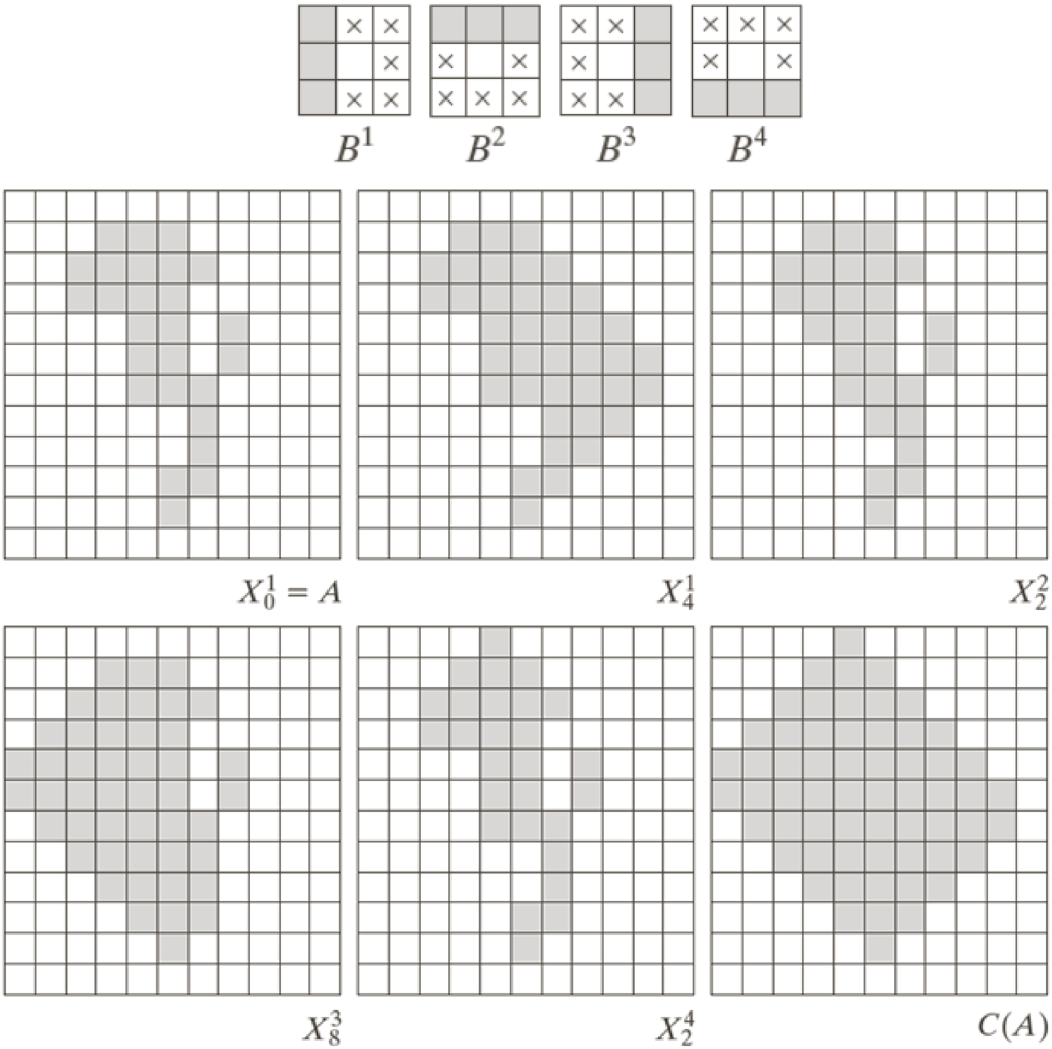
The hit-or-miss transform tries to find ("hit") these structures in the image.

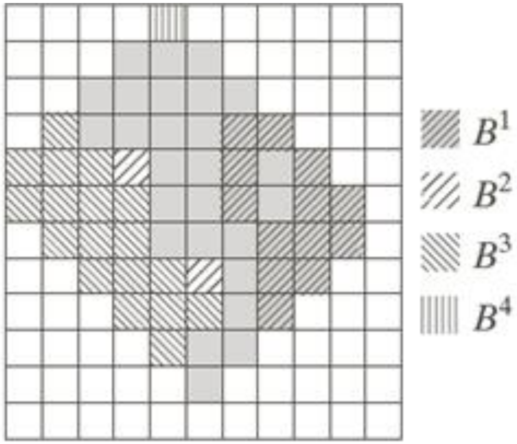
The SE has points with "don't care" condition. For all the SE, a match is found in the image when these conditions hold:

- the central pixel in the 3x3 region in the image is 0.
- the three shaded pixels under the mask are 1s.

The remaining pixels do not matter.

Convex Hull (cont.)



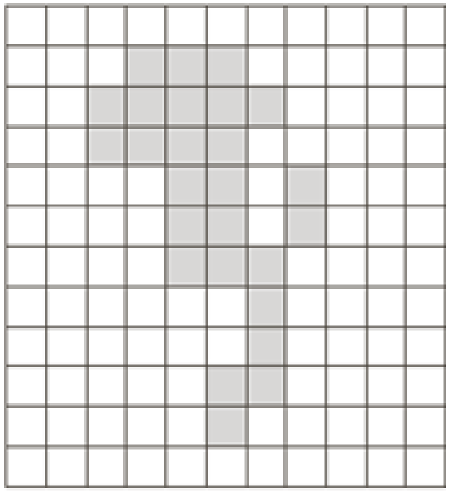


- Problem.

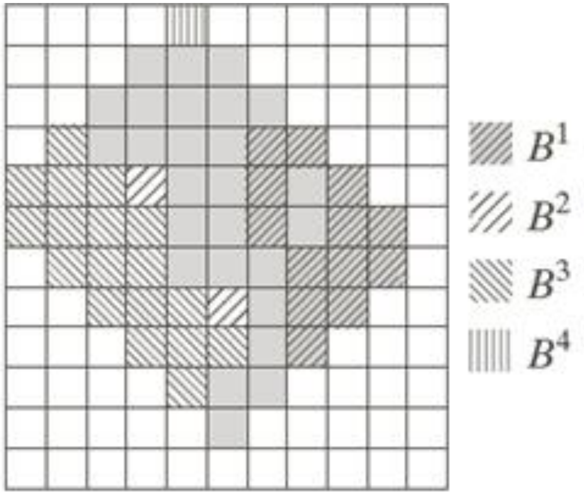
The result is convex but greater than the true convex hull.

Convex Hull (cont.)

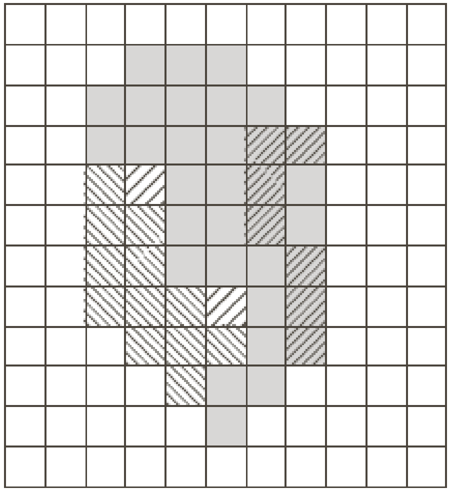
Solution: limit the growth so that it does not extend past the horizontal and vertical limits of the original set of points.



Original image



Initial convex hull



Refined convex hull

More complex boundaries have been imposed to images with finer details in their structure (e.g. the maximum of the horizontal vertical and diagonal dimensions could be used).

Thinning

The thinning of a set A, by a SE B may be defined in terms of the hit-or-miss transform:

$$A \otimes B = A - (A * B) = A \cap (A * B)^c$$

No background match is required and the hit-or-miss part is reduced to simple erosion.

A more advanced expression is based on a sequence of SE $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$,

where each B^i is a rotated version of B^{i-1}.

Thinning (cont.)

The thinning by a sequence of SE is defined by:

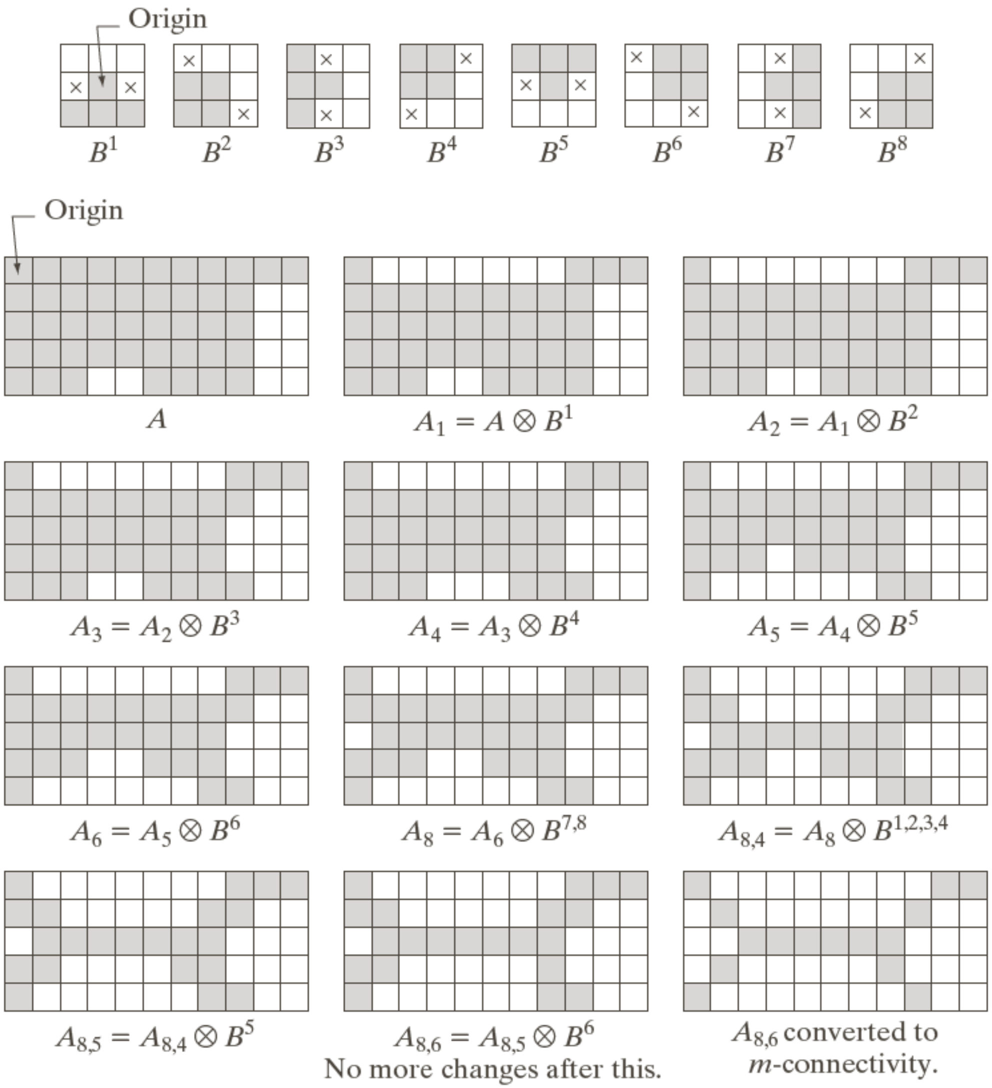
$$A \otimes \{B\} = ((...((A \otimes B^1) \otimes B^2)...) \otimes B^n)$$

The process is to thin A by one pass by B^1, then thin the result with one pass of B^2, and so on, until we employ B^n.

The entire process is repeated until no further changes occur. Each individual thinning is performed by:

$$A \otimes B = A - (A * B)$$

Thinning (cont.)



- No change between the result of B^7 and B^8 at the first pass.
- No change between the results of B^1, B^2, B^3, B^4 at the second pass.
- No change occurs after the second pass by B^6 .

- The final result is converted to *m*-connectivity to have a one pixel thick structure.

Thickening

Thickening is a morphological dual of thinning:

$$A \odot B = A \cup (A * B)$$

The SE have the same form as the ones used for thinning with the 1s and 0s interchanged.

It may also be defined by a sequence of operations:

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$

Thickening (cont.)

In practice, a separate algorithm is seldom used for thickening.

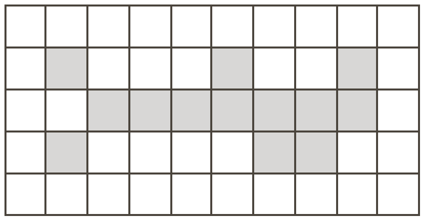
The usual process is to thin the background of the set in question and then take the complement of the result.

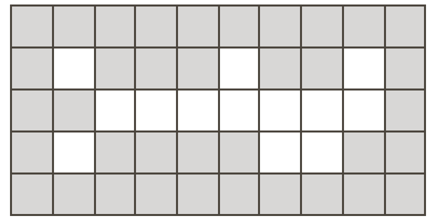
The advantage is that the thinned background forms a boundary for the thickening process. Direct implementation of thickening has no stopping criterion.

A disadvantage is that there may be isolated points needing post-processing.

Thickening (cont.)

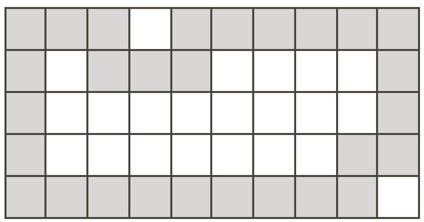
Original set A

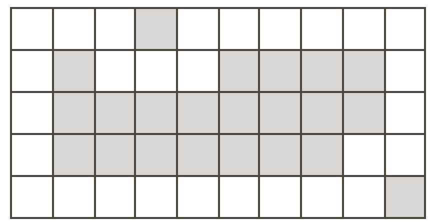




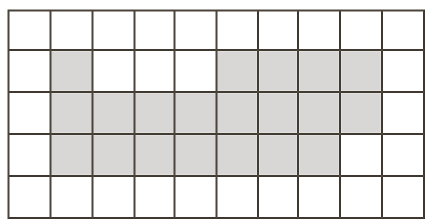
A^c

Thinning of A^c





Thickened set obtained by complementing the result of thinning.

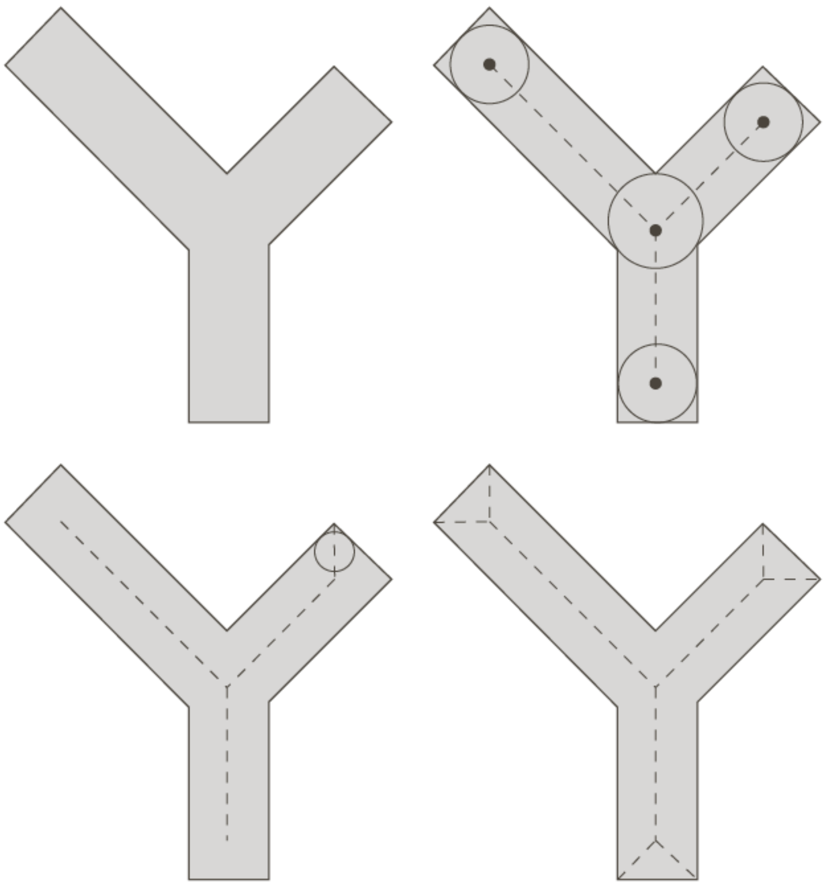


Elimination of disconnected points.

Skeletons

The notion of a skeleton $S(A)$ of a set A, intuitively, has the following properties:

- If z is a point belonging to $S(A)$ and $(D)_z$ is the largest disk centered at z and contained in A: one cannot find a larger disk (not necessarily centered at z) containing $(D)_z$ and included in A.
- $(D)_z$ is then called *maximum disk*.
- The maximum disk touches the boundary of A at two or more different points.



Skeletons (cont.)

It may be shown that a definition of the skeleton may be given in terms of erosions and openings:

$$S(A)=\bigcup_{k=0}^{K} S_k(A), \text{ with } S_k(A)=(A\ominus kB)-(A\ominus kB)\circ B$$

$$\text{with } (A\ominus kB)=\underbrace{((...(A\ominus B)\ominus B)\ominus ...)\ominus B)}_{k \text{ successive erosions}}$$

K is the last iterative step before A erodes to an empty set:

$$K=\max\{k \mid A\ominus kb\neq\varnothing\}$$

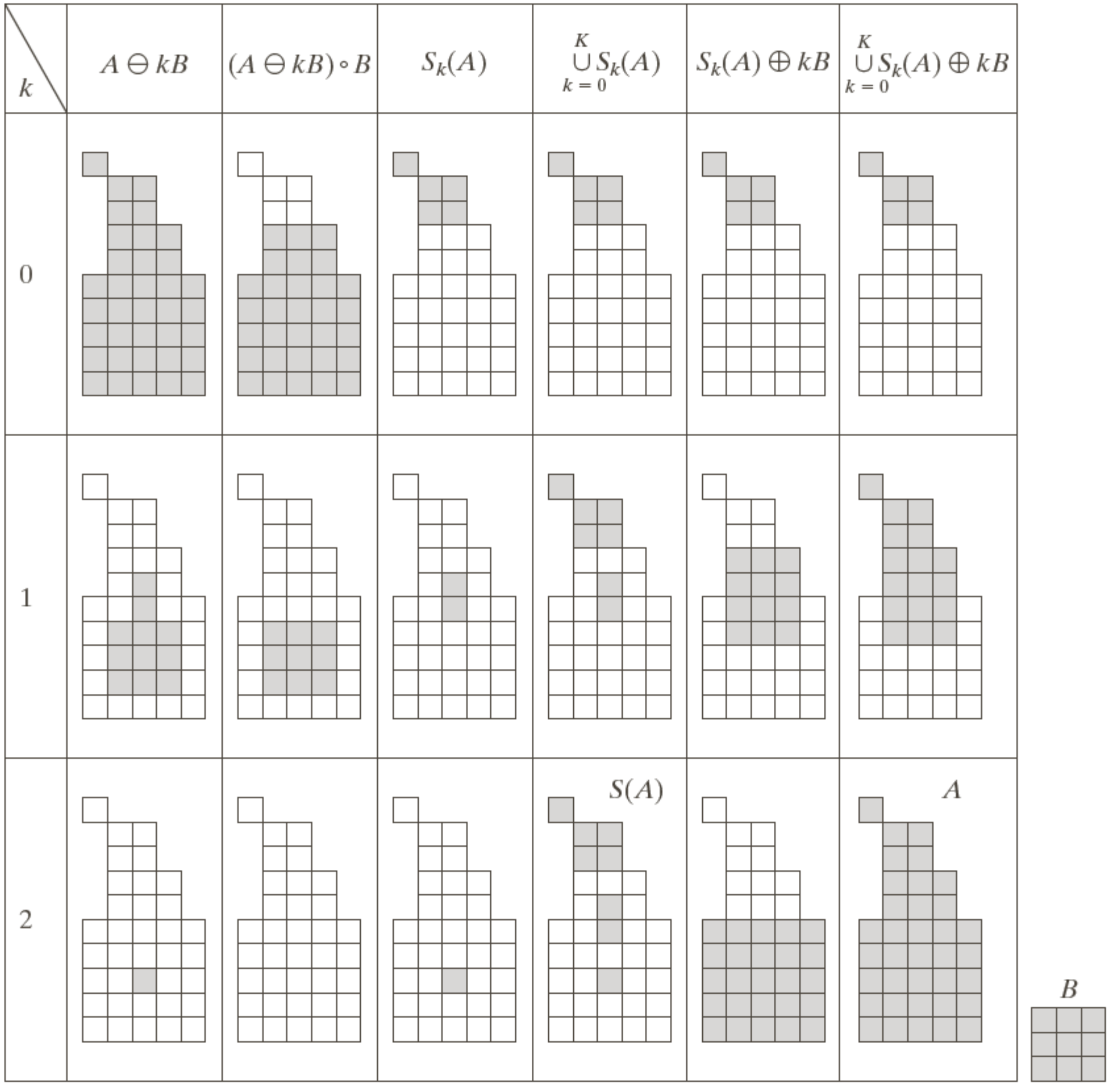
Skeletons (cont.)

The previous formulation allows the iterative reconstruction of A from the sets forming its skeleton by:

$$A = \bigcup_{k=0}^{K} \left(S_k(A) \oplus kB \right),$$

$$\text{with } S_k(A) \oplus B = \underbrace{((\ldots(S_k(A) \oplus B) \oplus B) \oplus \ldots) \oplus B)}_{k \text{ successive dilations of the set } S_k(A)}$$

Skeletons (cont.)



The skeleton is

- thicker than essential.
- disconnected.

The morphological formulation does not guarantee connectivity.

More assumptions are needed to obtain a maximally thin and connected skeleton.

Morphological Reconstruction

- The morphological algorithms discussed so far involve an image and a SE.
- Morphological reconstruction involves two images and a SE.
  - The marker image containing the starting point of the transformation.
  - The mask image, which constraints the transformation.
  - The SE is used to define connectivity.

Morphological Reconstruction (cont.)

The **geodesic dilation** of size 1 of a marker image F by a SE B, with respect to a mask image G is defined by:

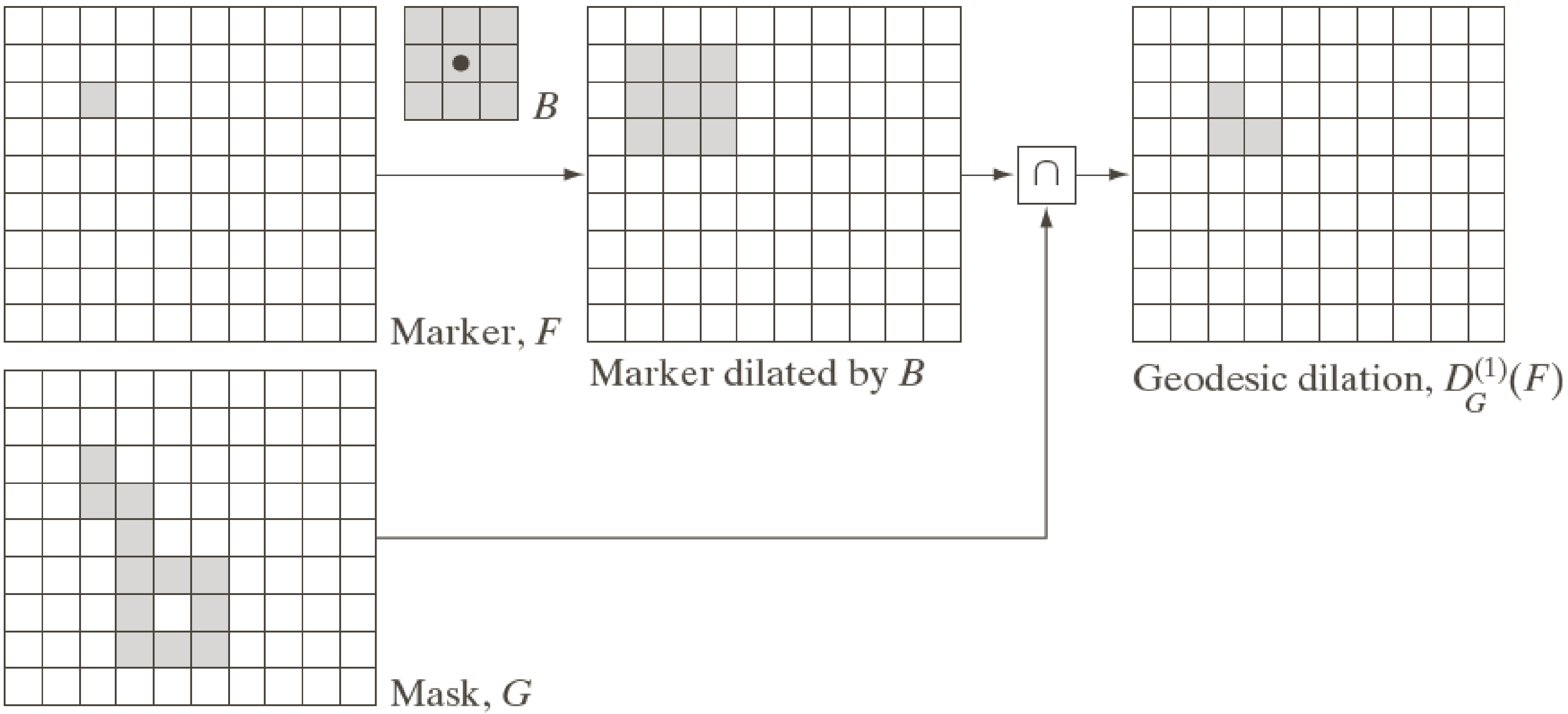
$$D_G^{(1)}(F) = (F \oplus B) \cap G$$

Similarly, the **geodesic dilation** of size n is defined by:

$$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right] \quad \text{with } D_G^{(0)}(F) = F$$

The intersection operator at each step guarantees that the growth (dilation) of marker F is limited by the mask G.

Morphological Reconstruction (cont.)



Geodesic dilation of size 1.

The result will not contain elements not belonging to the mask G.

Morphological Reconstruction (cont.)

The **geodesic erosion** of size 1 of a marker image F by a SE B, with respect to a mask image G is defined by:

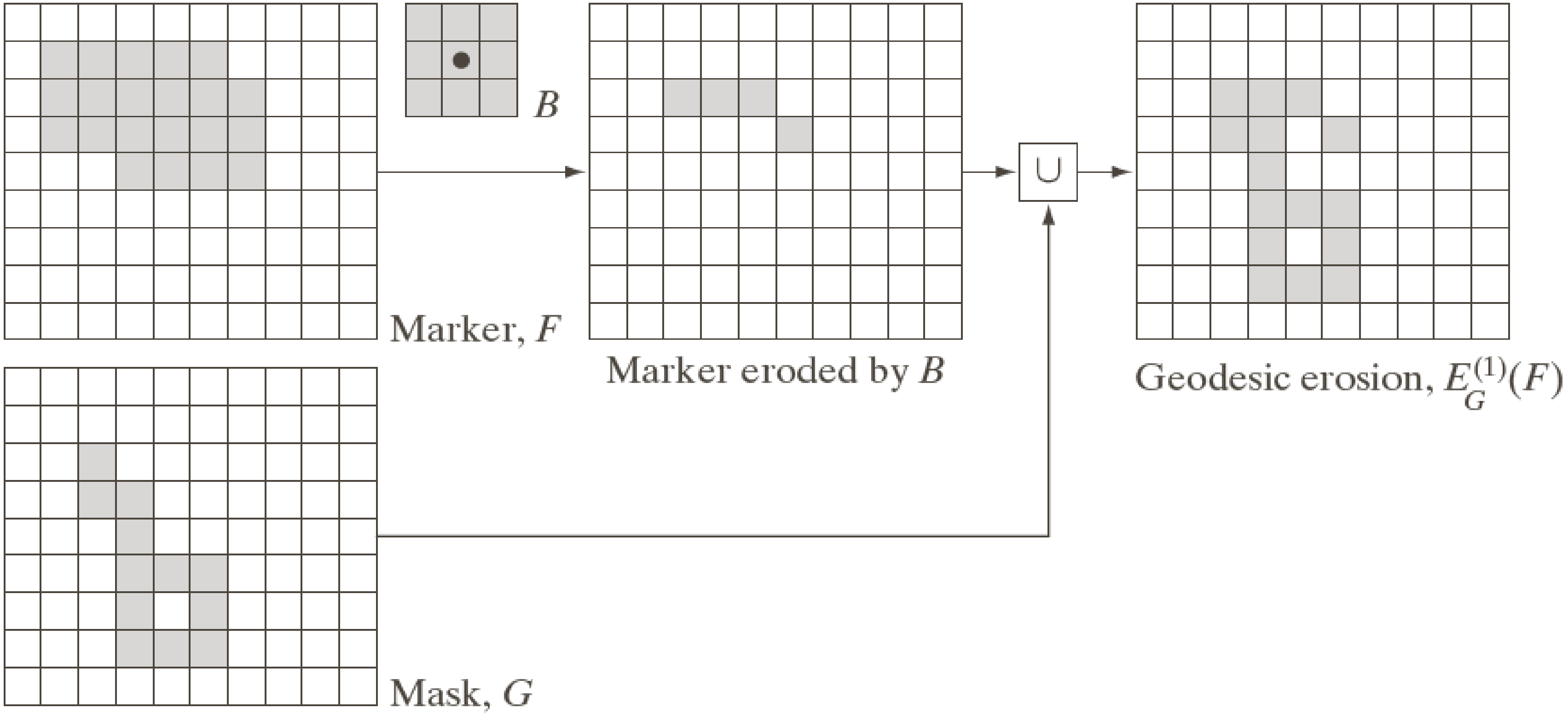
$$E_G^{(1)}(F) = (F \ominus B) \cup G$$

Similarly, the **geodesic erosion** of size n is defined by:

$$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right] \quad \text{with } E_G^{(0)}(F) = F$$

The union operator guarantees that the geodesic erosion of marker F remains greater than or equal to the mask G.

Morphological Reconstruction (cont.)



Geodesic erosion of size 1.

The result will at least contain the mask G.

Morphological Reconstruction (cont.)

- The geodesic dilation and erosion are duals with respect to set complementation.
- They always converge after a finite number of steps:
  - Geodesic dilation: propagation of the marker is constrained by the mask image.
  - Geodesic erosion: shrinking of the marker is constrained by the mask.

Morphological Reconstruction (cont.)

The **morphological reconstruction by dilation** of mask image G from a marker image F is defined as the geodesic dilation of F with respect to G, iterated until stability s achieved:

$$R_G^D(F) = D_G^{(k)}(F)$$

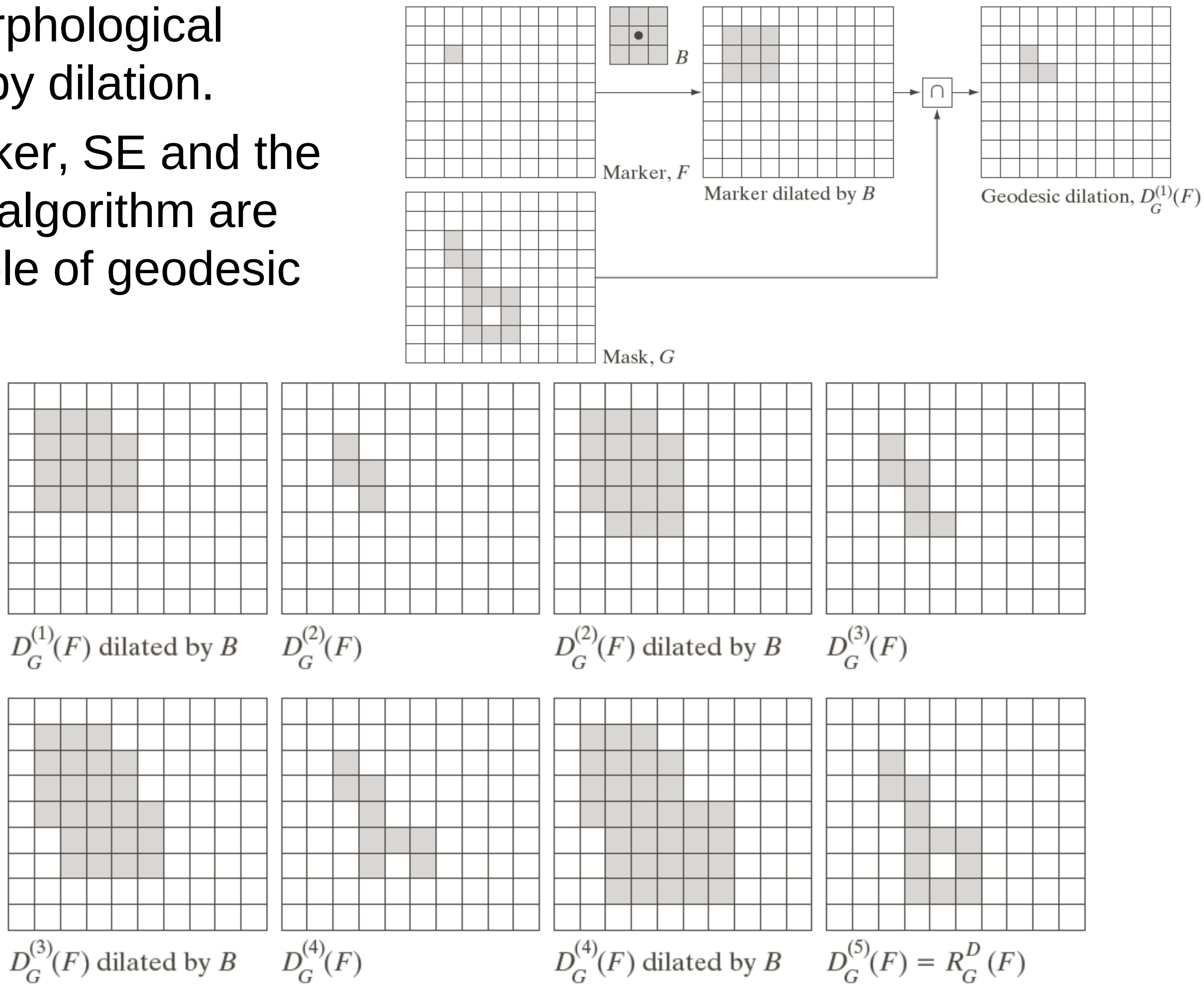
with k such that:

$$D_G^{(k)}(F) = D_G^{(k+1)}(F)$$

Morphological Reconstruction (cont.)

Example of morphological reconstruction by dilation.

The mask, marker, SE and the first step of the algorithm are from the example of geodesic dilation.



Morphological Reconstruction (cont.)

The **morphological reconstruction by erosion** of mask image G from a marker image F is defined as the geodesic erosion of F with respect to G, iterated until stability is achieved:

$$R_G^E(F) = E_G^{(k)}(F)$$

with k such that:

$$E_G^{(k)}(F) = E_G^{(k+1)}(F)$$

The example is left as an exercise!

Applications
Opening by Reconstruction

In morphological opening, erosion removes small objects and dilation attempts to restore the shape of the objects that remain without the small objects.

This is not accurate as it depends on the similarity between the shapes to be removed and the SE.

Opening by reconstruction restores exactly the shapes of the objects that remain after erosion.

Applications
Opening by Reconstruction (cont.)

The opening by reconstruction of size n of an image F is defined as the reconstruction by dilation of F from the erosion of size n of F:

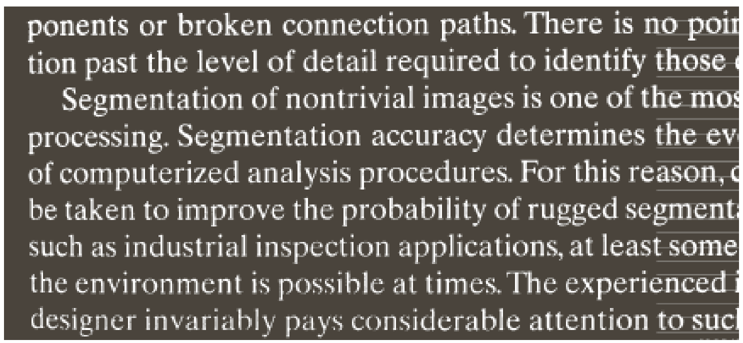
$$O_R^{(n)}(F) = R_F^D\left[(F \ominus nB)\right]$$

The image F is used as the mask and the n erosions of F by B are used as the initial marker image.

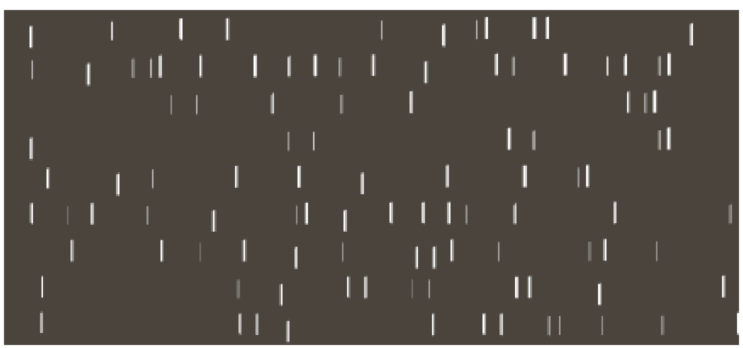
Applications
Opening by Reconstruction (cont.)

We are interested in extracting characters with long vertical strokes (~50 pixels high).

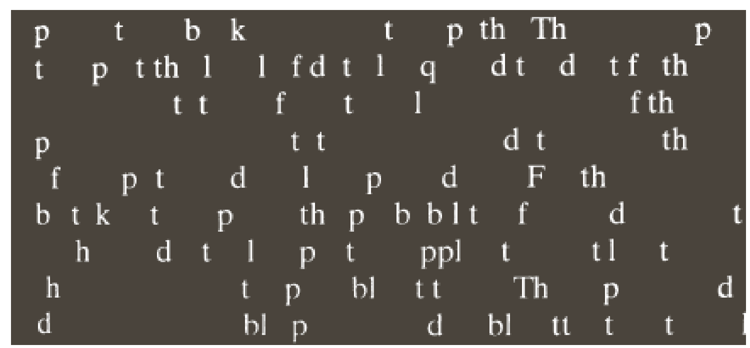
Original image


ponents or broken connection paths. There is no poi
tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mos
processing. Segmentation accuracy determines the ev
of computerized analysis procedures. For this reason,
be taken to improve the probability of rugged segment
such as industrial inspection applications, at least some
the environment is possible at times. The experienced
designer invariably pays considerable attention to suc

One erosion by a 51x1 SE



Opening



Opening by reconstruction

Applications
Region filling

No starting point is needed to be provided.

The original image $I(x,y)$ is used as a mask.

The marker image is

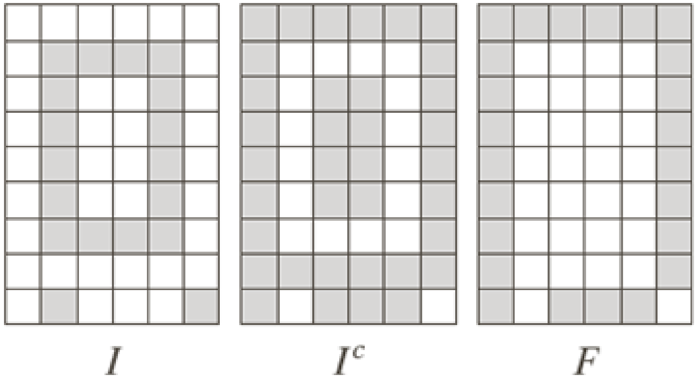
$$F(x,y)=\begin{cases}1-I(x,y) & \text{if } (x,y) \text{ is on the border of } I\\ 0 & \text{otherwise}\end{cases}$$

Only dark pixels of $I(x,y)$ touching the border have a value of 1 in $F(x,y)$.

The binary image with all regions (holes) filled is given by:

$$H=\left[R_{I^c}^{D}(F)\right]^c$$

Applications
Region filling (cont.)

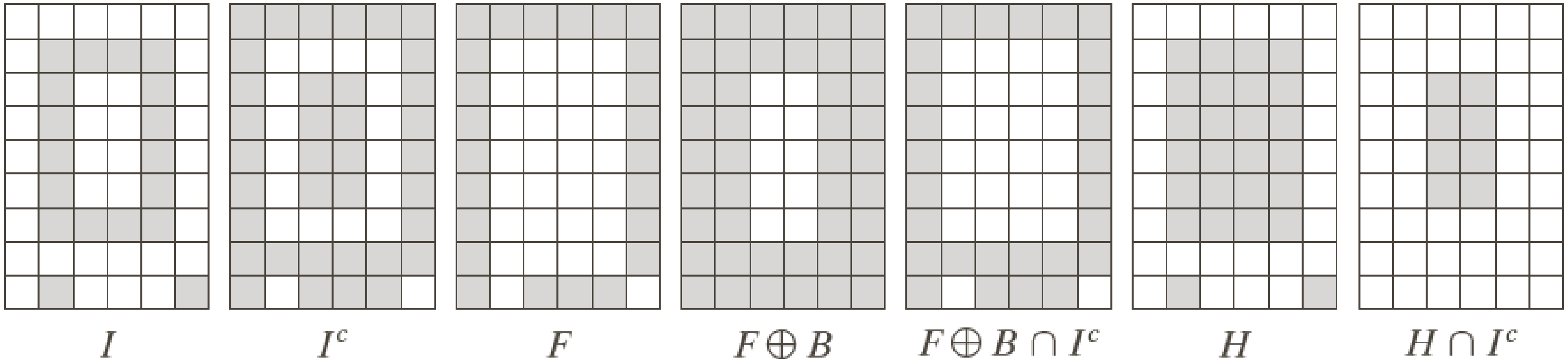


We wish to fill the hole of the image I.

The complement builds a wall around the hole.

The marker image F is one at the border except from border pixels of the original image.

Applications
Region filling (cont.)



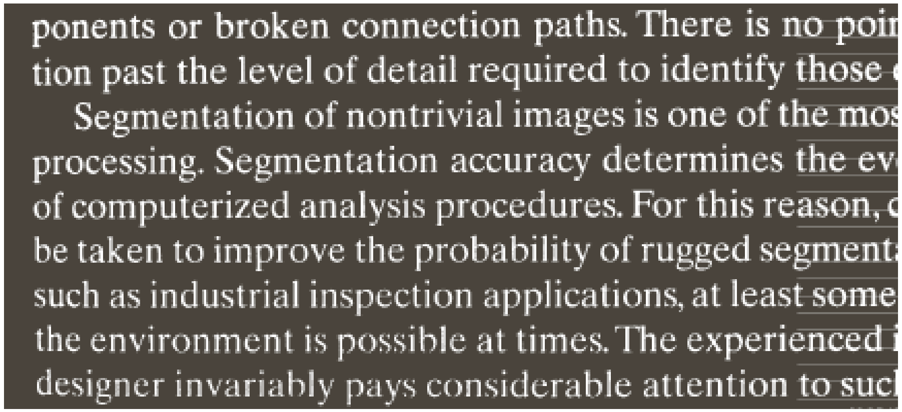
The dilation of the marker F starts from the border and grows inward.

The complement is used as an AND mask: it protects all foreground pixels (including the wall) from changing during the iterations.

The last operation provides only the hole points.

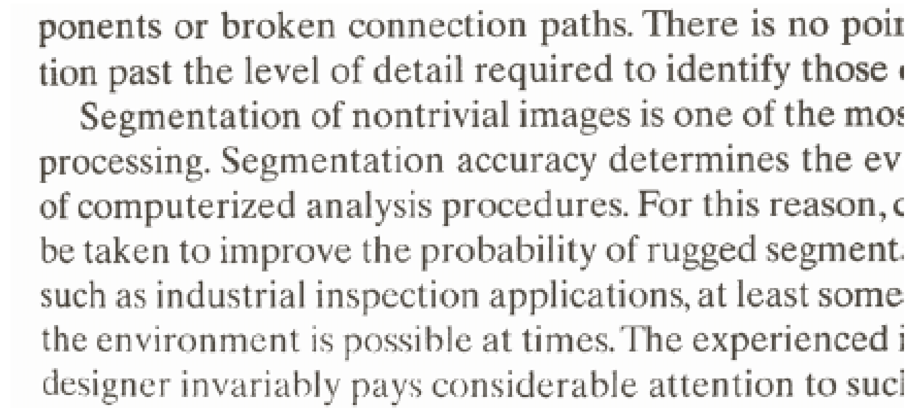
Applications
Region filling (cont.)

Original image



ponents or broken connection paths. There is no poi
tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mos
processing. Segmentation accuracy determines the ev
of computerized analysis procedures. For this reason,
be taken to improve the probability of rugged segment
such as industrial inspection applications, at least some
the environment is possible at times. The experienced
designer invariably pays considerable attention to suc

Complement of original image



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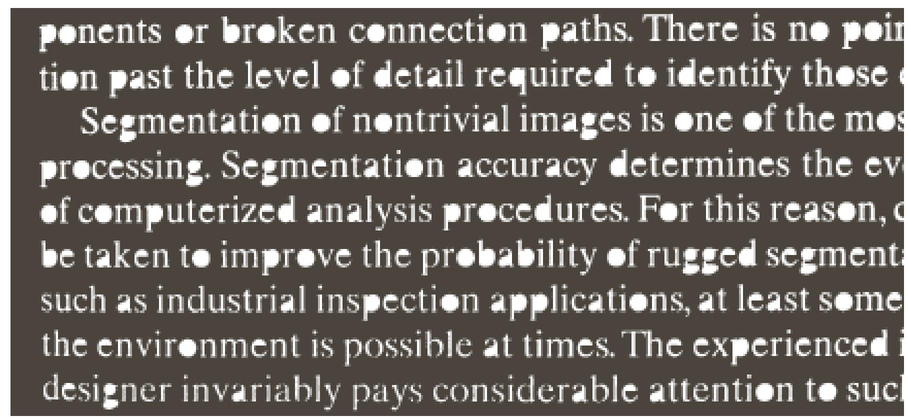
Marker image (1s almost everywhere on the border apart of some points on the right border)



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Result of hole filling

Applications
Border Clearing

The extraction of objects from an image is a fundamental task in automated image analysis. An algorithm for removing objects that touch (are connected) to the image border is useful because

- only complete objects remain for further processing.
- it is a signal that partial objects remain in the field of view.

Applications
Border Clearing (cont.)

The original image is used as a mask.

The marker image is

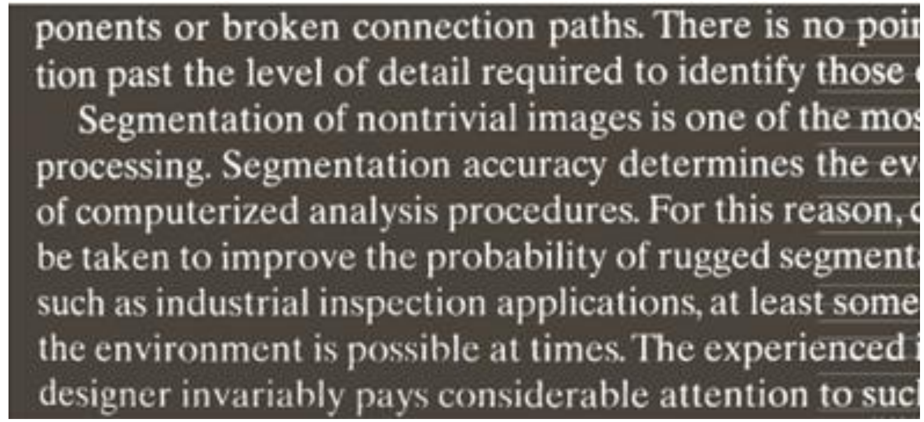
$$F(x,y)=\begin{cases} I(x,y) & \text{if } (x,y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases}$$

The border clearing algorithm first computes the morphological reconstruction $R_I^D(F)$,

which simply extracts the objects touching the border and then obtains the new image with no objects touching the borders $I - R_I^D(F)$.

Applications
Border Clearing (cont.)

Original image I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tion past the level of detail required to identify those
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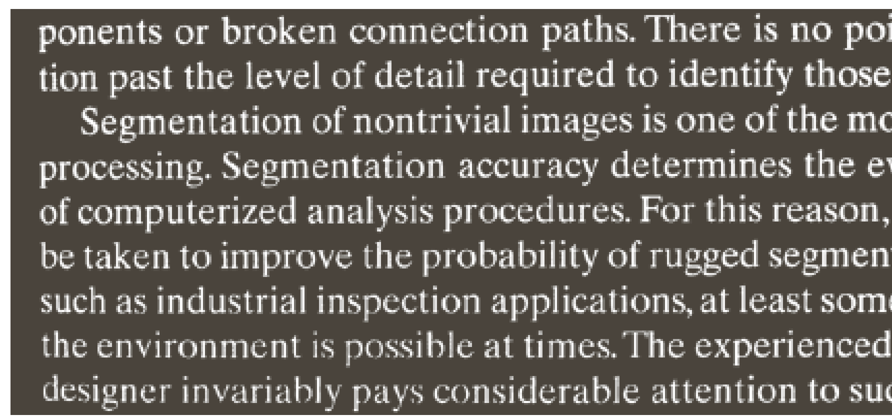
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tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mo
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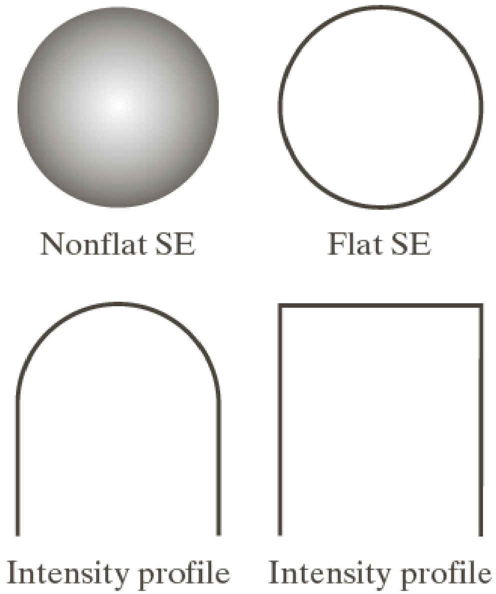
Reconstruction by dilation of the 1s touching the border

Reconstructed image $I - R_I^D(F)$

Gray-Scale Morphology

- The image $f(x,y)$ and the SE $b(x,y)$ take real or integer values.
- SE may be flat or nonflat.
- Due to a number of difficulties (result interpretation, erosion is not bounded by the image, etc.) symmetrical flat SE with origin at the center are employed.
- Set reflection: $\hat{b}(x, y) = -b(x, y)$



Gray-Scale Erosion

The erosion of image f by a SE b at any location (x,y) is defined as the minimum value of the image in the region coincident with b when the origin of b is at (x,y):

$$[f \ominus b](x,y) = \min_{(s,t)\in b}\{f(x+s, y+t)\}$$

In practice, we place the center of the SE at every pixel and select the minimum value of the image under the window of the SE.

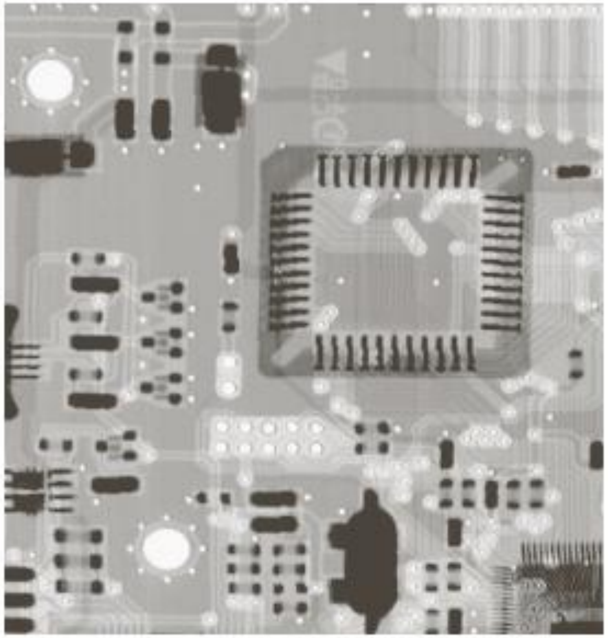
Gray-Scale Dilation

The dilation of image f by a SE b at any location (x,y) is defined as the maximum value of the image in the window outlined by b:

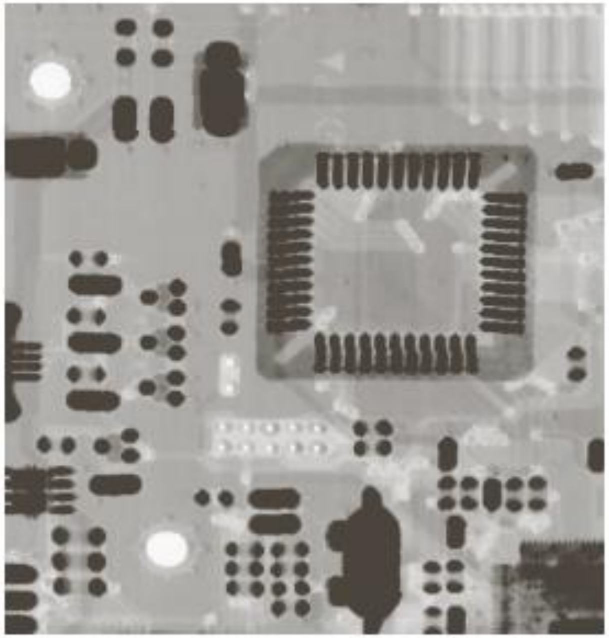
$$[f \oplus b](x,y) = \max_{(s,t)\in b}\{f(x-s, y-t)\}$$

The SE is reflected as in the binary case.

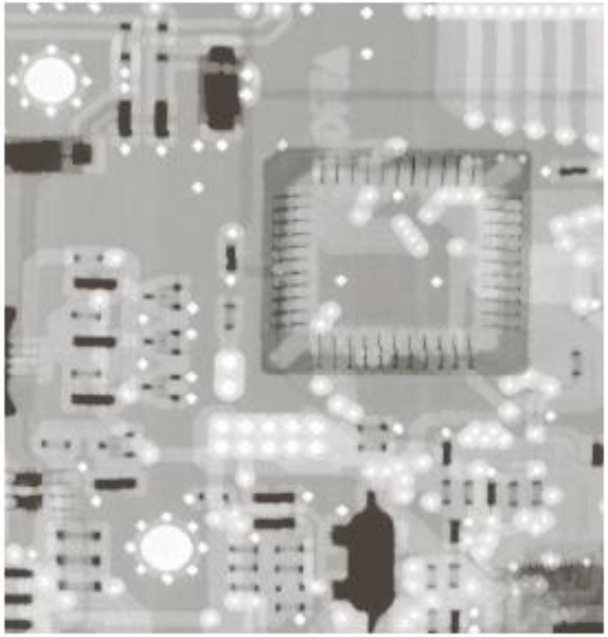
Gray-Scale Erosion and Dilation



Original image



Erosion by a flat disk SE of radius 2:
Darker background,
small bright dots reduced,
dark features grew.



Dilation by a flat disk SE of radius 2:
Lighter background,
small dark dots reduced,
light features grew.

Gray-Scale Morphology (nonflat SE)

The erosion of image f by a nonflat SE b_N is defined as:

$$[f \ominus b_N](x,y) = \min_{(s,t)\in b_N} \{ f(x+s, y+t) - b_N(s,t) \}$$

The dilation of image f by a nonflat SE b_N is defined as:

$$[f \oplus b_N](x,y) = \max_{(s,t)\in b_N} \{ f(x-s, y-t) + b_N(s,t) \}$$

When the SE is flat the equations reduce to the previous formulas up to a constant.

Duality

As in the binary case, erosion and dilation are dual operations with respect to function complementation and reflection:

$$(f \ominus b)^c(x,y) = (f^c \oplus \hat{b})(x,y)$$

Similarly,

$$(f \oplus b)^c(x,y) = (f^c \ominus \hat{b})(x,y)$$

In what follows, we omit the coordinates for simplicity.

Gray-Scale Opening and Closing

The opening of image f by SE b is:

$$f \circ b = (f \ominus b) \oplus b$$

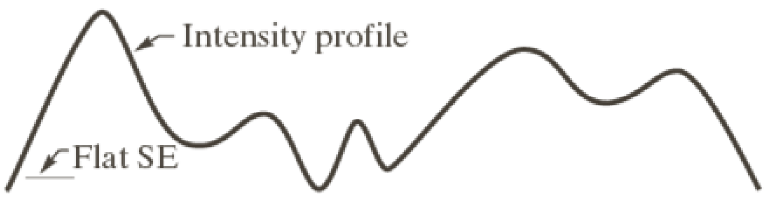
The closing of image f by SE b is:

$$f \bullet b = (f \oplus b) \ominus b$$

They are also duals with respect to function complementation and reflection:

$$(f \bullet b)^c = f^c \circ \hat{b} \qquad (f \circ b)^c = f^c \bullet \hat{b}$$

Gray-Scale Opening and Closing (cont.)



Geometric interpretation of **opening:**

It is the highest value reached by any part of the SE as it pushes up against the under-surface of the image (up to the point it fits completely).

It removes small bright details.

Notice the similarity with binary opening (smooths outward corners from the inside).

Gray-Scale Opening and Closing (cont.)

Geometric interpretation of **closing:**

It is the lowest value reached by any part of the SE as it pushes down against the upper side of the image intensity curve.

It highlights small dark regions of the image.

Notice the similarity with binary closing (smooths inward corners from the outside).

Gray-Scale Opening and Closing (cont.)

Properties of opening:

$$(1) \quad f \circ b \dashv f$$

$$(2) \quad \text{If } f_1 \dashv f_2 \text{, then } f_1 \circ b \dashv f_2 \circ b$$

$$(3) \quad (f \circ b) \circ b = f \circ b$$

The first property indicates that:

- the domain of the opening is a subset of the domain of f and
- $[f \circ b](x, y) \leq f(x, y)$

Gray-Scale Opening and Closing (cont.)

Properties of closing:

$$(1) \quad f \dashv f \bullet b$$

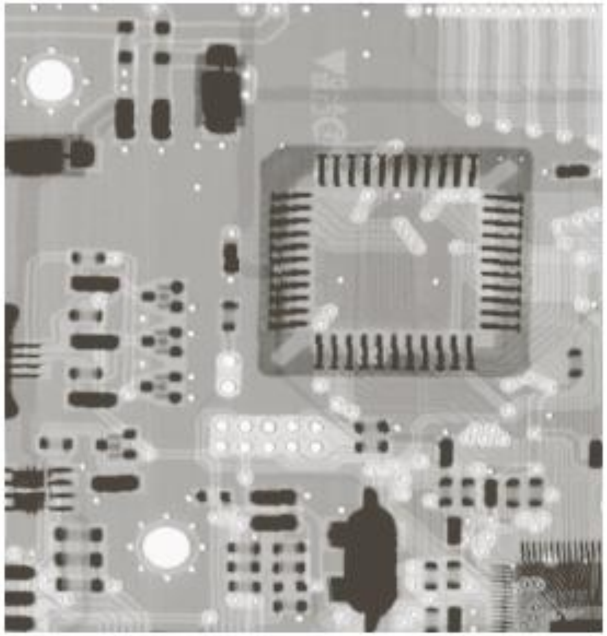
$$(2) \quad \text{If } f_1 \dashv f_2, \text{then } f_1 \bullet b \dashv f_2 \bullet b$$

$$(3) \quad (f \bullet b) \bullet b = f \bullet b$$

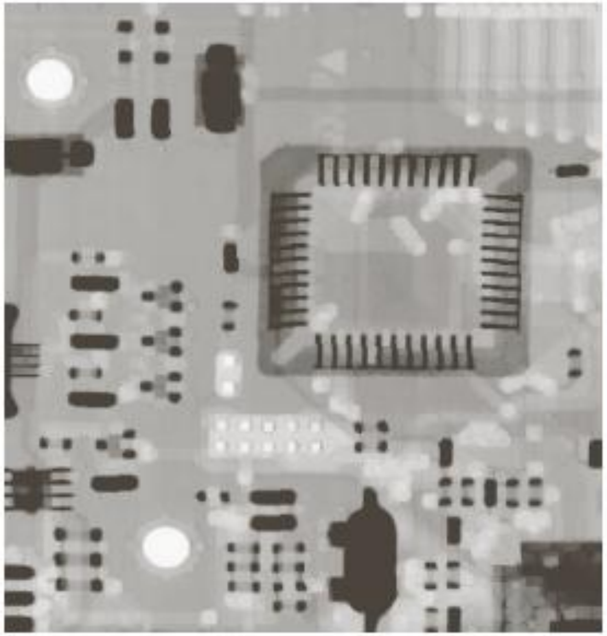
The first property indicates that:

- the domain of f is a subset of the domain of the closing and
- $f(x, y) \leq [f \bullet b](x, y)$

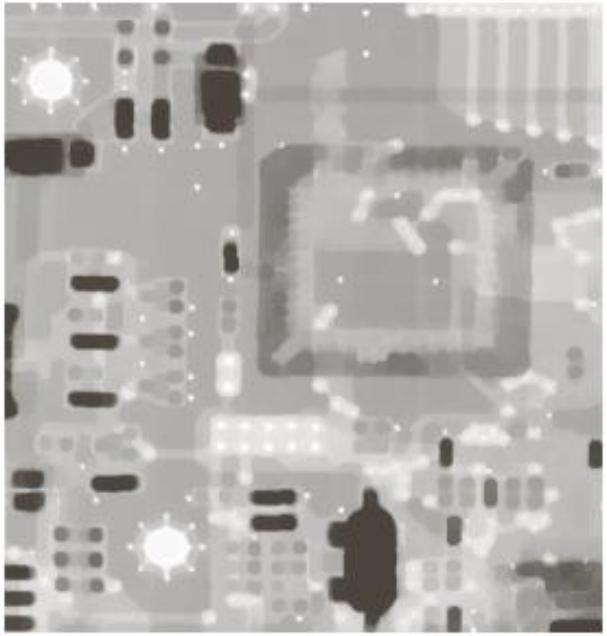
Gray-Scale Opening and Closing (cont.)



Original image



Opening by a flat disk SE of radius 3:
Intensities of bright features decreased,
Effects on background are negligible (as opposed to erosion).



Closing by a flat disk SE of radius 5:
Intensities of dark features increased,
Effects on background are negligible (as opposed to dilation).

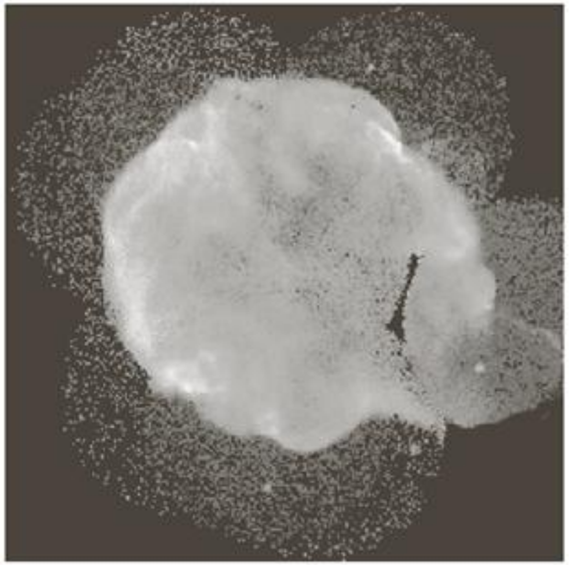
Gray-Scale Morphological Algorithms

- Morphological smoothing
- Morphological gradient
- Top-hat transformation
- Bottom-hat transformation
- Granulometry
- Textural segmentation

Morphological Smoothing

Opening suppresses light details smaller than the SE and closing suppresses (makes lighter) dark details smaller than the SE.

They are used in combination as *morphological filters* to eliminate undesired structures.

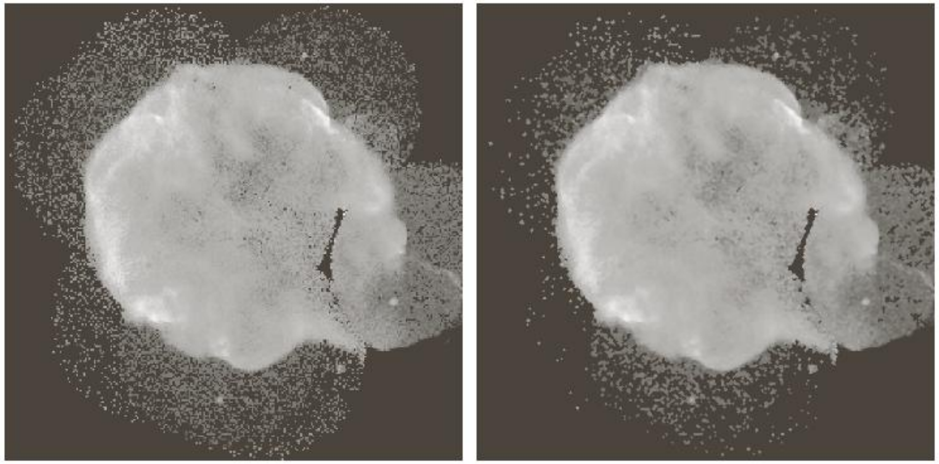


Cygnus Loop supernova. We wish to extract the central light region.

Morphological Smoothing (cont.)

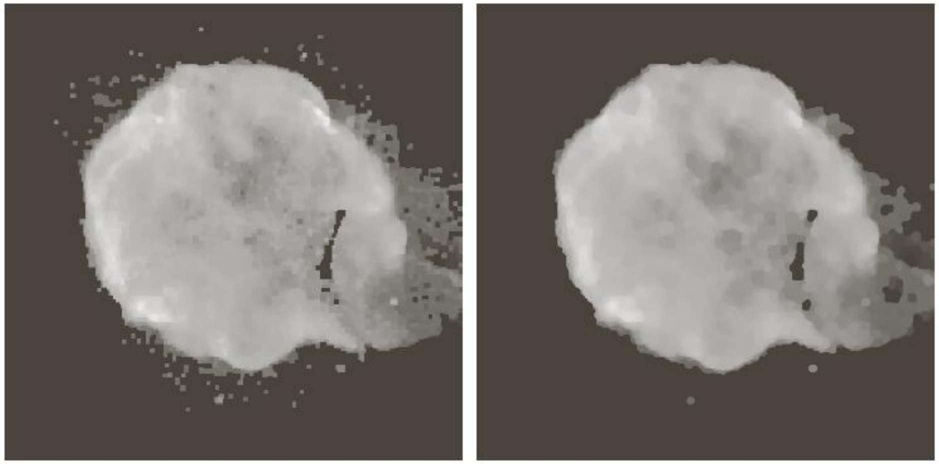
Opening followed by closing with disk SE of varying size

Original image



Radius 1

Radius 3



Radius 5

Morphological Gradient

The difference of the dilation and the erosion of an image emphasizes the boundaries between regions:

$$g = (f \oplus b) - (f \ominus b)$$

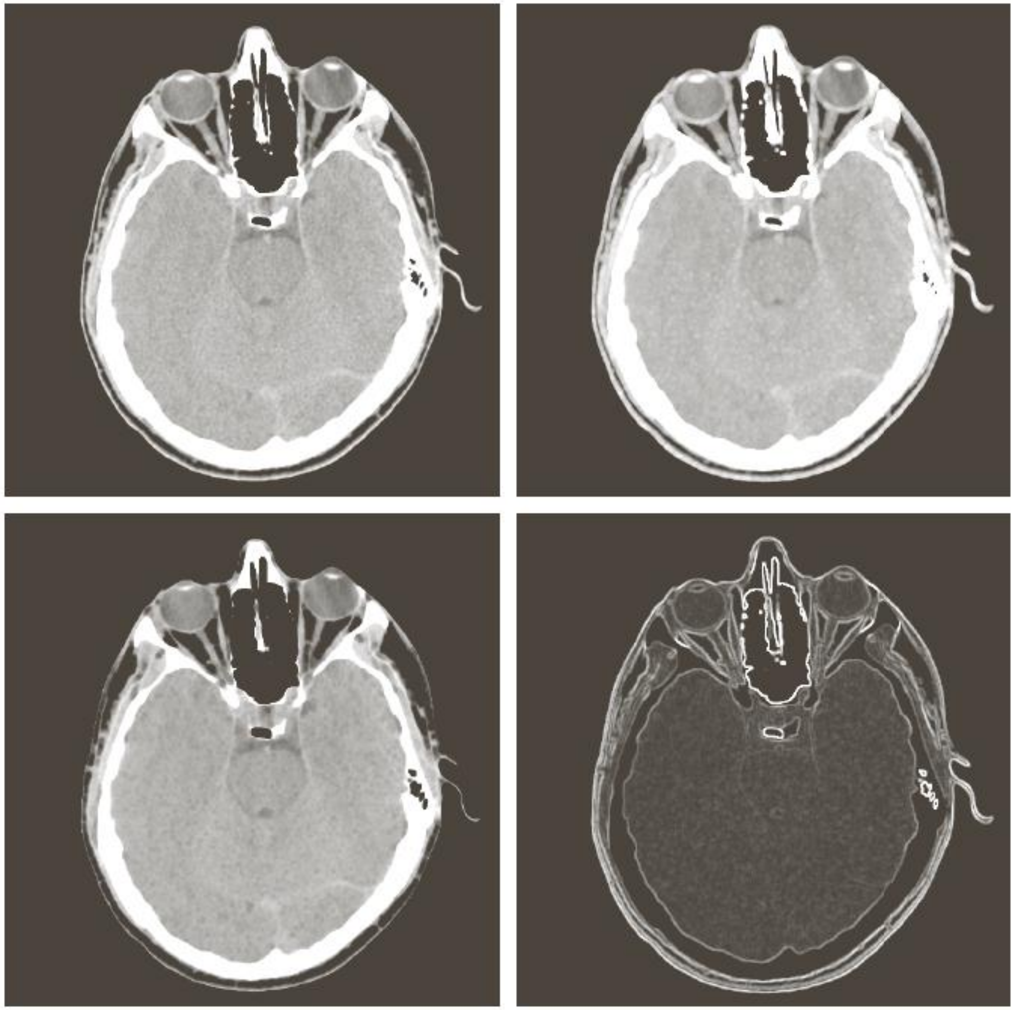
Homogeneous areas are not affected and the subtraction provides a derivative-like effect.

The net result is an image with flat regions suppressed and edges enhanced.

Morphological Gradient (cont.)

Original image

Dilation



Erosion

Difference

Top-hat and Bottom-hat Transformations

- Opening suppresses light details smaller than the SE.
- Closing suppresses dark details smaller than the SE.
- Choosing an appropriate SE eliminates image details where the SE does not fit.
- Subtracting the outputs of opening or closing from the original image provides the removed components.

Top-hat and Bottom-hat Transformations (cont.)

Because the results look like the top or bottom of a hat these algorithms are called **top-hat** and **bottom-hat** transformations:

$$T_{\text{hat}}(f) = f - (f \circ b)$$

Light details remain

$$B_{\text{hat}}(f) = (f \bullet b) - f$$

Dark details remain

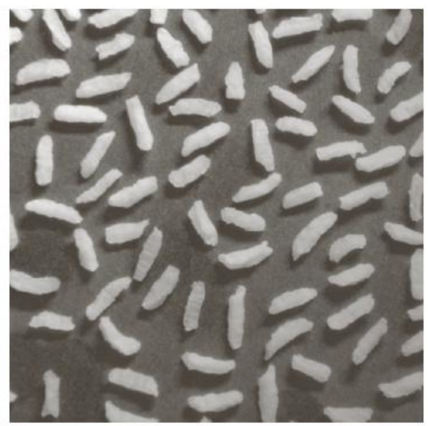
An important application is the correction of nonuniform illumination which is a pre-segmentation step.

Top-hat and Bottom-hat Transformations (cont.)

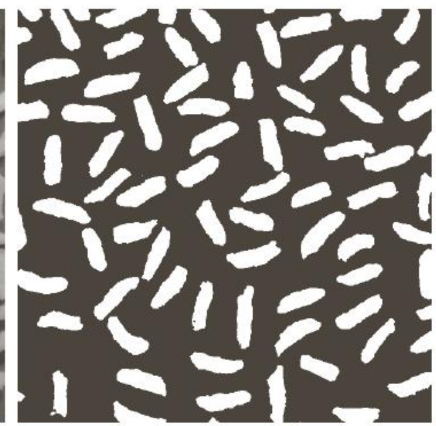
Original image

Thresholded image (Otsu’s method)

Opened image
(disk SE r=40)
Does not fit to grains and eliminates them



Top-hat reduced nonuniformity



Thresholded top-hat

Granulometry

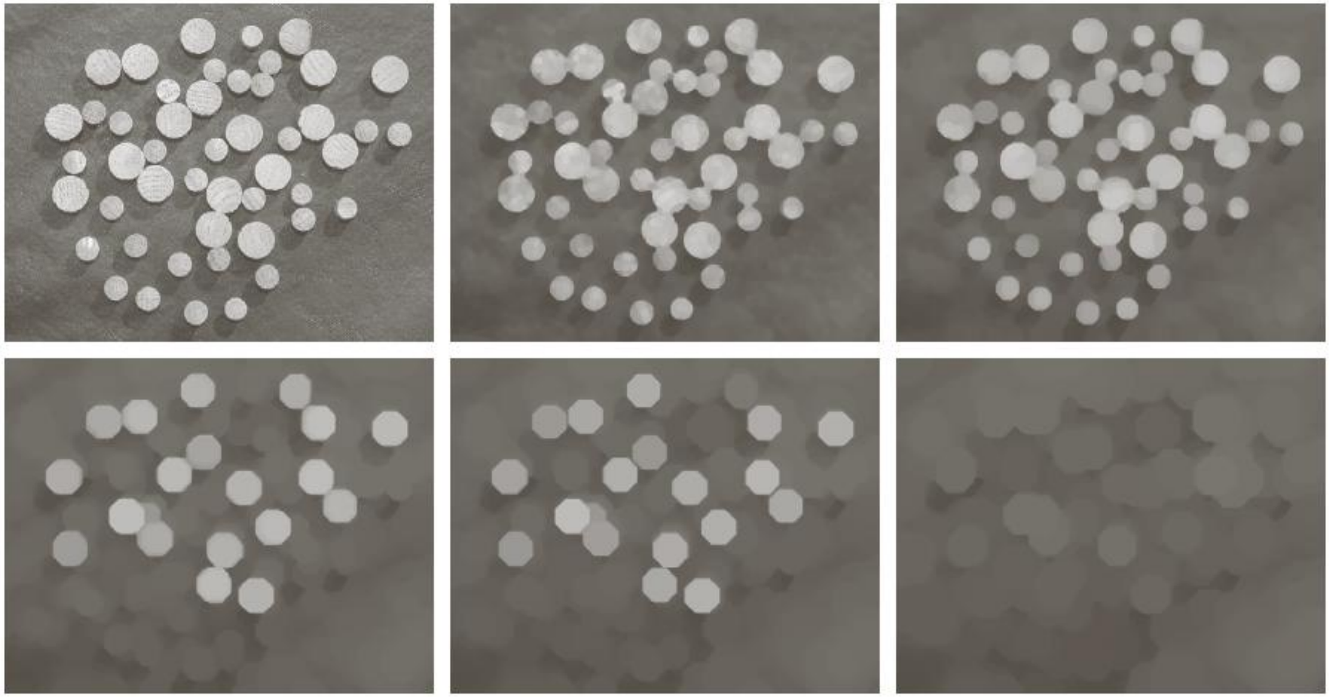
- Determination of the size distribution of particles in an image. Particles are seldom separated.
- The method described here measures their distribution indirectly.
- It applies openings with SE of increasing size.
- Each opening suppresses bright features where the SE does not fit.
- For each opening the sum of pixel values is computed and a histogram of the size of the SE vs the remaining pixel intensities is drawn.

Granulometry (cont.)

Image of wooden plugs

Smoothed image

Opening by SE of radius 10



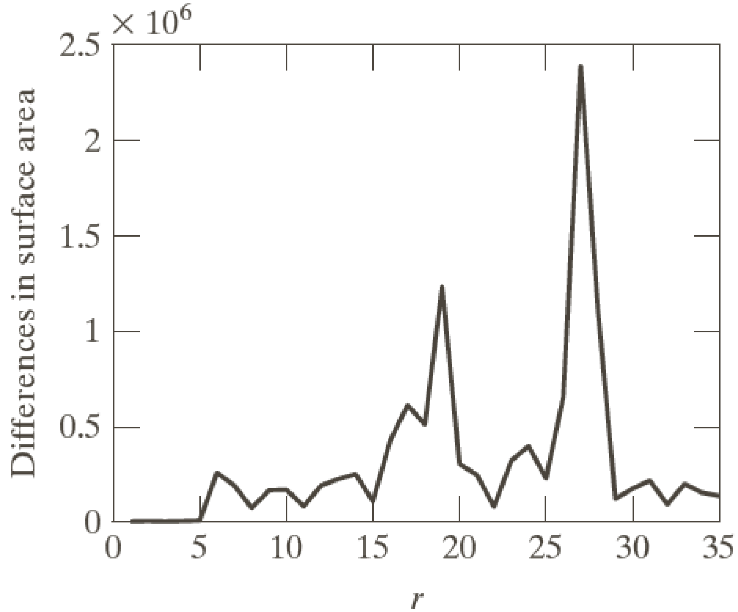
Opening by SE of radius 20. Small dowels disappeared.

Opening by SE of radius 25

Opening by SE of radius 30 Large dowels disappeared.

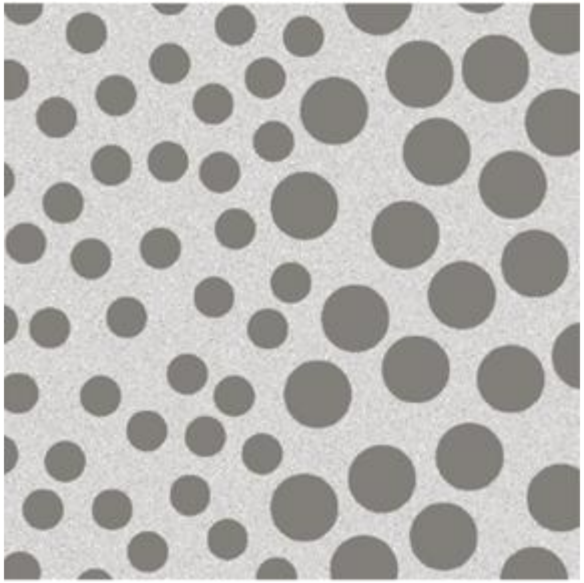
Granulometry (cont.)

- Histogram of the differences of the total image intensities between successive openings as a function of the radius of the SE.
- There are two peaks indicating two dominant particle sizes (of radii 19 and 27).



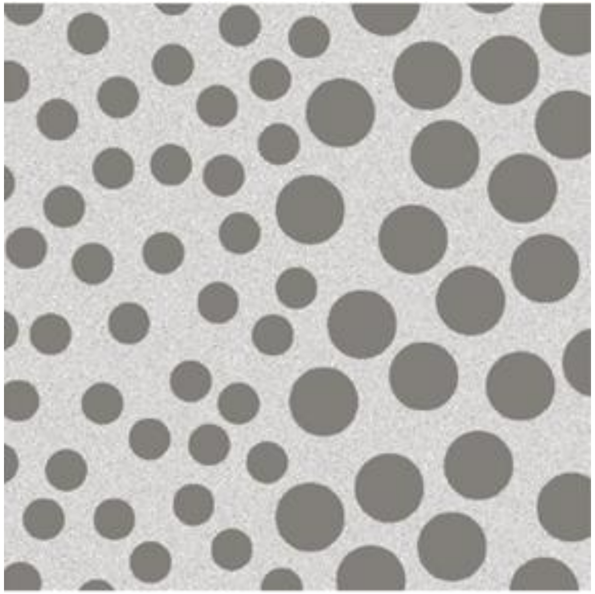
Textural segmentation

The objective is to find a boundary between the large and the small blobs (texture segmentation). The objects of interest are darker than the background. A closing with a SE larger than the blobs would eliminate them.



Textural segmentation (cont.)

- Closing with a SE of radius 30.
- The small blobs disappeared as they have a radius of approximately 25 pixels.




Textural segmentation (cont.)

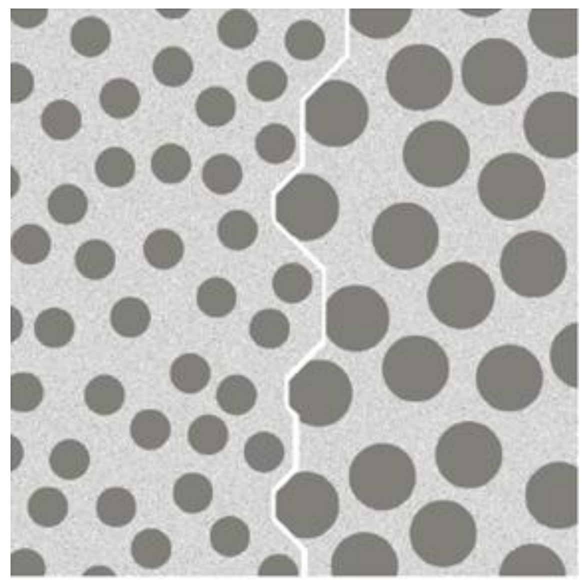
The background is lighter than the large blobs. If we open the image with a SE larger than the distance between the large blobs then the blobs would disappear and the background would be dominant.

Textural segmentation (cont.)

Opening with a SE of radius 60.
The lighter background was suppressed to the level of the blobs.

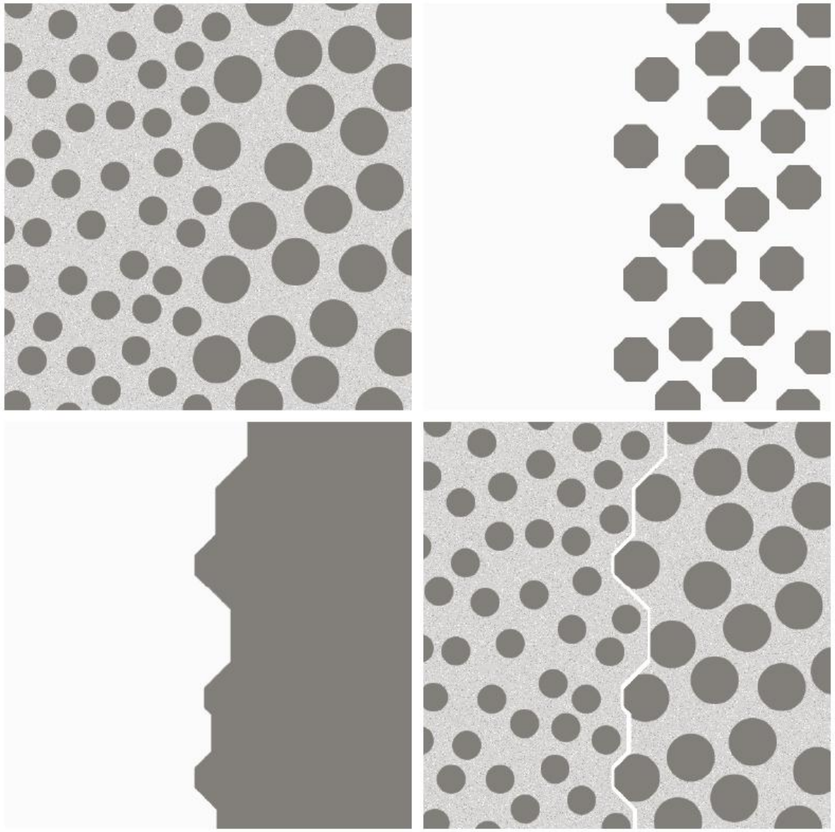
Textural segmentation (cont.)

A morphological gradient with a 3x3 SE gives the boundary between the two regions which is superimposed on the initial image.



Textural segmentation (cont.)

Original image



Closing with a SE of radius 30 (small blobs are removed)

Opening with a SE of radius 60 (large blobs flooded the background)

Morphological gradient superimposed onto the original image

Gray-Scale Morphological Reconstruction

The **geodesic dilation** of size 1 of a marker image f by a SE b, with respect to a mask image g is defined by:

$$D_g^{(1)}(f) = (f \oplus b) \wedge g$$

where $\wedge$ is the point-wise minimum operator.

This equation indicates that the geodesic dilation of size 1 is obtained by first computing the dilation of f by b and then selecting the minimum between the result and g at every point (x,y).

Gray-Scale Morphological Reconstruction (cont.)

The **geodesic dilation** of size n of a marker image f by a SE b, with respect to a mask image g is defined by:

$$D_g^{(n)}(f) = D_g^{(1)}\left[D_g^{(n-1)}(f)\right]$$

with $D_g^{(0)}(f) = f$

Gray-Scale Morphological Reconstruction (cont.)

The **geodesic erosion** of size 1 of a marker image f by a SE b, with respect to a mask image g is defined by:

$$E_g^{(1)}(f) = (f \ominus b) \vee g$$

where $\vee$ is the point-wise maximum operator.

This equation indicates that the geodesic erosion of size 1 is obtained by first computing the erosion of f by b and then selecting the maximum between the result and g at every point (x,y).

Gray-Scale Morphological Reconstruction (cont.)

The **geodesic erosion** of size n of a marker image f by a SE b, with respect to a mask image g is defined by:

$$E_g^{(n)}(f) = E_g^{(1)}\left[E_g^{(n-1)}(f)\right]$$

with $E_g^{(0)}(f) = f$

Gray-Scale Morphological Reconstruction (cont.)

The **morphological reconstruction by dilation** of gray scale image g from a marker image f is defined as the geodesic dilation of f with respect to g, iterated until stability is achieved:

$$R_g^D(F) = D_g^{(k)}(F)$$

with k such that:

$$D_g^{(k)}(F) = D_g^{(k+1)}(F)$$

Gray-Scale Morphological Reconstruction (cont.)

The **morphological reconstruction by erosion** of gray scale image g from a marker image f is defined as the geodesic erosion of f with respect to g, iterated until stability is achieved:

$$R_g^D(F) = E_g^{(k)}(F)$$

with k such that:

$$E_g^{(k)}(F) = E_g^{(k+1)}(F)$$

Gray-Scale Morphological Reconstruction (cont.)

The **opening by reconstruction** of size n of an image f is defined as the reconstruction by dilation of f from the erosion of size n of f:

$$O_R^{(n)}(f) = R_f^D\left[(f \ominus nB)\right]$$

The image f is used as the mask and the n erosions of f by b are used as the initial marker image.

Recall that the objective is to preserve the shape of the image components that remain after erosion.

Gray-Scale Morphological Reconstruction (cont.)

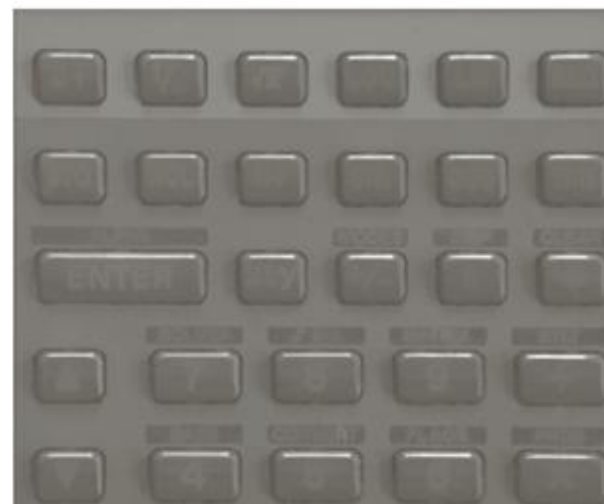
- The image has a size of 1134x1360.
- The target is to leave only the text on a flat background of constant intensity
- In other words, we want to remove the relief effect of the keys.

Gray-Scale Morphological Reconstruction (cont.)

At first we suppress the horizontal reflections on the top of the keys.

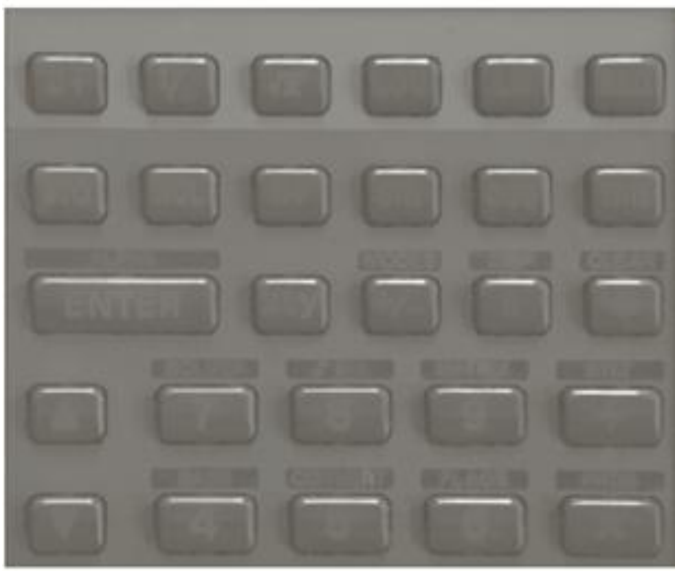
The reflections are wider than any single character.

An opening by reconstruction using a long horizontal line SE (1x71) in the erosion operation provides the keys and their reflections.

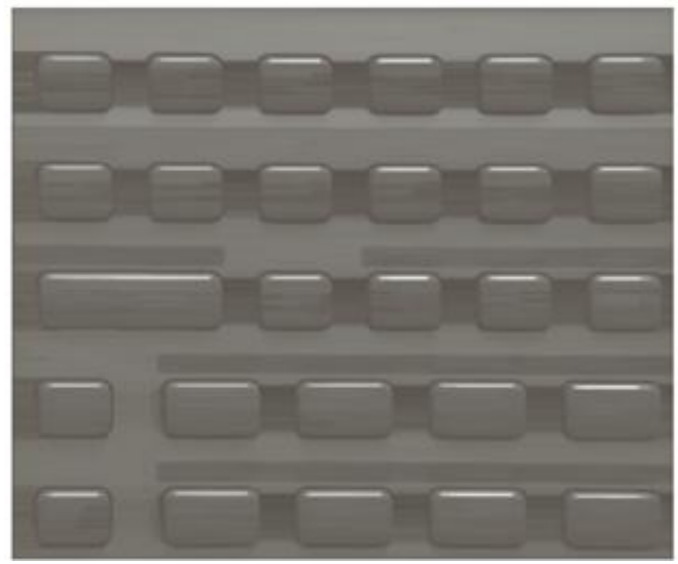


Gray-Scale Morphological Reconstruction (cont.)

A standard opening would not be sufficient as the background would not have been as uniform (e.g. look at the regions between the keys horizontally).



Opening by reconstruction



Standard opening

Gray-Scale Morphological Reconstruction (cont.)

Then, subtracting this result from the original image (*top-hat by reconstruction*) eliminates the reflections.

Original Image

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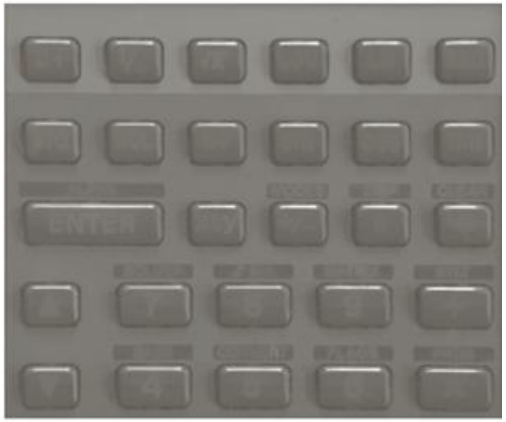

Opening by reconstruction

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Top-hat by reconstruction

Gray-Scale Morphological Reconstruction (cont.)

A standard top-hat transformation would not be sufficient as the background is not as uniform as in the top-hat by reconstruction operation.

Top-hat by reconstruction

Standard top-hat

Gray-Scale Morphological Reconstruction (cont.)

We now try to suppress the vertical reflections on the sides of the keys.

An opening by reconstruction using a horizontal line SE (1x11) in the erosion operation provides the keys and their reflections (after subtracting the result from the previous image).

Notice however that vertically oriented characters are eliminated (The “I” in the ”SIN” key)

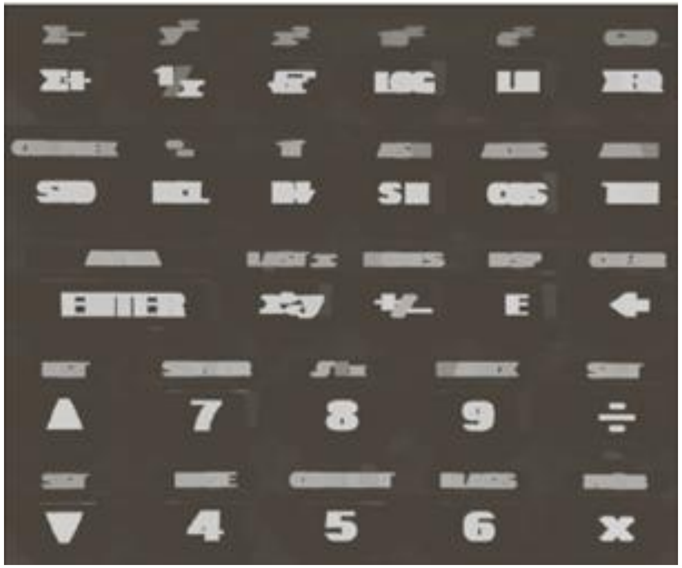
Gray-Scale Morphological Reconstruction (cont.)

How can we restore the suppressed character?

A dilation is not sufficient as the area of the suppressed character is now occupied by the expansion of its neighbors.

Dilation

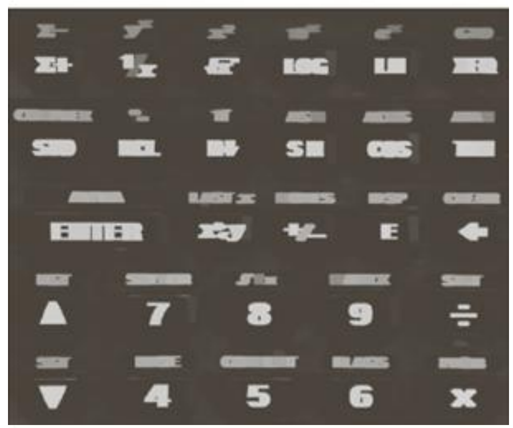
(SE 1x21)



Gray-Scale Morphological Reconstruction (cont.)

We form an image by taking the point-wise minimum between the top-hat by reconstruction image and the dilated image:

$\wedge$



$=$

Top-hat by reconstruction

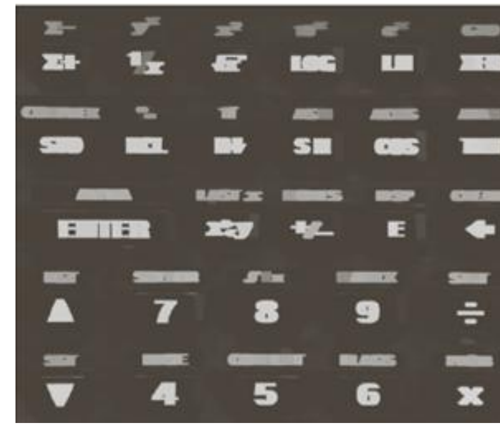
Dilated image

The result is close to our objective but the "I" is still missing

Gray-Scale Morphological Reconstruction (cont.)

Using the last image as a marker and the dilated image as a mask we perform a gray-scale reconstruction by dilation and we obtain the desired result.

Marker



Mask

Result