

# A Parametric Feature-based CAD System for Reproducing Traditional Pierced Jewellery\*

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## Abstract

In this paper, we introduce ByzantineCAD, a parametric CAD system for the design of pierced medieval jewellery, which is jewellery created by piercing, a traditional Byzantine technique. ByzantineCAD is an automated parametric system where the design of a piece of jewellery is expressed by a collection of parameters and constraints and the user's participation in the design process is through the definition of the parameter values. We present an approach to designing traditional pierced jewellery using a voxel-oriented feature-based Computer Aided Design paradigm: a large complex pierced design is created by appropriately placing elementary structural elements. We also present a scaling algorithm for enlarging pierced designs without altering the size of the elementary structural elements used to construct them.

## 1 Introduction

Computer Aided Design (CAD) systems are widely used in most industries and are increasingly used in jewellery manufacturing [1, 2]. While manual design of jewellery is still in wide use, this approach is both cumbersome and time consuming when compared to designing using 3D CAD systems. Furthermore with manual design, the designer sketches are 2-dimensional, thus they can only capture a limited part of his/her creative vision, whereas with 3-dimensional CAD systems, the model is realistically rendered from various viewpoints, giving the designer a clear optical understanding of the final result.

Editing and redesigning are feasible in a 3D CAD environment, as opposed to the difficult task of fixing 2D design sketches. 3D rendering helps the artist correct or redesign parts of the model that are unsatisfactory, or are to be determined by the customer [3]. In addition, in a 3D CAD system the designer has various tools to assist him/her in designing a piece of jewellery, including transformations, primitive solids, embedded libraries of jewellery parts such as cut stones and gems and ring settings. Finally, an important advantage of CAD systems is that models can be passed on directly to rapid prototyping machines for the manufacturing of jewellery using layering techniques.

However, there are categories of jewellery that are not easily designed even with modern CAD systems. An example of such a type is traditional Byzantine jewellery, which belongs to the family of pierced jewellery. This kind of jewellery was designed and created by hand using piercing, a traditional Byzantine technique, and the designs illustrated there were very detailed

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and complex. The design of pierced jewellery is a difficult task, not easily accomplished successfully with modern commercial CAD systems. In this paper we present ByzantineCAD, a parametric CAD system suitable for the design of pierced Byzantine jewellery. The system is automated and parametric [4, 5], meaning that the user-designer merely sets some parameter values and ByzantineCAD creates the jewellery model that corresponds to the specified values. This provides the designer with the ability to rapidly create custom-designed jewellery, based on the preferences of the customer. For instance, someone may wish to put their initials on a ring. We introduce feature-based [6] and voxel-based [7, 8] approach to designing jewellery, by defining elementary structural elements with specific attributes and properties that are used as building blocks to construct complex pierced designs.

This paper describes the development of a parametric feature-based system for designing and manufacturing jewellery. In particular, it makes the following technical contributions:

- introduces a novel feature-based approach to designing and manufacturing complex jewellery of a particular craftsmanship,
- presents a method for illustrating complex designs using non traditional voxels (such as elementary pierced solids),
- presents an algorithm for scaling design objects using fixed size pierced designs,
- describes our experience in the development of this software and in particular provides the reader with the heuristics adopted on how to optimize the time consuming solid manipulation operations and how to produce robust solid models ready for 3D printing.

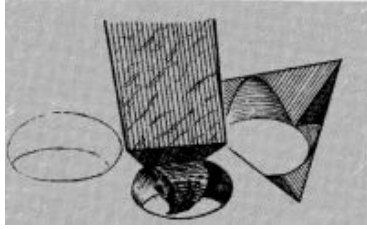
Section 2 describes pierced Byzantine jewellery and the particular way in which they were created. In section 3, we refer to the reasons for which current commercial CAD systems are not suitable for the design of this particular family of jewellery. Section 4 describes the approach used by ByzantineCAD to approximate a pierced design. Pierced designs are approached using a feature-based method, in which appropriate structural elements are defined and combined in “layout description files”. Section 5 presents a method for scaling pierced designs, while maintaining a fixed size for the basic components used to construct them. Section 6 describes an extension of ByzantineCAD, to include custom solid designs in a pierced surrounding. Section 7 presents our experience in developing this software. Section 8 provides conclusions.

## **2 Pierced Byzantine Jewellery**

Pierced Byzantine jewellery is gold jewellery with pierced designs that were made along the coastlines of the eastern Mediterranean Sea during the period 3<sup>rd</sup> –7<sup>th</sup> century A.D. Their originality is due to the particular processing technique that is used for their creation and the special esthetic effect [9].

Pierced jewellery was created from thin sheets of gold. The designs were engraved on these sheets of gold with a thin sharp tool. After the outlining of the designs, holes following their shape were created and these were decorated with triangular carvings, using an iron chisel (

Figure 1) [10, 11].



**Figure 1 - Using a chisel to create carvings around a hole**



**Figure 2 - A pierced Byzantine bracelet**

By studying pierced Byzantine jewellery one can distinguish certain types, shapes and characteristics. The piercing technique was applied to various types of jewellery, such as necklaces, earrings, rings and bracelets (

Figure 2) [9]. This technique was also used to create pierced crowns, pins used for holding tunics, belts and brooches. Jewellery such as earrings and pendants on the necklaces were usually shaped as rectangles, circles, hexagons, half moons (lunes) and teardrops. Many pieces of jewellery were decorated with sparsely-placed solid beads, or with a beadlike sequence wrapped around the piece of jewellery.

The designs used in Byzantine jewellery were specific. Usually, they were representations of nature or of human life. Frequently used nature inspired themes were different forms of birds, peacocks in particular, animals, especially dolphins, and plants, such as clovers and vine leaves. Many pieces of jewellery represented scenes combining nature and human beings (Figure 3) [9]. Byzantine jewellery had often representations of letters or religious symbols such as the cross. The letters used belonged either to the Greek or Latin alphabet and gave a personal touch to the jewellery.



**Figure 3 - A pierced bracelet featuring people and birds**

Pierced Byzantine jewellery is interesting for many reasons. There is a limited number of such jewellery preserved until today. The technique used to create them is a very sophisticated form of craftsmanship, as are the designs featured on them, and this technique has faded over the years. Furthermore, their esthetic effect is unique. There are other techniques that result in jewellery with engraved designs, but they differ in the way that they are processed and the final esthetic result. An example of such a technique is braiding plain or decorative wire to create the lace-like effect of pierced jewellery. The difference of the processing technique can be observed in the details of the designs and the back side of the jewellery. When the jewellery is pierced, the

traces of the tool used for the piercing are apparent, and the back side of the jewellery is an ensemble of holes in a solid surrounding.

Our goal is to deploy CAD techniques to reproduce this type of jewellery.

### 3 CAD systems for jewellery design

Many commercial CAD systems have been developed for the purpose of designing jewellery. Table 1 provides an overview of some of the most popular jewellery design systems. Most of these systems are parametric and feature-based [12]. They provide graphical interfaces with excellent rendering capabilities. Most of these systems provide built-in libraries of settings and cut gems and stones and advanced feature-based design tools [13]. Some systems provide advanced functionality such as Matrix [14] that provides the use of builders for recording design steps and for defining parameter values for parts to be used in the process. Also, the majority of these systems have the capability of exporting models to rapid prototyping machines [15]. All of these systems provide various tools for making jewellery design a simpler and less time-consuming process.

However, none of these systems is appropriate for designing and creating pierced Byzantine jewellery or other jewellery of sophisticated craftsmanship.

In most of these commercial systems, designing is performed manually using various tools and usually the design steps cannot be programmed to be executed automatically. Even in systems that step recording and parameterization is feasible, the tools provided are not adequate to provide for a voxel-based construction of objects, a method which is necessary for reconstructing pierced jewellery in an automated user friendly manner.

Our system is fully automated and easy to use even by the end users. In our system one defines the basic parameters that refer mostly to the appearance, size and content of the final product and then the construction of the specified model is carried out by the system. The advantage of such a system is that the end user need not have designing skills or knowledge of using CAD systems.

Furthermore, designing a pierced jewel using a traditional CAD system may lead to models with robustness problems which are inappropriate for manufacturing, creating therefore the need for repairing tools and techniques.

Finally, editing parts of a pierced design in commercial jewellery design systems requires in depth knowledge of feature-based design and solid modeling techniques.

CAD system name	Characteristics of the system
JewelCAD [16]	Very good rendering techniques, capability of modifying and repairing STL files. The user can define her/his own libraries, calculate the weight and volume of a piece of jewellery, export models to rapid prototyping machines
JewelSpace [17]	A feature-based system with: realistic rendering, built-in libraries of gems and settings, support of many file formats. It provides a variety of tools for designing jewellery, and the capability of exporting models to rapid prototyping machines.
Rhinoceros	A general purpose system for which plug ins have been developed for

[18]	designing jewellery. It is compatible with other jewellery design systems. It provides NURBS technology, repair tools and export operations directly to rapid prototyping systems.
Matrix [14]	A parametric feature-based jewellery design system based on Rhinoceros. It provides specialized libraries with gems, stones and jewellery settings, realistic rendering techniques, and builders that record and carry out a sequence of design steps. It can export models for rapid prototyping.
ArtCAM Jewelsmith [19]	ArtCAM provides powerful modeling tools, stl export capabilities for rapid prototyping, advanced ring creation tools (built-in settings, easy ring size adaption), precision manufacturing capabilities
TechJems 3.0 [20]	A plug-in for Rhinoceros with jewellery designing tools such as: built-in libraries of gems, cuttings, materials and settings, advanced tools such as a chain and ring creator, dummy tool (eg. a dummy bust for necklace designing), pave tools (for placing objects perpendicular to a surface), weight calculation tools, and STL export capabilities.
Jcad3/Takumi Pro [21]	Built-in libraries of cut gems, realistic rendering techniques and a variety of designing tools.
CADJewel 3D [22]	It is a system based on Rhinoceros and therefore has the same characteristics as Rhinoceros.

**Table 1- Popular Jewellery CAD systems and their characteristics**

#### **4 ByzantineCAD: A Parametric CAD System for the Design of Pierced Byzantine Jewellery**

By studying the craftsmanship of pierced Byzantine jewellery it is apparent that a pierced design can be represented by defining and combining appropriate structural elements. These structural elements are placed side by side, either on top, bottom, right or left of each other, and unioned into a new object. For two neighboring elements to be unioned, the facet of the first element must coincide exactly with the corresponding facet of the second element. A pierced design can be defined if we know which structural elements are required and in what order they have to be placed. The corresponding information for each pierced design is stored in a file called the “layout description file”.

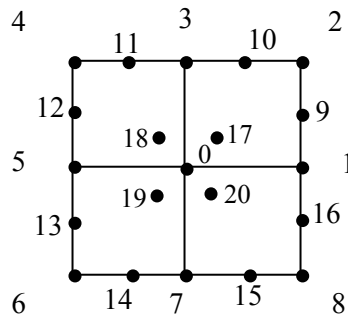
##### **4.1 An approach to designing pierced jewellery using structural elements**

###### *4.1.1 Description of a structural element*

By studying pierced Byzantine jewellery we discovered that the design is not just made up of cylindrical holes, but the holes have carvings around them. Each hole with the carvings around it can be considered a structural element. Each structural element is a solid made of a rectangular parallelepiped with a cylindrical hole and the corresponding carvings around the hole (Figure 5). The conformity of the structural elements with traditional jewellery was carefully examined in cooperation with Dr. Jack Ogden, a specialist in the materials and techniques of ancient and historic precious metals and jewellery [10, 11]. All structural elements have the same size but differ in the position of the hole and the carvings around it. The hole can be located either in the center of the parallelepiped or in any of the four quarters.

#### 4.1.2 The repertoire and naming scheme of the structural elements

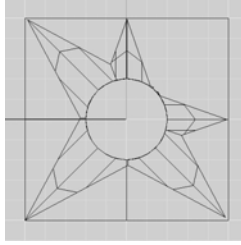
We introduce a complete naming scheme for the structural elements in the sense that a unique name is provided for every acceptable configuration. The name of a structural element describes the design created by the carvings and the cylindrical hole. The name is a string consisting of characters and numbers. The first two characters of the string describe the position of the cylindrical hole in the rectangular parallelepiped. The position of the hole is determined by positioning the circular area that corresponds to the top of the cylinder in the square upper face of the structural element. CT stands for “center” and corresponds to the circle is placed in the center of the square area. In the other cases, letters describe the quadrant where the circle is located (R= right, L= left, U= upper, L= lower). For instance, the name RU2 corresponds to a structural element whose cylindrical hole is located in the upper right quadrant of the solid. The letters describing the position of the hole are followed by a sequence of numbers describing the directions of the carvings. Specifically, we assume points in a structural element as depicted in Figure 4.



**Figure 4 - The valid direction points for carvings in a structural element**

Each carving is directed from the cylindrical hole towards one of these points. For instance, the carvings in the structural element of Figure 4 are directed towards points 1, 3, 4, 6 and 8. Therefore, the name of the structural element is CT13468. With this naming scheme we can label all valid structural elements and ensure that each different element will have a unique name. The order of the position numbers in the sequence is not significant, meaning that the structural element CT13648 is the same as CT13468. This is due to the fact that the name simply describes the set of the positions of the carving, without implying any order. However by convention, we assume that the carvings are described starting at point 1 and moving counterclockwise in the structural element.

In the case where the position of a point is described by a two-digit number, such as 19, this number is prefixed with the sign “\$”. The \$ symbol means that a two-digit number follows in the sequence. For instance, the name RL14\$19 refers to a structural element whose hole is located in the lower right corner and contains 3 carvings directed to positions 1, 4 and 19.



**Figure 5 - The structural element CT13468**

#### 4.1.3 Validity rules for structural elements

The number of different structural elements that can be created by a hole and carvings is quite large. Not all of these possible elements are valid for use in creating pierced designs. In order to determine which structural elements are valid for creating pierced designs, the following rules are applied:

- 1) If the hole is positioned in the center of the rectangular parallelepiped then,
  - i. Each carving starts from the hole.
  - ii. A carving cannot be directed towards the hole. Each carving is directed away from the hole.
  - iii. The valid points to which a carving can be directed are points 1-8.
  
- 2) If the hole is positioned in one of the four quadrants then,
  - i. Each carving starts from the hole
  - ii. Each carving is directed away from the hole
  - iii. The valid points to which a carving can be directed are 0-20. This however is constrained by the position of the hole.
    - a. The points that belong exclusively to the quadrant where the hole is located are not valid points for carving ends. For instance, if the hole is located in the quadrant RL (right lower), then points 8, 15, 16, and 20 are not valid end points for carvings, whereas 7 and 1 are, since they also belong to other quadrants.
    - b. Two carvings cannot be directed to points belonging to the same quadrant. In the previous example, if there is a carving pointing to position 2, then points 0, 1, 3, 9, 10 and 17 are not valid points for a second carving in the element.
    - c. There can be only one carving pointing to an internal point around the center point 0. This means, for example, that if a carving points to position 18, then there cannot be another carving pointing to positions 17, 19, 20 and 0.

Aside from the structural elements described above, there is also a structural element that is compact, with no hole or carvings. This structural element is called “SP” (Solid Parallelepiped), because it cannot be named according to the naming scheme.

From the group of structural elements that satisfy these conditions in the specific library of design that we have developed for ByzantineCAD we used only a subset, because the elements belonging to this subset are sufficient for the pierced designs that we have built. The rest of the valid structural elements can be used in developing alternative fonts and new patterns. Some characteristics of the structural elements that belong to the subset used so far in ByzantineCAD are a) the structural elements with holes located in the center have at most 5 carvings, and b) the

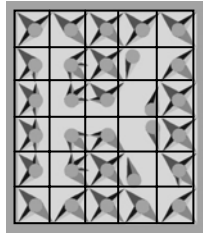
structural elements with the hole in one of the quadrants do not have more than 3 carvings. In Appendix A we present the figures of the structural elements translated and rotated in ByzantineCAD for creating pierced designs.

## 4.2 Coding a pierced design

### 4.2.1 The layout description files

Each pierced design is a combination of structural elements. Therefore, every design can be described using a “layout description file”, a file where the information needed to construct a specific pierced design is stored. Description files are simple text files containing in a row-wise format the names of the structural elements that the design consists of.

Each design can be thought of as a two-dimensional matrix (Figure 6) whose every entry corresponds to a structural element. The layout description file determines the structural element that must be placed in each position of the matrix.



**Figure 6 - The letter K as a 2-dimensional matrix**

The first line of a layout description file records the size of the matrix corresponding to the pierced design. The size of the design is expressed as [#rows] × [#columns], expressing the number of rows and the number of columns of structural elements used to create the overall design. The size is followed by the row-by-row description of the design, which is expressed by the names of the structural elements and the corresponding transformations (if any).

Each row is described from left to right. Rows are separated by empty lines. An example of a description file is shown in Figure 7. This file describes a pierced letter K. The size of the letter is 6×5 blocks. In the example of Figure 7 the first row of letter K is constructed as follows:

CT246:	is placed in position	[1][1]
CT248		[1][2]
CT2468		[1][3]
CT246		[1][4]
CT248		[1][5]



```

6*5

CT246
CT248
CT2468
CT246
CT248

CT467
CT1278
CT2468
LU6
CT268

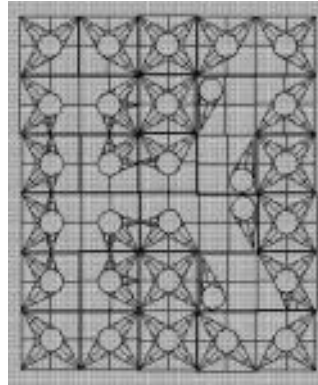
CT3467
CT123
CT245
RL2
CT268

CT3467
CT178
CT568
RU8
CT2468

CT346
CT1238
CT2468
LL4
CT2478

CT468
CT268
CT2468
CT468
CT268

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**Figure 7-(left) The description file for the letter K and (right) the pierced representation of letter K**

#### 4.2.2 *The process of constructing a pierced design*

A pierced design is created by using its layout description file. Each time the name of a structural element is read, it is created, transformed (if necessary) and then translated to the proper location. The horizontal and vertical translations of the element are calculated using the equations:

$$x = h \times l \quad (1)$$

$$y = v \times k \quad (2)$$

where  $x$ ,  $y$  are the horizontal and vertical translations respectively,  $h$  is the number of structural elements already placed horizontally in the current row,  $v$  is the number of structural elements already placed vertically in the current column and  $l$ ,  $k$  are the height and length of the structural element.

Every time a structural element is placed, it is unioned with the previous ones. Eventually, a pierced plate representing the design is created.

#### 4.2.3 The process of constructing a sequence of pierced designs

The pierced design on a piece of jewellery can be a sequence of individual designs. For instance, the design may be a sequence of letters forming a word. In this case, the process of creating the plate representing the word is the same as for a single design.

Specifically, the layout description files of each individual design are read in parallel and the plate is created row-wise (Figure 8). First the first line of the first letter is created, then the first line of the second letter is created and unioned with the first letter's first line and so on.

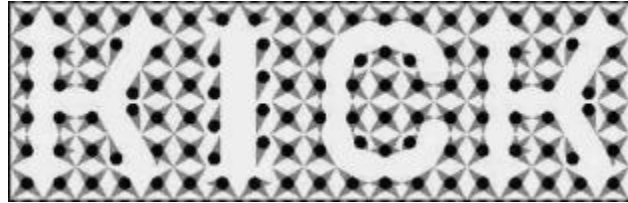


Figure 8 - A pierced plate

#### 4.3 The creation of different kind of jewellery

ByzantineCAD is capable of designing rings, bracelets, necklaces and earrings. Rings and bracelets are designed in a similar way: the pierced plate with engraved figures is constructed and then it is bended in a circle around the y-axis passing through the horizontal center of the plate. We consider a plane created by two direction vectors, the bending direction and the bending axis (Figure 9), that passes through the center of the pierced plate. The plane divides the plate into two sections. Each section is bended by  $180^\circ$  around the bending axis, towards the bending direction, on a circle. During the bending, the parts of the plate that are located on the circle are not altered, whereas the rest of the plate is compressed or stretched, in order to create the pierced cylinder (Figure 10). The points on the back surface of the plate are compressed whereas the points on the front surface are stretched. The points contained between these surfaces, excluding the ones that coincide with the circle, are stretched and compressed proportionally to their distance from the center surface.

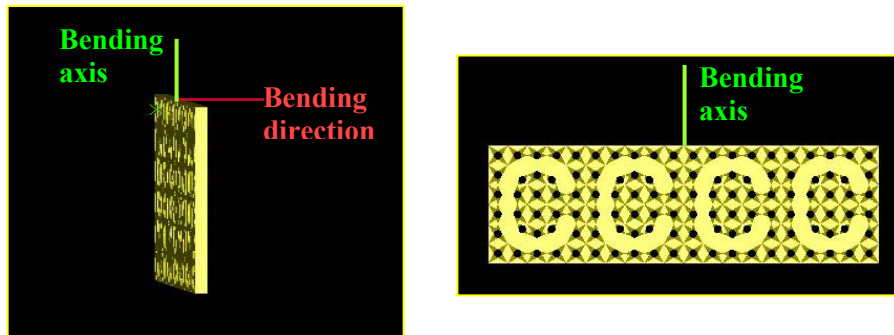
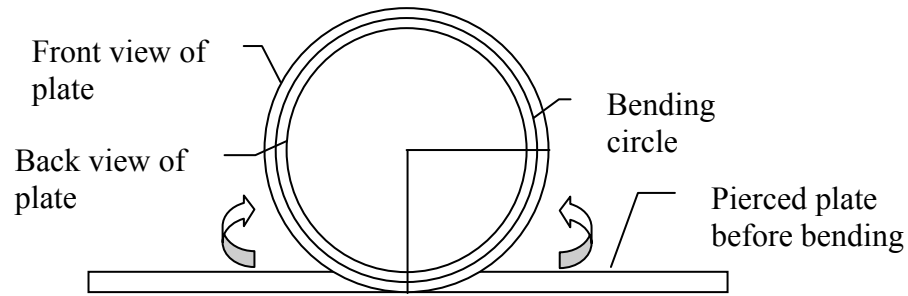


Figure 9 - The directions defined for bending the pierced plate



**Figure 10 - Cross section of pierced plate during bending**

For creating a ring or a bracelet, certain parameters have to be defined by the end user. Specifically, the user defines the size of the structural elements, the size of the ring and the designs that are to be engraved on the piece of jewellery. The designer may also choose to add a beaded border around the ring or bracelet. After the values for the parameters are set, the system checks the validity of the parameters [23], to see if a model can be created based on the specific values. The parameters are connected with the following equation:

$$p = c s \quad (3)$$

where  $p$  is the size of the plate,  $c$  is the number of columns in the plate and  $s$  is the size of the structural element.

The number of columns  $c$  is an integer, therefore the ration  $p/s$  should be integer. The system can slightly modify the given parameter so as to ensure that  $c$  is an integer. The parameter  $p$  cannot be modified, because rings and bracelets have a small fixed set of size values. Thus parameter  $s$  is adjusted accordingly.

Earrings and necklaces are created in a number of different shapes, and are decorated with a beaded border (Figure 11).



**Figure 11 - Some shapes of necklaces and earrings**

Each of the above shapes can be considered as a combination of two parts: a solid shape and a beaded border made up of bullet-like solids. Each bead of the border is created by unioning a cylinder with a sphere of the same diameter. The sphere is placed so that its center coincides with the center of the circle that creates the top surface of the cylinder. This creates a solid shaped as a bullet. These bullets are then placed almost tangent to the solid shape around the solid shape and side-by-side to each other, in such a way so that they slightly overlap. Then the bullets are unioned with the solid shape. We would like to create pierced jewellery with these shapes. This can be done by constructing a large pierced plate and by “cutting” out of the plate the desired shape. Therefore, we construct an enlarged pierced plate containing different designs, a solid

with the desired shape, and we perform an intersection operation between them (Figure 12). This results in a pierced jewel in the desired shape (Figure 13).

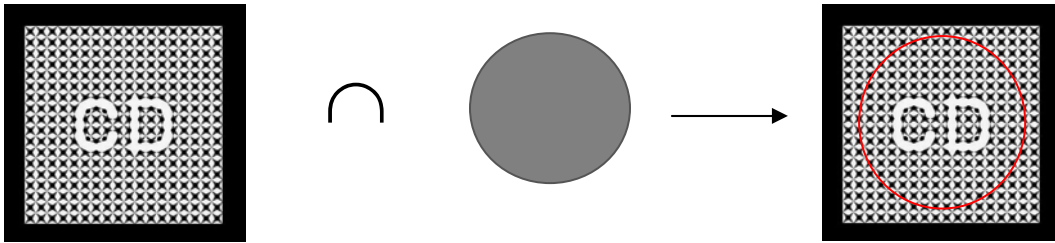


Figure 12 - Intersecting a cylinder and the pierced plate

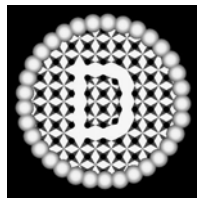


Figure 13 - A pierced earring in the shape of a circle

The parameters that have to be defined by the end user are: the shape of the jewel, its size, the size of the structural elements and the designs that are to be put on the jewel. The relation that is checked and modified for validity is:

$$j \geq c s \quad (4)$$

where  $j$  is the size of the piece of jewellery,  $c$  is the number of columns in the plate and  $s$  is the size of the structural element.

## 5 An algorithm for scaling pierced patterns

For designing jewellery depicting complex scenes it is important to have the capability of enlarging a pierced design without altering the size of the structural elements used to construct it. Having this capability we may include, for instance, different font sizes in the same design. For this reason we have developed a scaling method for pierced designs.

As mentioned earlier a pierced design is thought of as a 2-dimensional matrix whose every entry contains a structural element. Respectively, the scaled version of a design will be a larger 2-dimensional matrix (Figure 14).

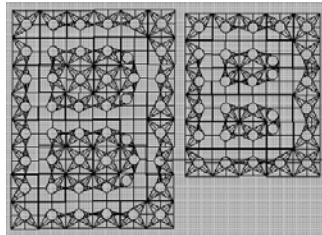


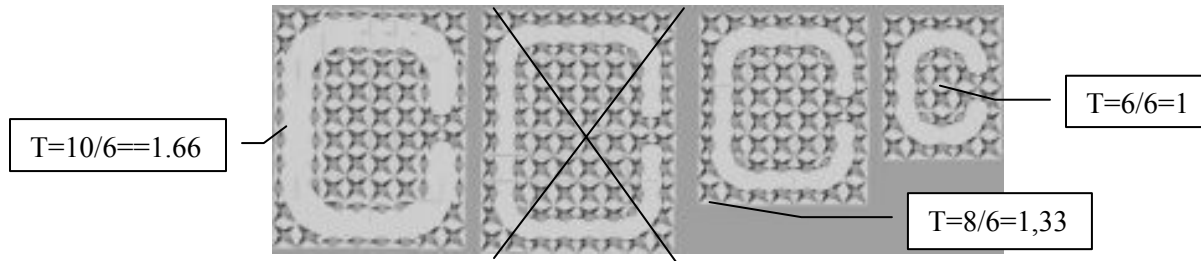
Figure 14 - The letter B in its original and scaled form

The percentage of scaling that can be achieved is discrete, because of the need to preserve symmetries that may exist in the original design. For instance, letter B is symmetric by a horizontal axis that goes through the middle of the design. A design is scaled by means of new rows and columns added to it. If we add only one new row to letter B, the letter becomes asymmetric, because if it's added to the upper half or the lower half of the design, then the letter's shape is altered unintuitively. Also if it is added in the middle, the design becomes unproportionally thicker at that point and therefore alters its original shape. These restrictions are best expressed by the following rules for upwards scaling:

- avoid adding one row, or one column, and
- the number of rows and columns is integer

As a consequence of the first rule we choose to perform discrete scaling at a fixed factor. We choose a scale factor of 1.33 because it always results in adding two or more rows or columns.

Therefore, from now on, we will refer to levels of scaling and not the scaling factor. Level 1 corresponds to scaling the design by a factor of 1.33, Level 2 corresponds to a scale factor of 1.66 and so on.



**Figure 15 - Different scaled versions of the letter C**

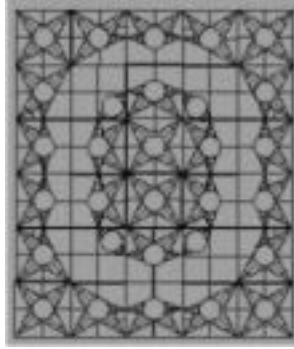
We observe that while scaling a design (Figure 15), as the design gets larger, there is a need for thickening the engraved shape, so as to preserve its original form. In Figure 15 we see an original design of the C (rightmost design), which is scaled to two different levels (Level 1: 8 rows and Level 2: 10 rows). There are two different versions of Level 2 scaling. The crossed out version of letter C is not valid, because there is a distinct difference in the font style, compared to the original design. Therefore, the interior shape in Level 2 has to be thickened. We define as thickness factor T the ratio:

$$T = \frac{\text{Number of rows in scaled design}}{\text{Number of rows in original design}} \quad (5)$$

If  $T < 1.5$ , the thickness of the curves inside the design are not altered. If  $1.5 < T \leq 2$  the thickness is increased by 50%, whereas if  $T > 2$  the shape thickness is doubled (100% increase).

### 5.1 The scaling algorithm

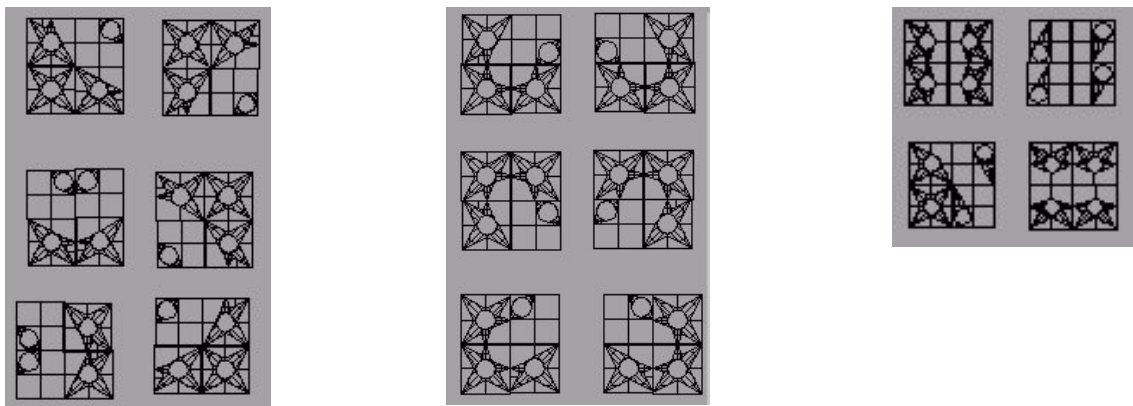
A pierced design is represented by a 2-dimensional matrix whose every entry contains a link to a structural element. For instance let us consider the Level 1 scaling of a letter of font size  $6 \times c$ . When scaled to Level 1 a design is transformed from a  $6 \times c$  matrix to an  $8 \times k$  matrix (8 is the closest integer to  $6 * 1.33 = 7.98$ ), where  $c$  and  $k$  are the number of columns of each matrix. The number of columns in the scaled design depends on the original number and is calculated in the same manner. For example, let us consider the scaling of letter O (figure 16).



**Figure 16 - The pierced letter O**

The size of the design, according to its description file, is  $6 \times 5$ . We would like to scale it up to Level 1, which means that the new design will have 8 rows. The number of columns, based on the original design, is calculated to be 7 (7 is the closest integer to  $5 * 1.33 = 6.65$ ), therefore a  $8 \times 7$  matrix is created and it remains to be filled in with the structural elements of the scaled design.

The idea behind the scaling method is to gradually scan the design row by row using a sliding  $2 \times 2$  window of structural elements, scale individually the  $2 \times 2$  windows of the design and then integrate smoothly the scaled overlapping parts to create the scaled version of the design. The combinations of the structural elements form different designs that can be categorized accordingly. An example is shown in Figure 17.



**Figure 17 - Different categories of  $2 \times 2$  structural element windows. Categories: (From L-to-R) "CURVES", "CURVE ENDS" and "LINES"**

The original design is thus scanned using a  $2 \times 2$  window that starts scanning the design row-wise from the upper left corner. The design is scanned from left to right, and from top to bottom. At each step the window is shifted to the right by one position, and when an entire row has been scanned, the window is initialized at the beginning of the next row.

Below we present an outline of the algorithm. Steps 1 to 4 are explained in more detail later:

*The Scaling Algorithm*

```
for  $i = 1$  to  $n$ 
  for  $j = 1$  to  $m$ 
    step 1: Consider the  $2 \times 2$  window of structural elements:  $W[i, j] = [D[i, j],$   

       $D[i+1, j], D[i, j+1], D[i+1, j+1]]$ 
    step 2: Determine the new magnified window  $W'$ . This window will be  

       $2 \times 2, 2 \times 3, 3 \times 2$  or  $3 \times 3$  according to the category and position of the  

      original window.
    step 3: Update the corresponding positions of the new scaled matrix  $D_s$   

      by placing the magnified window  $W'$  so as its upper left corner goes to  

       $[i, j]$ . If any such value conflicts with previous values of  $D_s$ , then integrate  

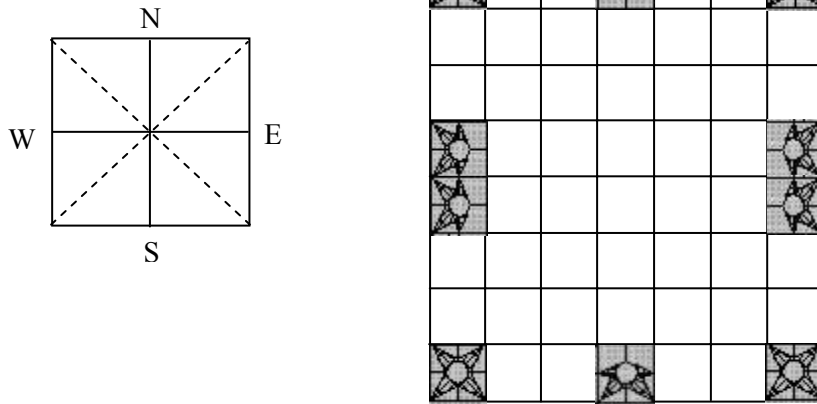
      them so that the two overlapping windows join smoothly with each other
  end for
end for
```

step 4: Go through  $D_s$  searching for empty entries and fill them in with the neutral structural element.

where  $n$  is the number of rows and  $m$  is the number of columns,  $D$  is the matrix describing the original design,  $D_s$  is the matrix describing the scaled design,  $W$  is the sliding window, and  $W_s$  is the scaled sliding window.

The time complexity of the algorithm is  $O(nm)$ .

Before scanning and scaling, datum positions are marked in the scaled design matrix. We consider the structural elements positioned at North, South, East, West, South-East, South-West, North-West and North-East as our datum “points”. These reference points are useful for making sure that we preserve the symmetries and that the various proportions of the shapes within the design are maintained. Figure 18 depicts the eight structural elements used as reference points. When the number of rows of the scaled design is even reference points E and W are duplicated. Respectively, when the number of columns of the scaled design is even reference points N and S are duplicated.



**Figure 18 - Datum positions are marked in the scaled design matrix**

**Step 1:** Every time a window scan is performed, a combination of 4 structural elements is returned.

**Step 2:** This combination is scaled individually and placed appropriately in the scaled design matrix. The scaling of the  $2 \times 2$  block of structural elements, is determined by the following principles:

- The relative position of the block in the original design should be maintained in the scaled design.
- The datum points should be respected.
- If the block contains part of a curve of a shape the corresponding curve should be scaled appropriately.

The size of the scaled combination is normally  $3 \times 3$ . However, according to the above principles the size of the scaled window may be reduced to  $3 \times 2$  or  $2 \times 3$  (one column or one row truncated), or  $2 \times 2$  (one row and one column truncated).

**Step 3:** The appropriate scaling for the specific combination determined in Step 2 is used to fill in the corresponding entries in the scaled matrix. This is placed in the new scaled matrix so as to overlap previous scaled windows. The overlapping is used to ensure that the connection among neighbor cells is a valid one.

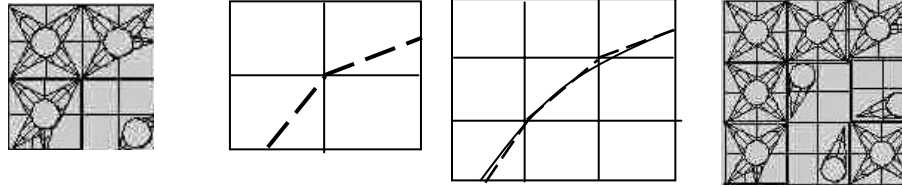
**Step 4:** The above steps are carried out row-wise until all of the design is scanned and scaled. If there are empty spaces in the scaled design matrix, then these are filled with the neutral structural element CT2468.

In the following we explain how the algorithm will scale the design of Figure 16. In general, the scaling method can be thought of as a transformation function through which a  $6 \times c$  design is scaled up to a larger one. The input of the function is the  $2 \times 2$  scan window and the output is a 2-dimensional matrix of various sizes, according to the placement of the window combination.

The way a  $2 \times 2$  combination of structural elements is scaled depends on the pattern created by the elements. Specifically, the structural elements can create curves, straight lines (horizontal,



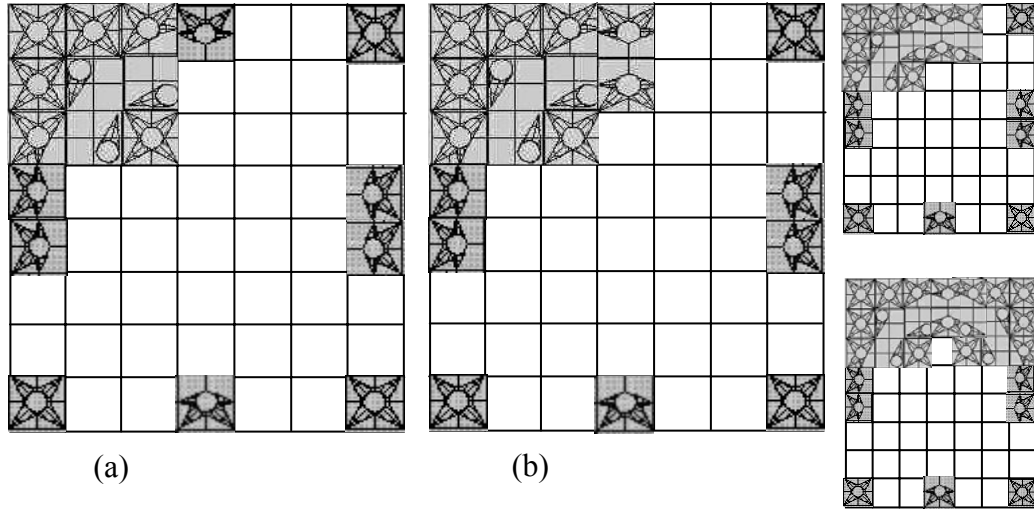
vertical and diagonal), line segments, “line intersections” and other designs. The scaled form of a combination tries to approximate the original form as much as possible. Let’s consider, for instance, the first window combination of the letter O. This combination creates an approximately  $\pi/3$  curve. The scaled version of this combination must also be a curve with the same angle.



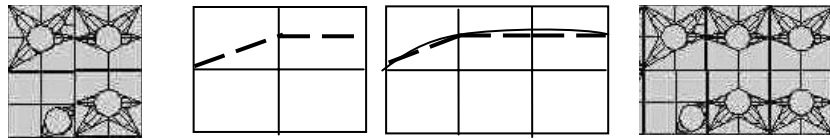
**Figure 19 - The process of scaling a curve combination**

By analyzing the original curve, we can see that it is created by two structural elements whose carvings create specific angles (Figure 19). If we consider that each structural element has a north, south, east and west orientation, then we can describe the angles through this orientation. Specifically, the structural element in position [2, 1] of the original design matrix, which marks the beginning of the curve, contains an angle with a S→NE orientation, whereas the element in position [1, 2] which continues the curve, contains a SW→E angle. The starting orientation of the curve is S and the ending orientation is E. Therefore the scaled version of this curve must have the same start and end orientation. After marking these points, the curve connecting them is drawn and then approximated with structural elements, as best as possible.

After the scaled version of the 2×2 combination window is determined, it is placed in the correct position in the scaling matrix (Figure 20 (a)). The window is shifted one position to the right and the next combination is examined. This combination is the extension of the previous curve to a horizontal line. Theoretically, the specific form could be scaled to the form shown in Figure 21. However, each quadrant of the design is scaled up until the datum point is reached and then it is connected with the datum’s element. In our example, the element in position [1, 2] of the scan window is the datum component, therefore the scaled version of this combination must not exceed that mark. Consequently, the scaled version of the specific form is the same 2×2 combination (Figure 20 (b)). The combination is placed in the design matrix by using overlapping. The combination to be placed is always positioned in such a way as to overlap the last column or row of the previous combination with the first column or row respectively of the newly placed one. This overlap however results in a conflict of overlapping elements. The conflicting element is located in position [2, 3] of the scaled design matrix (Figure 20 (b)). This position is currently occupied by a RL6 structural element. The orientation of this element is SW→E. The overlapping element in this position is the structural element RL17, with an orientation of S→E. To determine the structural element that will fill the position, we examine the neighboring elements already placed. From Figure 20 (a), (b) we observe that position [3, 2] is occupied by a structural element whose orientation is S→NE. Therefore, the element in position [2, 3] must have an orientation beginning with SW. The structural element that satisfies this condition is the one already in this place, therefore is not replaced.



**Figure 20 - The scaling of the letter O**



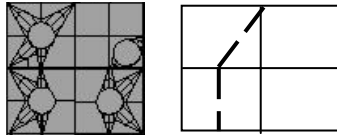
**Figure 21 - A combination of structural elements forming a curve end and a horizontal line, and its possible scaled version**

The window is again shifted in the design matrix and the next combination is a horizontal line ending with the beginning of a curve. This form is the mirror image of the previous combination, meaning that we can scale it in two different ways. In this case the next  $2 \times 2$  combination is examined. If the next combination is scaled to 3 columns, then the current combination is kept  $2 \times 2$ . Otherwise it is scaled to its  $2 \times 3$  version. In our case, it is used in its  $2 \times 2$  form and it is placed in the scaled design matrix.

The window is shifted for the last time in the first row. The last combination is analogous with the first one, and the same methodology is followed.

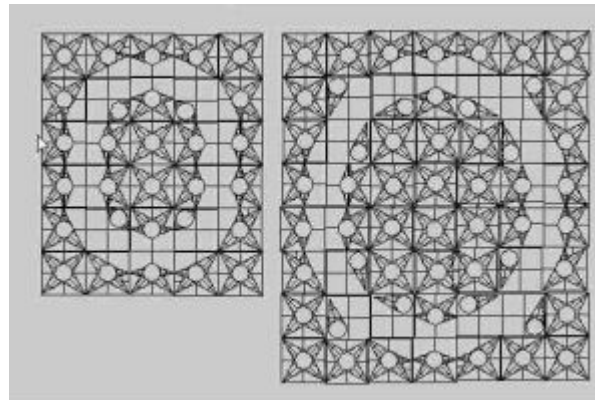
The scaling window is shifted down one row and it is initialized to the first column. The combination examined is a curve end continued by a vertical line. The scaling process for this combination is again analogous with that of the second step in the first row, only that in this case the form can be scaled downwards by one row (Figure 22). However, the position  $[2, 1]$  of the combination window is marked by a datum point element, therefore the accepted form is that of the  $2 \times 2$  window. This combination must now be placed in the scaled design matrix. The combination to be added is placed so that its first row overlaps the last row of the combination above it. This overlap however results in a conflict of overlapping elements. The conflicting element is the one placed in the position  $[3, 2]$ . This position is filled with the element RL2, whereas the overlapping element is RL17. To determine which element will finally occupy the position, we examine their orientation. The first element is oriented from S to NE, whereas the second is oriented S to E, so their difference is due to their neighboring element on their east side. The elements beside (east side) and above diagonally (northeast side) the conflicting element are examined and based on its orientation, the proper one is chosen. In our case, the

element on the eastern side is empty (to be filled later on with a neutral element) and the northeastern element has an SW to E orientation. Therefore the element that wins the conflict is the one already occupying the corresponding entry.



**Figure 22 - The analysis of a combination of a curve and a vertical line**

This method is continued until all of the design is scanned and scaled. If there are empty spaces in the scaled design matrix, then these are filled with the neutral structural element CT2468 (Figure 23).



**Figure 23 - The letter O and its scaled version**

## **6 Extending ByzantineCAD to encapsulate custom solid figures in a pierced surrounding**

In traditional Byzantine jewellery there were pieces in which solid non-pierced designs were placed in a pierced environment (Figure 24) [9]. Therefore we have expanded our system to have the capability of embedding solid designs in pierced surroundings.



**Figure 24 - Byzantine jewellery showing 2 peacocks and a cross**

To integrate a solid design with a pierced surrounding, certain conditions have to be satisfied:

- a) The design must be provided in a format that is compatible to the system. For ByzantineCAD the internal solid modeling format used is the ACIS [24] SAT format. The solid design can be created using CAD or modeling systems capable of exporting designs in ACIS SAT format. The designs demonstrated in this section were created using the general purpose CAD system Rhinoceros.
- b) The center of the design imported is by convention located at the origin in the global coordinate system.
- c) The dimensions of the solid design imported to ByzantineCAD are predefined. We consider the predefined dimensions of the solid to be  $10 \times 10 \times 2$  units. ByzantineCAD users must be able to express in a simple manner the desired dimensions of the solid design in the piece of pierced jewellery. Therefore we assume that the dimensions of the solid design are expressed through the number of structural elements that correspond to the volume the non-pierced design will occupy.

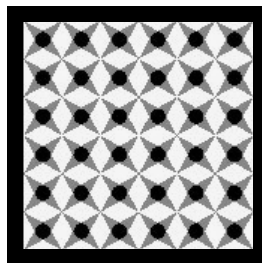
For instance, suppose we want to include the design in Figure 25 as part of a necklace.



**Figure 25 - A solid design featuring a cross**

Originally, the dimensions of the imported solid design are  $10 \times 10 \times 2$  units. We provide the system with the dimensions of the design, referring to the number of structural elements needed to create the matrix it will be embedded in. For example, if the desired size of the design is  $3 \times 3$  structural elements (we assume that the width of the design is constant), then the design is scaled appropriately so that its horizontal size is equal to  $3 \times$  (structural element length) and its vertical size is equal to  $3 \times$  (structural element height).

Suppose we would like to insert a solid design into pierced jewellery. Initially, the designer selects the designs, pierced or solid, to be include in the piece of jewellery. ByzantineCAD creates the plate replacing the positions of the solid designs with neutral structural elements CT2468, as is shown in Figure 26.



**Figure 26 - A complex structure made up of neutral structural elements**

After the creation of the plate, the files containing the solid designs are loaded by the system and each design is scaled to the appropriate size. A rectangular parallelepiped with the same size as the plate is created and placed at the center of the neutral plate. It is subtracted from the plate (Figure 27) and the solid design is placed in the empty space that is left and unioned with the rest of the plate (Figure 28).

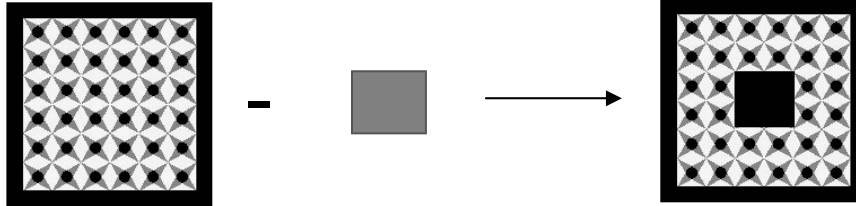


Figure 27 - Subtracting a cubic solid from the pierced plate. The custom solid design is inserted in the empty space

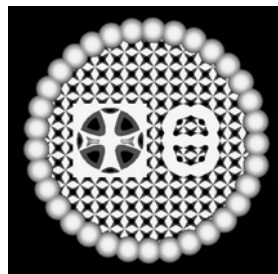


Figure 28 - An example of pierced jewellery with a solid design

## 7 Implementation Issues

ByzantineCAD was implemented under the Microsoft Visual C++ programming environment using ACIS solid modeling libraries by Spatial [24].

ByzantineCAD is a user friendly system that provides the designer with various capabilities. The user-designer interacts with the GUI of the system and provides the parameter values and then the system creates the model based on these values (Figure 29).



Figure 29 – The GUI of ByzantineCAD: (left) The necklace parameter window, (right) the rendered model

The CAD system uses the ACIS (SAT) solid modeling format for the internal representation and is capable of exporting to stereolithography (STL) format. The system renders the SAT models, whereas the STL model is ready to be submitted to a rapid prototyping machine, for manufacturing the wax model.

A few problems were encountered during the implementation of ByzantineCAD. The main problems were the long execution time the system needed to create the final model and the robustness of the STL model.

Increasing the memory of the system improved considerably the execution time, since disk swapping was avoided.

In the case of constructing a ring, on a Windows 2000 Server platform with two 2GHz processors and 1 GB RAM, the execution time break down is approximately 20% for plate creation, 60% for the bending process and 20% for the STL file creation.

Some of the robustness problems were resolved by changing the tolerances of the system. Specifically, the system variables modified are the smallest representable number and the computer precision. The smallest number representable, that was initially  $10^{-6}$ , was changed to  $10^{-8}$ , whereas the computer precision was changed from  $10^{-11}$  to approximately  $2.22 \times 10^{-16}$ .

Initially, the structural elements were designed manually using the CAD system Rhinoceros (version 1.1), and exported in SAT format. These elements were imported in our system and used to create the pierced design plates. However, this approach proved both time consuming and error prone. Specifically, during the creation of pierced rings, the data structures used to represent the plate (which was later on bended to form the ring) required a lot of memory leading to disk swapping during the bending operation. Also the ring created from the above plate, with the bending process, had various flaws, such as naked edges, blind holes, and overlapping ring ends. Both of these problems of the models were handled by constructing the structural elements using solid modeling operations.

Also, certain heuristics were adopted to create robust models. Initially, the process for constructing a plate of pierced designs was based on the idea of creating the plate for each design separately, and then unioning all these individual plates to form the final one. When this method was implemented, the final plate model had various robustness flows, meaning that there were either gaps or seams among the individual subplates, which later caused flaws in the STL model. Therefore, the process presented in section 4.2.3 was finally adopted for the creation of a sequence of pierced designs. However, when the plate is very large (for example in large bracelets and earrings), other techniques are used for the creation of the plate. For instance, in the case of a large neutral plate, one row may be created and then replicated to create the remaining rows, which are then all unioned together.

ByzantineCAD was tested by creating STL models and by sending these models to a wax 3D printer. Wax models were manufactured and from these, metal prototypes were created (Figure 30). From the prototypes created, certain issues regarding the system parameters were re-evaluated and corrections were made so that we have a better final aesthetic result.



**Figure 30 - A metal prototype of a pierced ring**

## **8 Conclusions**

Pierced Byzantine jewellery is a unique kind of jewellery, mainly due to the special piercing technique through which it is created. This paper has introduced a novel CAD approach to designing handmade objects of complex and sophisticated craftsmanship. We have presented ByzantineCAD, a parametric system for the design of pierced Byzantine jewellery. This system provides the user with the capability of designing custom pierced jewellery in an easy-to-use and efficient manner, using a parametric feature-based design concept. The final piece of jewellery is produced by applying a sequence of operations on a number of elementary solids. An algorithm for scaling pierced patterns and designs has been introduced to enlarge pierced figures without altering the size of the structural elements used to construct them. We have also presented a number of heuristics for enhancing the robustness of the models and for increasing the efficiency of the system.

## **Acknowledgements**

We would like to thank the personnel of Geoanalysis and in particular Sophia Theodoridou, Christina Edipidi and Dimitris Avramidis for testing ByzantineCAD and for manufacturing wax and metal prototypes of the STL models. Also, we would like to thank Jack Ogden for his cooperation in designing the repertoire of structural elements, so that it conforms with traditional jewellery. For more information on the prototype system, its commercial version and copyright issues the reader may contact Geoanalysis SA (<http://www.geoanalysis.gr>).

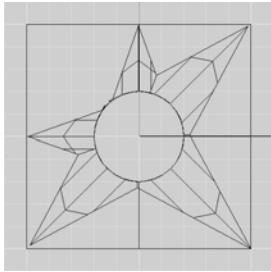
## References

1. Malagoli MG. "CAD/CAM Technology – Transforming the Goldsmith's Workshop", Gold Technology; No. 34, Spring 2002, pp.31-35
2. Molinari LC and Megazzini MC. "The role of CAD/CAM in the modern jewellery business", Gold Technology; No. 23, Spring 1998, pp. 3-7
3. Bohm W. "Design Opportunities through production technology", Gold Technology; No. 23, Spring 1998, pp. 8-11
4. Hoffmann C.M and Rossignac J, "A Road Map to Solid Modeling", IEEE Transactions on Visualization and Computer Graphics, 2(1), March 1996, pp. 3-10
5. Anderl R and Mendgen R. "Parametric Modeling and its Impact on Solid Modeling Applications", Solid Modeling 1995, Salt Lake City, Utah USA
6. Hoffman C and Joan-Arinyo R. "On User-defined features", Computer-Aided Design; 30(5), 1998, pp. 321-332
7. Chiu W.K. and Tan S.T, "Using dexels to make hollow models for rapid prototyping", Computer-Aided Design, 30(7), June 1998, pp. 539-547
8. Jense G.J, "Voxel-based methods for CAD", Computer-Aided Design, 21(8), October 1989, pp. 528-533
9. Geroulanou A. "Diatrita", Benaki Museum Publications, Athens 1999
10. Ogden J and Schmidt S. "Late Antique Jewellery: Pierced Work and Hollow Beaded Wire", Jewellery Studies 4 (1990), 5-12
11. Ogden J. "Ancient Jewellery", London 1992
12. Shah JJ and Mantyla M. "Parametric and Feature-Based CAD/CAM", John Wiley & Sons Inc., 1995
13. Salomons O, van Houten F and Kals H. "Review of research in feature-based design", Journal of Manufacturing Systems, Vol. 12, No. 2, 1993, pp. 113-132
14. Gemvision, "Matrix", <http://www.gemvision.com/html/matrix.html>
15. Folini F, Galloni M and Cugini U. "Rapid Prototyping and Parametric CAD systems", 27<sup>th</sup> International Symposium on Automotive Technology & Automation (ISATA) (Aachen, Germany 31 October – 4 November 1994), pp. 51-57
16. Jewellery CAD/CAM Ltd. "JewelCAD", <http://www.jcadcam.com>
17. Caligari Software, "JewelSpace", <http://www.jewelspace.net>
18. Robert McNeel & Assoc. "Rhino3D – NURBS modeling for Windows", <http://www.rhino3d.com>
19. Delcam, "ArtCAM Jewelsmith", <http://www.artcamjewelsmith.com>
20. TechJewel, "TechGems 3.0", <http://www.techjewel.com>
21. Takumi Pro, "JCAD 3", <http://www.jcad3.com>
22. Cadsources, "CADjewel", <http://www.cadsources.com.au/software>
23. Hoffman C and Kim K. "Towards Valid Parametric CAD models", Computer-Aided Design; 33(1), January 2001, pp. 81-90
24. Spatial, "ACIS", <http://www.spatial.com>

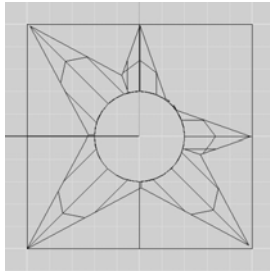


## Appendix A

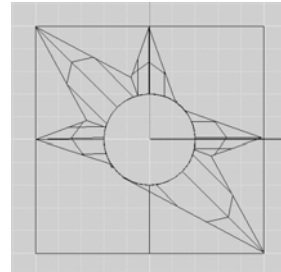
In this appendix we present the set of structural elements that are used in ByzantineCAD to create traditional pierced Byzantine jewellery. The structural elements in this set can be rotated to obtain more structural elements.



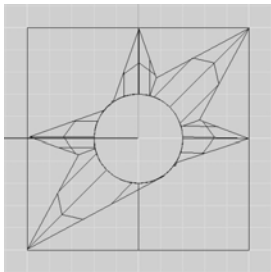
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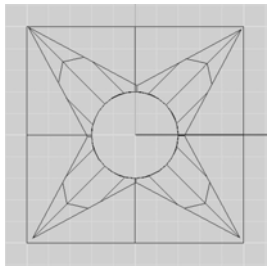
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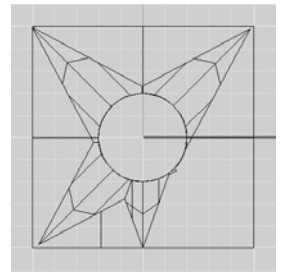
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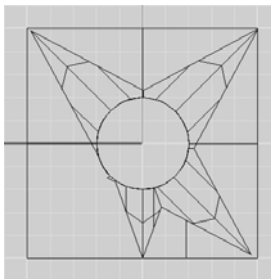
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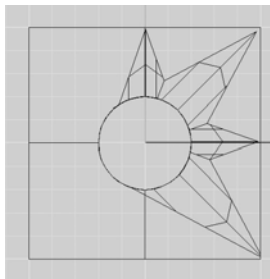
CT2468 (the neutral element)



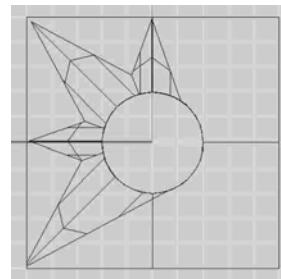
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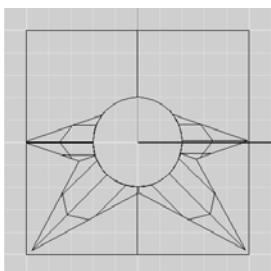
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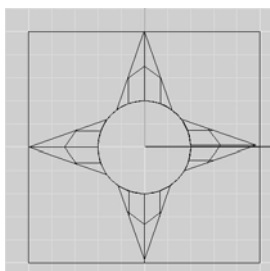
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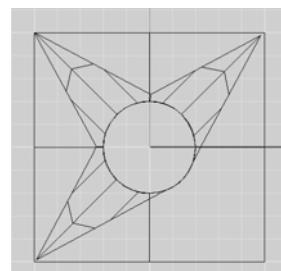
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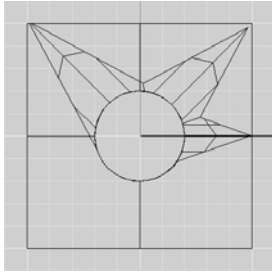
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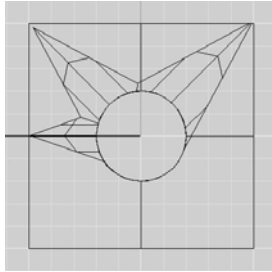
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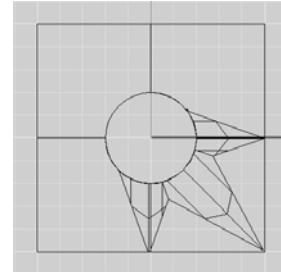
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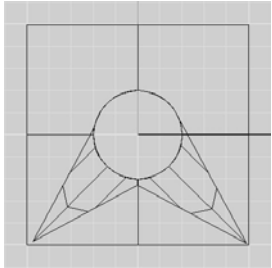
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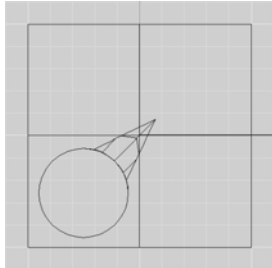
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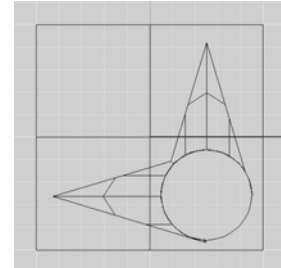
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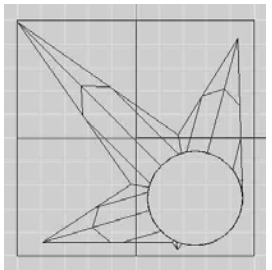
CT68



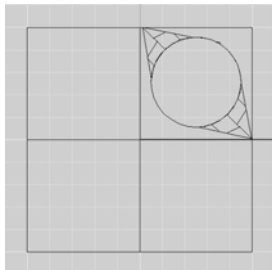
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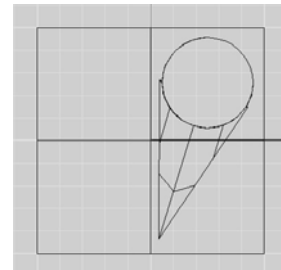
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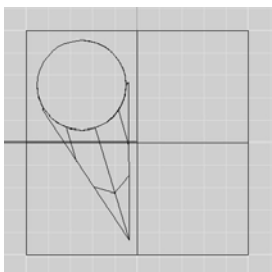
RL246



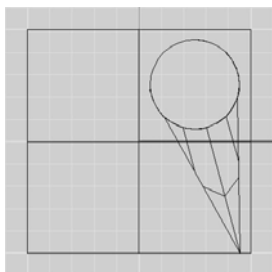
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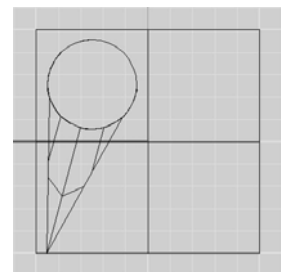
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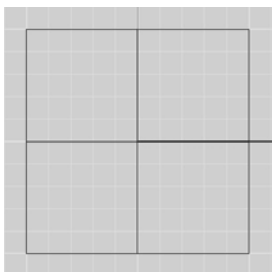
LU7



RU8



LU6



SP