MODELING HYSTERESIS CURVES OF MAGNETIC AND MAGNETOSTRICTIVE MATERIALS

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ABSTRACT

A hysteresis model, following the Preisach formalism, is used to reproduce major and minor $M(H)$ and $\lambda(H)$ curves. The output sequence is obtained by integrating the characteristic probability density function, $\rho(a,b)$, of the elementary hysteresis operators, γ_{ab}, operating on the input sequence over the Preisach plane. The model can be one- or two-dimensional depending on the dimensionality of the hysteresis operator chosen. The identification method is using data from a major hysteresis curve and a least-squares fitting procedure for the parameters of a given bi-variate probability density function, namely a normal, a lorentzian, the first-order derivative of a sigma function and a mixture of gaussians. Results using one-dimensional (1D) and two-dimensional (2D) implementations of the Preisach hysteresis formalism are compared against data for a thin film sample which when annealed at different temperatures exhibits a distinctively different major loop characteristic.

1 INTRODUCTION

Hysteresis may be a desired effect, when the stability of information or energy storage is of interest or an undesired effect when a material is used as a sensor or an actuator. In either case, the modeling of hysteresis is highly desired. Magnetic materials exhibit hysteresis in the magnetization response with respect to the applied field, $M(H)$, while in magnetostrictive materials, the dependence of strain on the applied field, $\lambda(H)$, may be hysteretic. In the case of quasistatic hysteresis treated here, the output is delayed with respect to the input and depends on the current as well as on previous inputs, but it is not affected by the input rate.

The Preisach formalism, a favorite in hysteresis modeling because of its abstract formulation and speed of the resulting calculations, postulates that hysteresis is the aggregate response of a distribution of elementary hysteresis operators. The resulting model is computationally efficient, and, for systems that fulfill certain necessary and sufficient conditions it is quite reliable [1]. The hysteresis operator of the classical Preisach model (CPM) is a relay (Figs. 1a, 1b) that switches between two states, (+1, -1) or (0,1), at two critical input values, (a,b) where $a>b$. An extensive discussion on the mathematical properties, the hysteresis operator, the identification, and the invert of the CPM can be found in Refs. [1-2]. However, the inherently scalar nature of the CPM has drawn a considerable amount of criticism on the grounds that the one-dimensional treatment of a hysteresis process is not always valid and a lot of information is lost when modeling the one-dimensional projection of a vector process. To address the issue of vector hysteresis modeling, vector extensions of the original formalism have been developed [3-6]. In the scalar case, the identification of the model can be carried out through detailed measurements of the characteristic density of the system being modeled [1]. In the vector case alternative methods need to be considered. One such approach is based on a major loop measurement and consists in determining the parameters of the probability density function chosen to model the system under consideration [4-6].

2 THE HYSTERESIS MODEL

According to the Preisach formalism, the response of the system, $f(t)$, to an input, $u(t)$, is the integral of the output states of each elementary loop weighed by the probability density function $\rho(a,b)$:

$$f(t) = \iint_{a\geq b} \rho(a,b)\gamma_{ab}u(t)dadb. \tag{1}$$

If the operator γ_{ab} is a scalar one, like the ones shown in Figs. 1a-c then the model of Eq. (1) is also scalar. The response is aligned with the input allowing only for irreversible switching. If hysteresis needs to be treated as a vector process including rotations as well, the 2D model using a 2D operator (Figs 1d-e) is preferred:

$$f(t) = \iint_{a \geq b} \rho(a,b)\gamma_{ab} u(t) da db . \quad (2)$$

The model of Eq. (2) is able to respond to vector inputs and allows for reversible as well as irreversible rotations of the output vector [3,7].

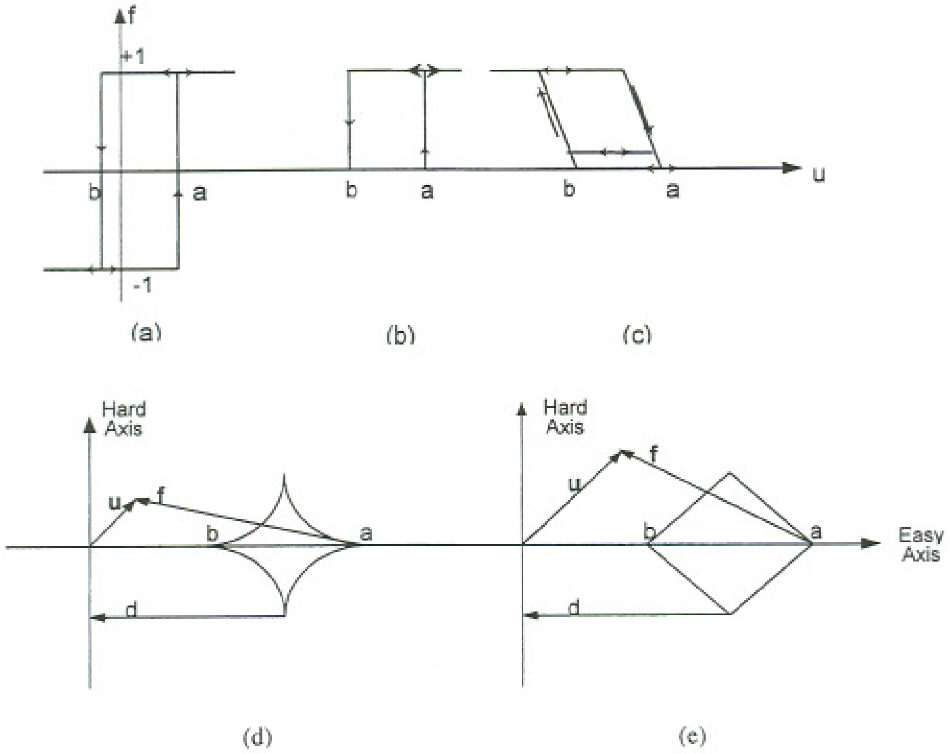

Fig. 1. Hysteresis operators: (a) the classical, 'cpm1' (b) the modified classical, 'cpm2' (c) the 'kp' (d) the sw-astroid and (e) the diamond

1D operators

The classical operator "cpm1", (Fig. 1a), is a simple relay with output ±1 and upper and lower switching points a and b, respectively. It is used to model hysteresis in ferromagnets where the magnetization, M vs. the applied field, H relationship is given by the $M(H)$ loop (Fig. 2). The output, M, varies between positive and negative saturation M_s. A modification of the classical operator (Fig. 1b), "cpm2", can be used to model hysteresis in magnetostrictive materials where the deformation, λ vs. the applied field H, relationship is described by the $\lambda(H)$ loop (Fig. 5). The output, λ, varies between 0 and a maximum value, λ_s, which can be a positive or negative number depending on the type of magnetostriction. The third type of scalar operator is the so-called "kp" operator (Fig. 1c). It allows for a linear transition between the minimum and maximum values, and bi-directional horizontal movement at any point of the ascending or descending curve. This operator is appropriate for hysteresis modeling in SMAs [7].

2D operators

The Stoner-Wohlfarth astroid (Fig. 1d), called the "sw" operator, is the locus of $u_x^{2/3} + u_y^{2/3} = 1$ where u_x and u_y are the components of the input u along the easy and the hard axes of the pseudoparticle, respectively [6]. The diamond (Fig. 1e), called the "dm" operator, is the first order approximation of the sw-astroid. Both vector operators are used for hysteresis modeling in ferromagnets. Note that both mechanisms assume uniaxial anisotropy. For inputs along the easy direction, the vector operators respond identically to the classical scalar operator of Fig. 1a. For imperfectly aligned materials, a distribution of easy axes must be included in Eq. (2) by superimposing the responses of angularly dispersed models [3].

Identification

In the classical Preisach model the characteristic density $\rho(a,b)$ can be directly measured [1]. When this is not possible, as in the case of vector hysteresis, an alternative approach is used. This consists of fitting the parameters of a known probability density function (pdf) to some points on a major hysteresis curve using a least-squares procedure. This method, relying on a major and/or virgin curve measurement does not depend on the 1D or 2D treatment of the problem or the ability to carry out detailed measurements.

RESULTS AND DISCUSSION

Three different pdfs have been used to reproduce the $M(H)$ curve (Fig. 3).

The normal:

$$\rho(a,b)=\frac{1}{2\pi\sigma_a\sigma_b\sqrt{1-r^2}}\exp\left[-\frac{1}{2(1-r^2)}\left(\left(\frac{a-\mu_a}{\sigma_a}\right)^2-2r\left(\frac{a-\mu_a}{\sigma_a}\right)\left(\frac{b-\mu_b}{\sigma_b}\right)+\left(\frac{b-\mu_b}{\sigma_b}\right)^2\right)\right]. \tag{3}$$

If the correlation coefficient $r=0$, as is the case shown in this work, $\rho(a,b)=\rho(a)\rho(b)$. For symmetric loops where, as in the case of ferromagnets, it can be shown [5] that $\mu_a=\mu_b$ and $\sigma_a=\sigma_b$ where μ_a, μ_b and σ_a, σ_b are the mean values and standard deviations of a and b, respectively.

A product of lorentzians:

$$\rho(a,b)=\frac{1}{\pi^2\sigma_a\sigma_b}\left(\frac{1}{1+\left(\frac{a-\mu_a}{\sigma_a}\right)^2}\right)\left(\frac{1}{1+\left(\frac{b-\mu_b}{\sigma_b}\right)^2}\right). \tag{4}$$

A product of the first order derivatives of the sigma function:

$$\rho(a,b)=\left(\frac{\exp\left(-\frac{a-\mu_a}{\sigma_a}\right)}{1+\exp\left(-\frac{a-\mu_a}{\sigma_a}\right)}\right)\left(\frac{\exp\left(-\frac{b-\mu_b}{\sigma_b}\right)}{1+\exp\left(-\frac{a-\mu_b}{\sigma_b}\right)}\right). \tag{5}$$

Fig. 2 shows the effect that the choice of the pdf has on the shape of the $M(H)$ loop. The major loops have been generated using the vector model of Eq. (2) and the diamond operator of Fig. 1e. The parameters are the same for all three pdfs of Eqs. (3)-(5).

It is also possible to tailor the shape of the major loop using a weighed sum of pdfs, *e.g.* a weighed sum of normal pdfs [5]. Fig. 3 shows major descending curves obtained using the scalar model of Eq. (1) and the classical operator of Fig. 1a. The pdf used is the weighed sum of two gaussians having the same mean but different standard deviations. Interesting results are also obtained for both different means and standard deviations [5]. Using a mixture of pdfs in the identification algorithm introduces more degrees of freedom making the convergence of the fitting algorithm harder.

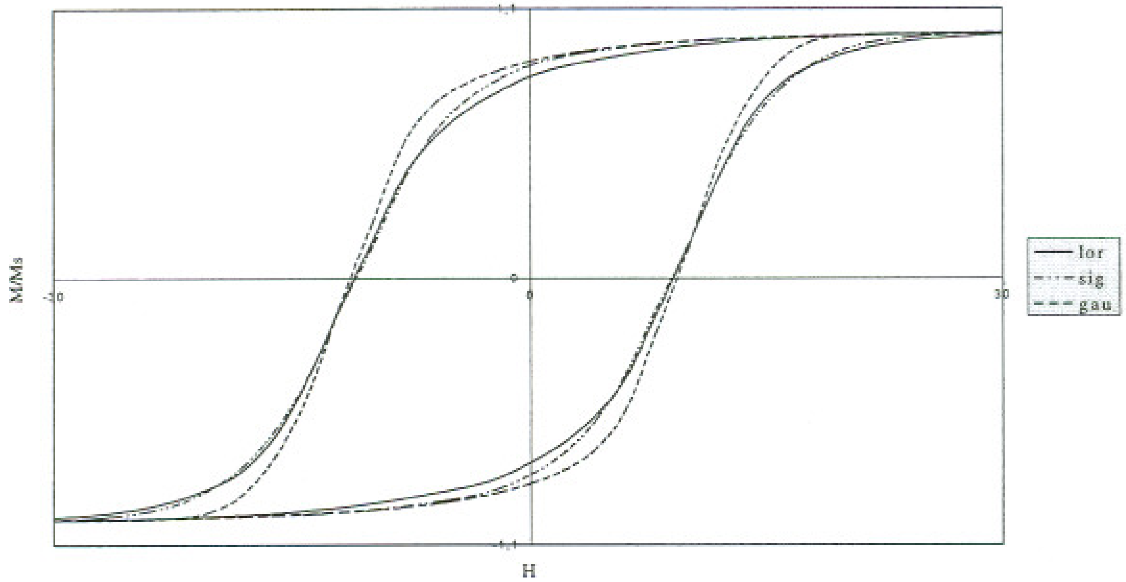

Fig. 2. Calculated $M(H)$ curves based on three different pdfs: a gaussian, a lorentzian and the first order derivative of a sigma pdf.

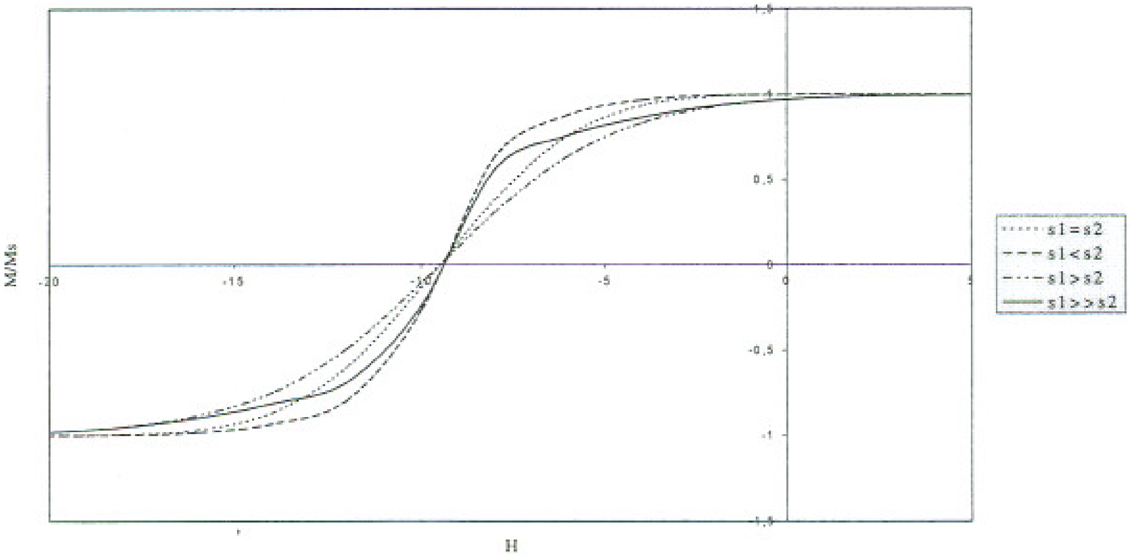

Fig. 3. Major descending curves for a weighed sum of two gaussians with different standard deviations s1, s2.

In order to model the $\lambda(H)$ curve, the dependence of the deformation of the magnetostrictive material on the applied magnetic field, the scalar model of Eq.1 in conjunction with the modified operator of Fig. 1b is used. The underlying distribution is obtained as a sum of two gaussians $\rho_1(a,b)$ and $\rho_2(a,b)$ of opposite signs and equal standard distributions. Their means are such that, $\mu_{a1} = \mu_{b2}$ and $\mu_{b1} = \mu_{a2}$ (Fig. 4). Fig. 5 shows calculated major and minor $\lambda(H)$ loops obtained using the density shown on Fig. 4.

The 2D model of Eq. (2) and the identification method described is tested using experimental data from descending curves measured on Gd-film samples annealed at 610 °C and 560 °C prior to the hysteresis measurement. Annealing sharpens the anisotropy distribution of the sample a fact that is reflected in the phenomenology of the curves [7]. At lower annealing temperatures, the anisotropy distribution width is larger and a 2D model seems more appropriate. As it can be seen in Fig. 6, the 610 °C curve is accurately reproduced by the 1D-model using the classical operator (cpm1) while the 560 °C curve is better modeled by the 2D-model and the 'sw' operator. For square loops like the 610 °C curve the classical 1D model can be quite accurate and the high S^* of the curve is reflected in the low σ-value obtained [5]. At lower annealing temperatures, the loops become less square both at the remanence and the coercivity and the 2D model is needed. In general, for low S^* loops, like the 560 °C curve, 'sw' performs better than the 'dm' [6-7].

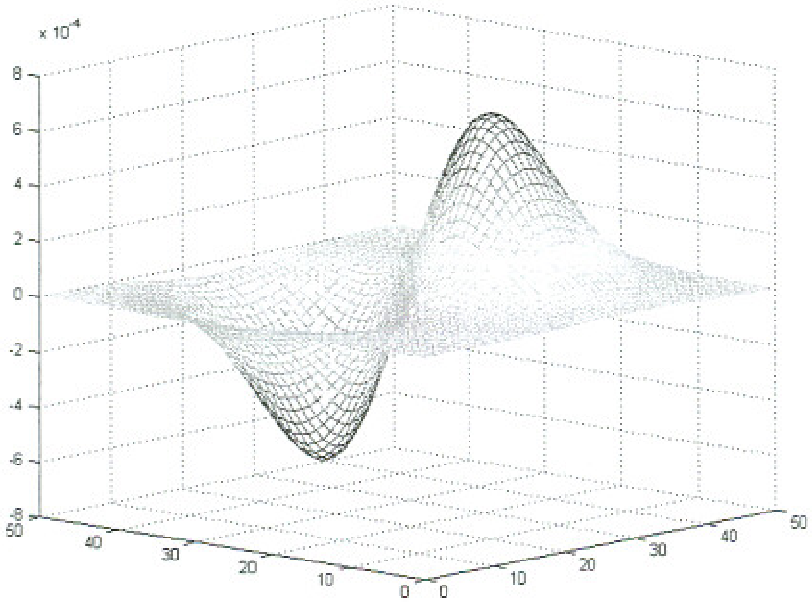

Fig. 4. The density used for the $\lambda(H)$ curve.

6 CONCLUSIONS

Hysteresis models based on the Preisach formalism have been used to model the hysteretic response of the magnetization and the deformation of magnetic and magnetostrictive materials on the applied magnetic field. The dimensionality (1D or 2D) of the model and the appropriate operator is first chosen depending on the type of material and the phenomenology of its hysteresis characteristic. Options for the underlying pdf include the normal, the gaussian, the sigma or a mixture of gaussians. The identification method consists in determining the parameters of the chosen pdf based on a least-squares fitting algorithm using data from an experimental major loop curve. The model is shown to be able to respond to any sequence of applied fields and tune in to the material been modeled regardless of the undelying hysteresis mechanism or physics.

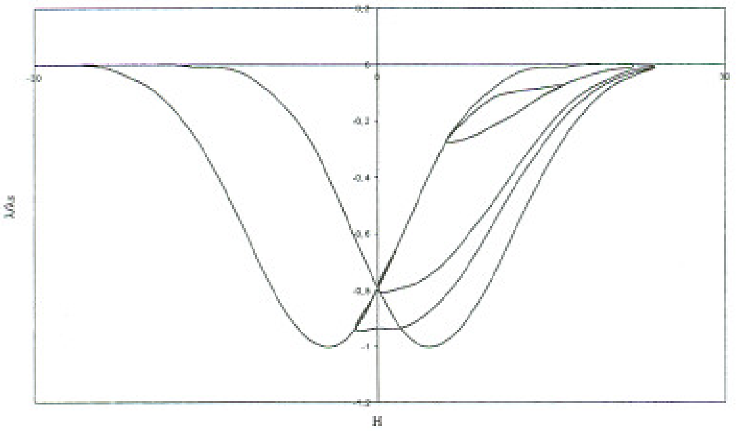

Fig.5. Calculated major and minor λ(H) curves.

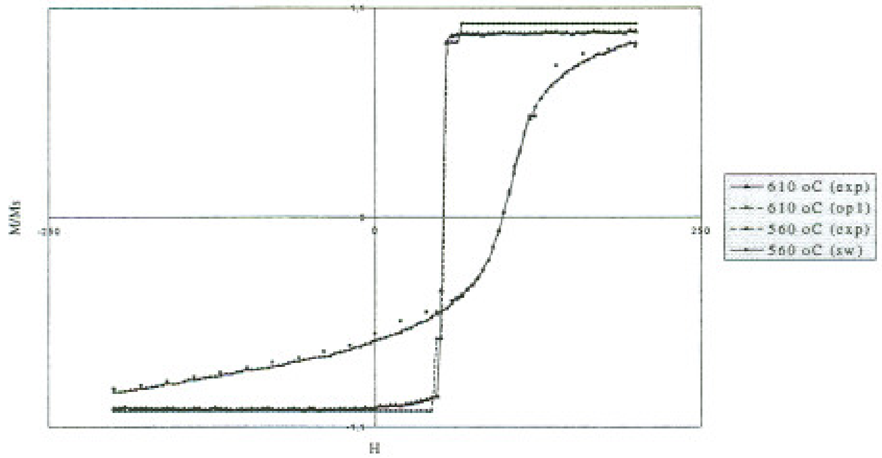

Fig. 6: Experimental and calculated ascending major loops of two thin film ferromagnetic samples.

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