

**USE OF NOVEL RULE-BASED EXPERT SYSTEM IN THE
DETECTION OF CHANGES IN THE ST SEGMENT
AND THE T WAVE IN LONG DURATION ECGs**

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Use of a novel rule-based expert system in the detection of changes in the ST segment and the T wave in long duration ECGs

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Aims The development of a new fast and robust computerised system is examined in detecting ECG changes in long duration ECG recordings. The system distinguishes these changes between ST segment deviation and T wave alterations and can support the produced diagnosis by providing explanations for the decisions made.

Methods and Results A rule-based expert system was developed for the detection of ST segment and T wave episodes, based on a four-stage algorithm. The European Society of Cardiology ST-T Database was utilised for evaluating the performance of the system. Sensitivity and positive predictive accuracy were the performance measures used and the proposed system scored 92.02% and 93.77%, respectively, in detecting ST segment episodes and 91.09% and 80.09% in detecting T wave episodes. Using the chi-square test we also compared the performance of the system between ECG recordings with minimal and substantial amount of noise. The designed system can operate in real time since it exhibits very short decision delay time.

Conclusion The sensitivity of the proposed system is higher than of other algorithms reported in the literature and the positive predictive accuracy is comparable to, or better than, most of them. The current approach is also able to clarify the type of each detected episode.

Abstract An expert system with novel architecture is proposed that can detect ST segment and T wave episodes in long duration ECG recordings. It is based on a four-stage rule-based algorithm that runs in real time and is able to produce explanations for the decisions made. The data from the ESC ST-T database were used to test the performance of the proposed system. The obtained sensitivity was 92.02% in detecting ST segment episodes and 91.09% in detecting T wave episodes while the obtained positive predictive accuracy was 93.77% and 80.09%, respectively.

Key Words: Computerised detection of ECG changes, ST segment episodes, T wave episodes, medical expert system.

Introduction

Diagnosis of ischaemic episodes based upon 24-hour ambulatory electrocardiograms (ECGs) or ECG monitoring in the coronary care units is useful in the management of coronary artery disease ^[1-3]. Several techniques that automate the detection of ischaemic episodes in long duration ECGs have been developed during the last decade. These techniques, depending upon the computational paradigm utilised, can be categorised to rule-based expert systems, fuzzy expert systems, artificial neural networks, statistical methods, systems based on wavelets theory, principal component analysis, etc ^[4-13]. The currently used techniques however, have either rather low performance in detecting ischaemic episodes (fuzzy systems, etc.) or they are unable to specify the nature of the ECG changes (artificial neural networks). Recently published literature suggests that the prognosis of unstable angina patients who on admission have ECGs with ST segment changes is worse than of those with T wave changes only ^[14]. This finding is also possibly important, in terms of prognosis, for those patients with coronary artery disease being monitored on the coronary care unit or investigated with 24 hours ambulatory ECG.

We have developed a new rule-based expert system for the detection of ECG changes suggestive of ischaemia (ST segment deviation and/or T wave changes) in long duration ECG recordings. This system is able to distinguish between episodes of ST segment deviation and T wave changes. In the present study we examined the performance of this system in long duration ECG recordings.

Methods

Description of the rule-based expert system

Expert systems are intelligent computer applications that provide decision support through acquisition and processing of human experts knowledge. A rule-based expert system is an

expert system based on sets of rules that are utilised to make decisions. To design an expert system a knowledge engineering process is needed, where the rules used by human experts are accumulated and translated into an appropriate for computer processing form. The under evaluation system is implemented on a 450 MHz processor and has four stages: a) ECG signal processing and analysis, b) beat classification, c) window characterisation and d) identification of episodes with ECG changes (ST segment deviation and T waves changes) (Figure 1). Initially the ECG recording is pre-processed in order for the noise to be removed and all ECG features (isoelectric line, QRS complex, ST segment and T wave), that will be subsequently used, to be extracted. For this reason we developed a module that handles the power line interference, inactivates the electromyographic contamination and eliminates the baseline wandering, while the original signal remains unaffected ^[15]. Furthermore for faster QRS complex detection and more accurate isoelectric line and ST segment definition we used previously described algorithms after been modified ^[16, 17]. The peak of the T wave was defined as the point with the maximum amplitude difference from the J80 point (80ms after the J point). T wave peaks of the first 30 s of each recording were averaged and used as the reference in order to detect changes in T wave amplitude.

After all the ECG characteristics (relevant to the detection of changes in the ST segment or the T wave) were recognised by the system, each beat was classified as normal, abnormal or artefact. The beat classification was based upon: a) negative ST deviation (≥ 0.8 mm below the isoelectric line and with a slope $\geq 65^\circ$ from the vertical line), b) positive ST deviation (≥ 0.8 mm above the isoelectric line), c) T wave inversion, d) T wave flattening ($\geq 50\%$ amplitude decrease) and e) amplitude increase of a negative T wave ($\geq 50\%$ and ≥ 0.2 mV). The ST deviation was measured at the J80 (heart rate < 120 beats min^{-1}) or the J60 point (heart rate ≥ 120 beats min^{-1}). The ST slope was defined as the slope of the line connecting the J and J80

(or J60) points. The peak of the T wave was used for the T wave measurements. The rules described above are shown graphically in Figure 2.

During the window characterisation stage all the cardiac beats were examined in groups (windows). The first window was the sequence of the first cardiac beats of approximately 30 s in duration. The next window was created by subtracting the first beat and adding in the end the number of beats needed in order another interval of 30 s to be created. This was repeated to the end of the recording. The beats of each window were examined and the window was classified as normal or abnormal. A window was characterised as abnormal when it contained more than 75% of abnormal beats with the same ECG pattern changes (ST segment deviation or T wave changes).

In the fourth stage, consecutive windows classified as abnormal due to similar ECG changes (ST segment deviation or T wave changes) were concatenated in order to define the starting and ending points of an ST (ST deviation) or T (T wave changes) episode. In cases where normal intervals of duration <30 s appeared between two abnormal windows, the process of merging was continued not allowing thus the fragmentation of the ST or T episodes. After the merging procedure was applied in each of the recorded leads separately, the findings of each lead were combined to identify the duration of each ST or T episode.

Study population

The European Society of Cardiology (ESC) ST-T database was utilised. This was developed especially for the evaluation of algorithms designed to analyse ST and T wave changes^[18]. It consists of 90 continuous two-channel recordings, two hours each, taken from 79 different ambulatory ECGs. The ECGs were provided by 13 research groups of eight different countries. Each record of the database was submitted to two participating groups of experts for beat-by-beat annotation and then the annotated records returned to the co-ordinating group

for the final annotation. The database records refer to each lead separately outlining the starting and ending points of the episodes and defining the type of ECG changes for every lead (e.g. ST segment deviation of $\geq 0.1\text{mV}$ and T wave amplitude change of $\geq 0.2\text{mV}$). We applied a merging procedure for the margins of the episodes to refer to the findings of both leads in order to obtain an overall annotation (lead independent) of the ischaemic episodes^[15]. Also, for the purposes of the present study the T episodes in which the polarity of the positive T waves was increased and were annotated in the database as T episodes were excluded as they refer to myocardial infarction rather than myocardial ischaemia^[19,20].

The original analogue signals were recorded using a variety of two-channel ambulatory ECG recorders: ICR 7200 (37 records), Del Mar Avionics 445B (14 records), Oxford Medilog 4-24 (12 records), Oxford Medilog MR-14 (2 records), Oxford Medilog MR-20 (14 records), Oxford Medilog MR-35 (2 records), Oxford Medilog MR-40 (2 records), Ela Medical 2448 (3 records), Reynolds Tracker (3 records) and Applied Cardiac Systems (1 record).

Using the noise information provided by the ESC ST-T database, we separated the 90 ECG recordings in two groups: group A (ECGs with minimal amount of noise: number of noisy beats $< 10\%$ of the total number of beats) and group B (ECGs with substantial amount of noise: number of noisy beats $\geq 10\%$ of the total). Group A consists of 64 ECGs and Group B of 26 records (e0107, e0115, e0118, e0119, e0121, e0133, e0139, e0148, e0155, e0159, e0170, e0205, e0213, e0406, e0415, e0515, e0601, e0607, e0611, e0612, e0613, e0614, e0801, e0808, e0817 and e0818).

Performance assessment of the rule-based expert system

The performance of the new rule-based expert system in detecting episodes of ST segment deviation and T wave changes was assessed in the total number of ECG recordings of the ESC ST-T database and in the groups A and B. To assess the performance of the under

evaluation system we used the sensitivity and the positive predictive accuracy. Sensitivity was defined as the ratio of the number of episodes correctly detected as abnormal to the total number of the abnormal episodes, while positive predictive accuracy as the ratio of the number of correctly detected abnormal episodes to the total number of the detected episodes. The time needed for the processing of an ECG recording was reported in each case.

Statistical Analysis

The new system was validated, regarding its sensitivity and positive predictive accuracy, in detecting episodes of ST segment deviation and T wave changes in the total number of ECG recordings and in the ECGs with minimal and substantial amount of noise. In every case aggregate gross statistics was used. To compare the performance of the under evaluation system in ECGs with minimal and substantial noise the chi-square test was used. The processing time for ECG recordings was expressed as mean value \pm one standard deviation.

Results

The sensitivity and positive predictive accuracy of the new system in the three groups of ECG recordings (total number of ECG recordings, groups A and B) are shown in Table 1. In the Appendix the performance of the system in each recording is shown. Figure 3 depicts a graphical evaluation of the system's performance when the presence of noise is taken into account.

Total number of ECG recordings

The ESC database consists of 90 ECG recordings with 589 ST and 393 T episodes. Our system detected 578 ST and 447 T episodes and correctly identified 542 and 358, respectively. The sensitivity and positive predictive accuracy of the system for the ST

episodes were 92.02% and 93.77%, respectively, and for the T wave episodes 91.09% and 80.09%. The time needed for the processing of each ECG recording was 439s±93s.

ECG recordings with minimal noise (Group A)

The ESC database contains 64 minimal noise ECG recordings with 409 ST and 266 T episodes. Our system detected 393 ST and 303 T episodes and correctly identified 377 and 242, respectively. The sensitivity and positive predictive accuracy of our system for the ST episodes was 92.18% and 95.93%, respectively, and for the T episodes 90.98% and 79.87%. The time needed for the processing of each ECG recording was 433s±94s.

ECG recordings with substantial noise (Group B)

The 26 ECG recordings in the ESC database with substantial noise have 180 ST and 127 T episodes. Our system detected 185 ST and 144 T episodes. The correctly identified ST and T episodes were 165 and 116, respectively. The sensitivity and positive predictive accuracy of our system for the ST episodes was 91.67% and 89.19%, respectively, and for the T episodes 91.34% and 80.56%. The time needed for the processing of each ECG recording was 455s±91s.

The sensitivity of the system in detecting ECG episodes of ST deviation was not statistically different in ECGs with and without noise, although its positive predictive accuracy was superior in ECGs without noise ($p=0.003$). The new system was able to detect ECG episodes of T wave changes equally well in ECGs with and without noise (sensitivity and positive predictive accuracy non significantly different).

Discussion

Various algorithms based on sets of rules, fuzzy logic, artificial neural networks, statistics, wavelet theory, principal component analysis and other signal processing techniques have been proposed for the automated detection of ischaemia in long duration ECGs [4-13]. Of these

algorithms neural networks perform better than the other systems. Their main drawback is that they cannot discriminate ST segment from T wave changes as the reason for the diagnosis. Systems based on sets of rules or fuzzy logic, on the other hand, provide information regarding the type of the detected ECG changes but do not have the high diagnostic performance of the artificial neural networks.

The techniques mentioned above have been tested either on the data of the ESC ST-T database ^[4-8] or on various independent datasets ^[9-13]. In cases where the ESC ST-T database was used the sensitivity and positive predictive accuracy ranged from 71% to 85.2% and 66% to 90%, respectively. These techniques used only partially the ESC ST-T database when tested. In particular they either used certain ECG recordings ^[4-6], or been tested on detecting ischaemic beats with ST segment changes ^[5] or ST segment episodes ^[6-8]. The techniques tested on independent datasets had sensitivity and specificity, ranging from 92% to 97.2% and 69% to 96.2%, respectively ^[9-12]. Most of them tested on their ability to diagnose “ischaemic patients” rather than detect ischaemic episodes and subsequently their performance was judged by their specificity instead of positive predictive accuracy. ^[9-11].

The currently under evaluation rule-based expert system was tested on all the ECG recordings of the ESC ST-T database. Its sensitivity and positive predictive accuracy in detecting ST segment episodes was 92.02% and 93.77%, respectively, while in detecting T wave episodes 91.09% and 80.09%. This sensitivity is higher than of the previously described algorithms (except those tested on individual datasets) while the positive predictive accuracy is comparable to, or better than, most of them. The current approach is able to clarify the type of each detected episode (different types of ST segment vs. T wave changes) with high rates of sensitivity and positive predictive accuracy. Furthermore, the performance of the system is not affected by the presence of noise due to its novel noise-handling module, while it exhibits

very short decision delay time. The later enables real-time operation providing on line decision support to the medical personnel.

The innovations of the studied system are the use of certain rules for beat classification, the noise handling module and the utilisation of a sliding adaptive window. The set of rules for beat classification differentiates T wave inversion from flattening or increase in the negative amplitude. This was incorporated for the first time in a system for automated diagnosis and possibly increased the positive predictive accuracy of the system. The noise-handling module has the advantage of successfully removing from the recorded ECG signal the three basic types of noise (power line interference, electromyographic contamination and baseline wandering) without altering any of the ECG characteristics. This is important as even slight modifications, especially to the ST segment and the isoelectric line, can lead to inaccurate diagnosis. Also, the sliding adaptive window, applied in the third stage of the algorithm, offers flexibility in the decision making process. Intervals of ECG with ST segment deviation or T wave changes containing normal beats were characterised as abnormal whereas stricter algorithms would either miss the episode or produce more episodes of shorter duration.

The performance of the proposed system could be further improved, especially in terms of positive predictive accuracy, through further refinement of the noise-handling procedure. Recordings with very low signal-to-noise ratio often resulted in problematic detection of the J point, the isoelectric line and the T peak. Incorrect T peak detection at the beginning of the ECG recordings led to false definition of the sign and amplitude of the T wave. This explains the better performance of the system in detecting ST segment than T wave episodes. Modern ECG recorders and Holter devices include filtering modules and the output ECG signals have higher signal-to-noise ratio than the recordings of the database. Our method would be expected to perform better if the ECG recordings were obtained using contemporary equipment.

In conclusion this new rule-based expert system rapidly and reliably detects ECG changes suggestive of ischaemia in long duration ECGs. Its performance is not affected by the presence of noise and clarifies the type of each detected episode. Whether the advantages of this new system will increase the diagnostic and prognostic accuracy requires further evaluation.

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Appendix

System's performance for all ECG recordings of the ESC ST-T database

Record	ST episodes		T episodes		Record	ST episodes		T episodes	
	Se	PPA	Se	PPA		Se	PPA	Se	PPA
103	6/6	6/6	4/4	4/4	211	2/2	2/2	0/0	0/1
104	12/12	12/12	14/14	14/15	212	0/1	0/0	0/0	0/0
105	6/6	6/6	2/2	2/7	213	2/4	2/2	0/0	0/0
106	11/12	11/11	1/3	1/1	302	8/8	8/8	3/3	3/6
107	4/4	4/7	10/10	10/10	303	1/1	1/1	0/0	0/1
108	22/22	22/22	7/8	7/7	304	5/5	5/5	7/7	7/8
110	3/3	3/3	0/0	0/0	305	1/1	1/1	1/2	1/1
111	11/11	11/11	5/5	5/10	306	2/3	2/2	0/5	0/0
112	6/6	6/6	6/7	6/8	403	22/22	22/22	8/8	8/10
113	4/8	4/4	10/11	10/11	404	9/9	9/9	8/8	8/8
114	13/14	13/13	5/5	5/5	405	8/8	8/8	9/9	9/10
115	5/6	5/5	4/5	4/6	406	1/2	1/1	0/0	0/4
116	½	1/3	14/14	14/17	408	2/2	2/2	1/1	1/1
118	8/8	8/10	10/11	10/13	409	2/2	2/2	3/3	3/3
119	9/9	9/9	11/11	11/11	410	1/2	1/1	0/0	0/1
121	3/3	3/6	5/6	5/6	411	4/4	4/4	2/2	2/5
122	1/1	1/5	6/6	6/7	413	4/4	4/9	1/1	1/3
123	3/3	3/3	0/0	0/0	415	7/8	7/7	1/4	1/1
124	4/7	4/4	2/2	2/4	417	6/6	6/6	0/0	0/7
125	1/3	1/1	6/8	6/6	418	11/13	11/11	0/0	0/1
126	1/1	1/1	8/9	8/8	501	4/4	4/4	0/0	0/1
127	1/2	1/1	7/7	7/8	509	1/1	1/1	3/3	3/3
129	10/10	10/10	3/3	3/5	515	3/4	3/4	8/10	8/8
133	0/0	0/0	1/1	1/1	601	0/2	0/2	4/4	4/4
136	7/7	7/7	7/7	7/7	602	11/11	11/11	7/7	7/7
139	3/3	3/3	13/13	13/16	603	9/9	9/9	1/1	1/1
147	7/7	7/7	10/10	10/11	604	1/10	1/1	5/5	5/5
148	11/11	11/11	1/1	1/2	605	2/2	2/2	2/2	2/2
151	15/16	15/15	0/0	0/0	606	5/5	5/5	2/3	2/2
154	1/1	1/1	2/2	2/2	607	14/14	14/14	4/4	4/4
155	9/9	9/9	11/11	11/11	609	3/3	3/3	1/1	1/1
159	2/2	2/2	0/0	0/0	610	4/4	4/4	3/3	3/3
161	15/15	15/15	0/0	0/1	611	0/0	0/0	4/4	4/4
162	8/8	8/8	5/5	5/5	612	4/4	4/7	0/1	0/0
163	4/5	4/4	8/8	8/8	613	9/13	9/10	0/0	0/2
166	16/16	16/21	1/1	1/5	614	19/19	19/19	2/2	2/2
170	3/3	3/7	6/6	6/10	615	7/8	7/7	0/0	0/3
202	8/8	8/8	8/9	8/9	704	8/8	8/8	7/8	7/7
203	2/2	2/2	0/0	0/0	801	0/2	0/0	0/2	0/0
204	0/2	0/0	2/2	2/2	808	14/14	14/14	15/15	15/15
205	5/5	5/6	2/2	2/3	817	17/17	17/17	2/2	2/2
206	5/5	5/5	1/5	1/2	818	13/14	13/13	2/2	2/9
207	4/4	4/4	8/9	8/8	1301	8/8	8/8	4/4	4/5
208	9/9	9/9	0/0	0/0	1302	4/4	4/4	8/9	8/10
210	4/4	4/4	1/1	1/1	1304	1/1	1/1	3/4	3/3

Se: Sensitivity, PPA: positive predictive accuracy

Table 1 Performance of the new rule-based expert system in the three studied groups.

ECG change		Sensitivity		PPA	
		episodes	%	episodes	%
ST segment	All ECGs	542/589	92.02	542/578	93.77
	Group A	377/409	92.18	377/393	95.93
	Group B	165/180	91.67	165/185	89.19
T wave	All ECGs	358/393	91.09	358/447	80.09
	Group A	242/266	90.98	242/303	79.87
	Group B	116/127	91.34	116/144	80.56

PPA: positive predictive accuracy

Figure 1 The four-stage algorithm of the proposed rule-based expert system.

Figure 2 Five cases of ECG signal alterations suggesting ischaemia that correspond to each one of the five rules used by the proposed rule-based expert system.

Figure 3 Graphical representation of the performance of the system (sensitivity and positive predictive accuracy: PPA) in detecting ST segment and T wave episodes in long duration ECGs with and without substantial noise.

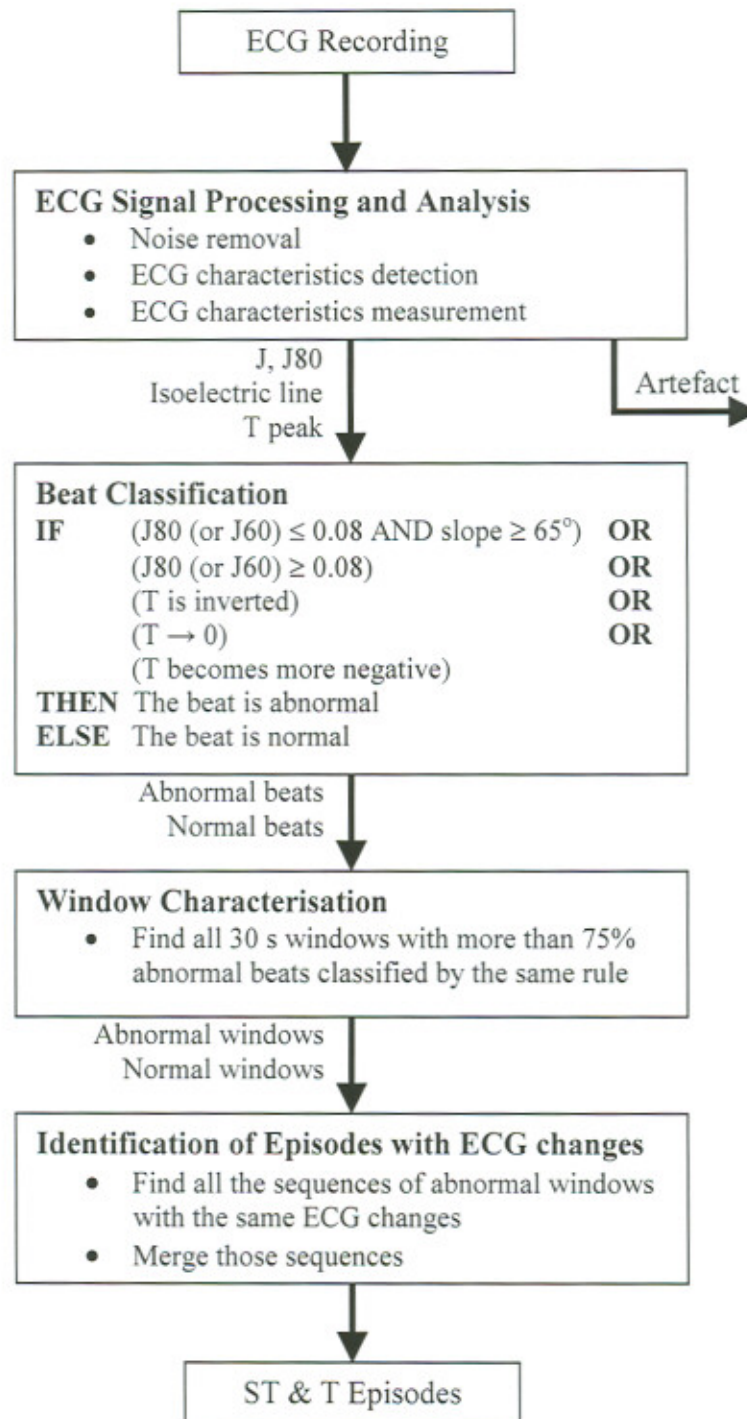


Figure 1

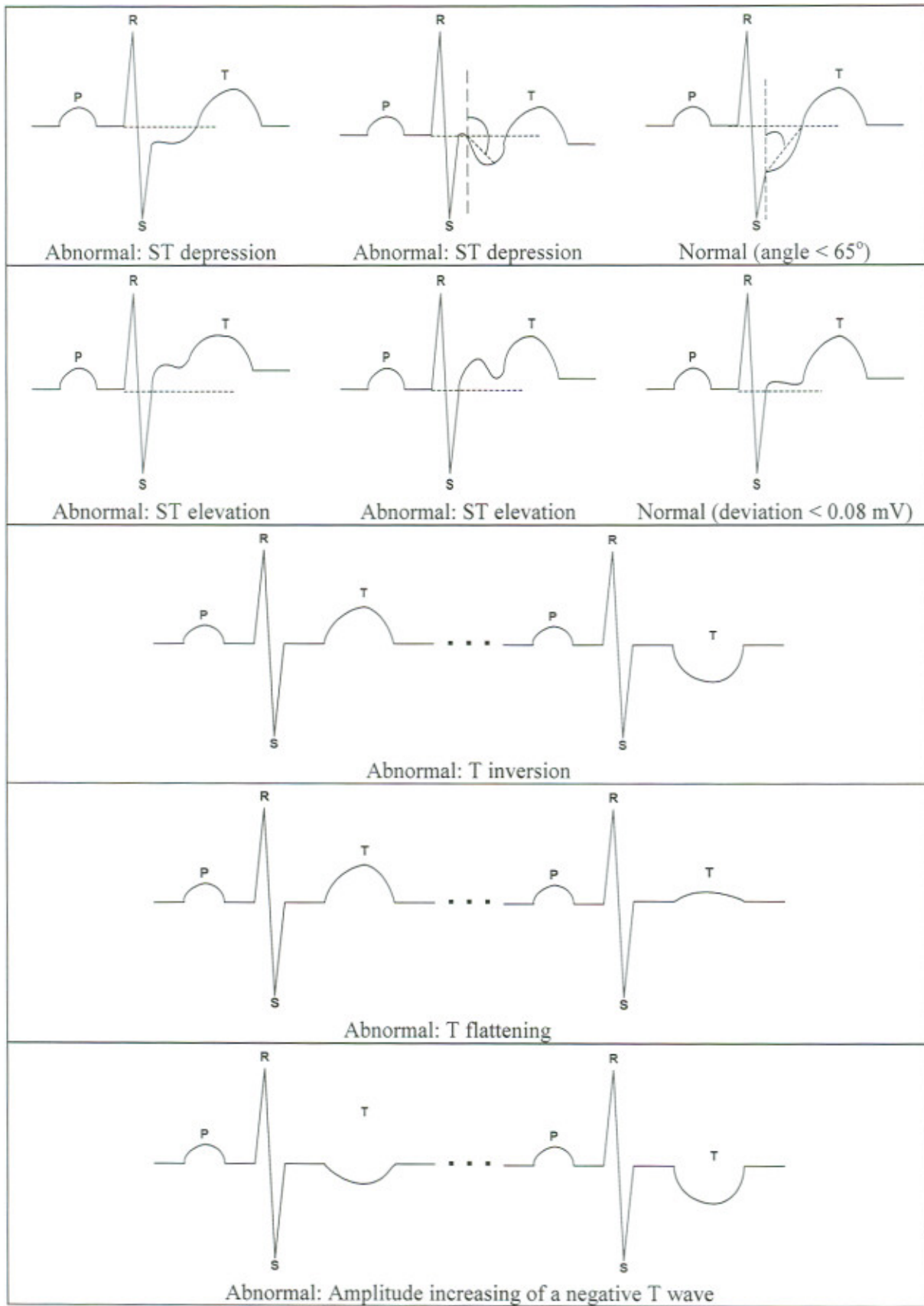


Figure 2

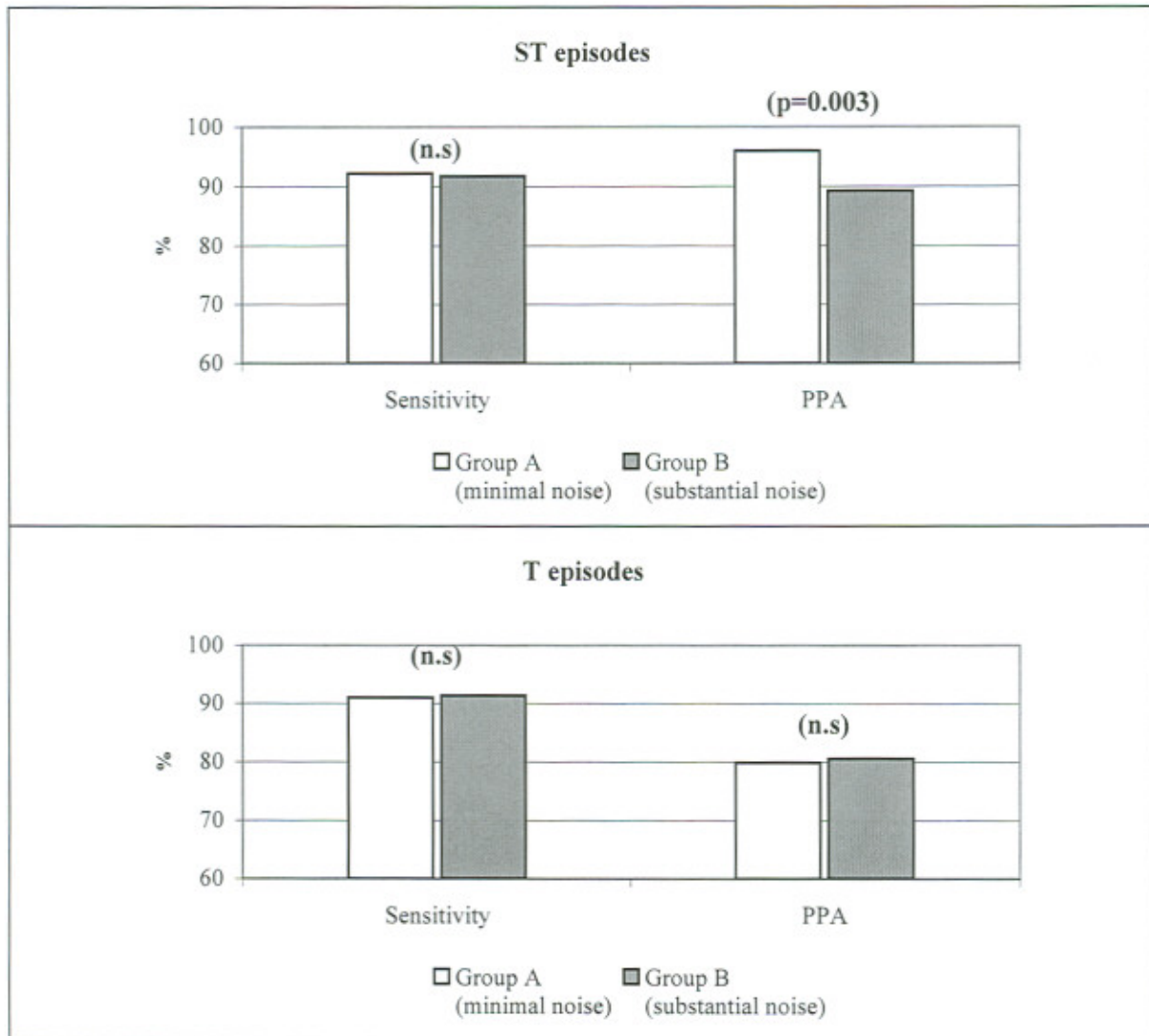


Figure 3