USE OF PREISACH MODELS TO STUDY HYSTERESIS

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Use of Preisach Models to Study Hysteresis

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Abstract

A vector Preisach-type model is presented to study hysteresis phenomena in various physical systems. The advantages of abstraction, simplicity and fast algorithms of the classical scalar model are preserved while the incorporation of vector properties waives the limitations of the classical approach. The model accounts for reversible and irreversible processes predicting minor and noncongruent loops in accordance with experimental evidence. The vector properties of the model are also in qualitative agreement with experimental observations. The classical Preisach model is also reviewed.

Key words: Hysteresis; Preisach Models.

AMS subject classifications: 93A30, 47H99, 81T80.

1 Introduction

Hysteresis is a non-linear phenomenon encountered in systems characterized by the long (persistent) memory effect. problems in magnetism, in biology, in plasticity, in economics even. The present output $f(t)$ always lags the present $input u(t)$ hysteresis is a Greek word meaning delay and is a non-linear function of the present as well as past values of $u(t)$. A system exhibiting hysteresis is often related to a major loop curve like the one shown in Fig. 1. The major loop curve is the outer loop enclosing all other trajectories. For example, the set of trajectories shown in Fig. 1 corresponds to the path leading from the state of positive saturation, $[\max\{u(t)\} = +u_s, \max\{f(t)\} = +f_s]$, to the state [0,0]. For each input there are several possible outputs. The choice of the output each time depends on the input history of the system.

The nonlinear lag of the output with respect to the input is in accordance with the second law of thermodynamics and suggests that there is only one way to trace a major loop curve: Starting at the $+f_s$-state and with a monotonically decreasing input $u(t)$ the $-f_s$-state is reached tracing the left-hand side branch of the curve, called the major descending curve. The ascending branch on the right is the trajectory obtained when the input increases from u_s to $+u_s$.

The shape and size of the major and minor curves depends on the physics of the system which in most cases are quite complicated taking into account the internal microstructure and interactions developed.

Hysteresis is a property of several systems in nature such as ferromagnetism (u=applied field, f=magnetization), plasticity (u=stress, f=strain), absorption (u=pressure, f=absorbed volume) etc. There are hysteresis models specifically designed for each class of systems which are beyond the scope of this article which is focused on the Preisach-type modeling of hysteresis. Preisach models have the advantage that they may be applied to different systems regardless of the specifics of the underlying physics.

F. Preisach developed, in the thirties, a scalar model of hysteresis in ferromagnets [1]. treating the magnetization response of a magnet to an applied field as the superposition of responses of local hysteresis operators appropriately distributed. About twenty years later, the mathematical abstraction of the model was noticed and its properties have been investigated [2] allowing the model to be used in control systems theory. The original Preisach formalism,

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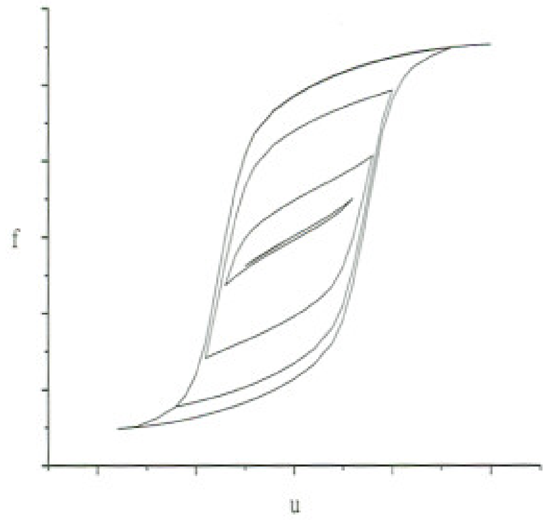

Figure 1: The hysteresis loops.

reviewed in section 2, has been extensively studied [3] - [4]. The formalism is abstract, elegant and simple leading to fast algorithms and reliable results in spite of the complexity of the phenomenon it models. However, it has limitations some of which are waived by the introduction of vector properties to the model. Even though vector Preisach-type models are relatively recent [5] - [7],they have already been successfully applied to ferromagnets. A two-dimensional approach is presented in section 3 and results from application to ferromagnets are discussed in section 4.

2 The Original Model

In the Preisach approach, hysteresis is the result of superposition of scalar local memory operators. Lets assume a bistable operator,$\gamma_{\alpha,\beta}$, like the one shown in Fig. 2 b:

(1) $$\gamma_{\alpha,\beta} = \begin{cases} +1, & u(t) > \alpha \\ -1, & u(t) < \beta \end{cases} .$$

When the input becomes greater than α the operator $\gamma_{\alpha,\beta}$ assumes the value $+1$ which it retains until the input, $u(t)$, becomes smaller than β in which case it switches to 1. The function $\gamma_{\alpha,\beta}$ is discontinuous at the switching points α and β. The system being modeled can then be viewed as a collection of subcomponents each of which has a hysteresis characteristic $\gamma_{\alpha,\beta}$ with different switching points α and β. The displacement of the elementary loop from the origin corresponds to the effective interactions, u_{int}, experienced by the given component: $u_{\text{int}} = \frac{\alpha+\beta}{2}$.

If the subcomponents are isolated or the sum of interactions one of them experiences is zero the corresponding loop is centered at the origin and $\alpha = -\beta$. The interactions are added to the applied input yielding an effective input value: $u_{\text{eff}}(t) = u(t) + u_{\text{int}}(t)$ Another quantity of interest is the loop halfwidth, $u_{\text{hw}} = \frac{\alpha-\beta}{2}$. When the degenerate loop of zero halfwidth, $u_{\text{hw}} = 0$, is obtained.

The system is then modeled by a distribution of α and β, as a collection of elementary loops of various switching points, or equivalently, of various interactions and halfwidths. The distribution is obtained from the characteristic density of the system, $\rho(\alpha, \beta) = \rho(u_{\text{hw}}, u_{\text{int}})$, which is defined over the Preisach plane (Fig. 2b) [7].

The plane is bounded by $u_{\text{hw}} = 0$ (otherwise the lower switching point β would be greater than the upper switching field which violates the second law of thermodynamics), $u = +u_s$ and $u = -u_s$ where $+u_s$ and $-u_s$ are the input values leading to positive and negative saturation respectively: $\forall \alpha, \beta \quad \alpha \leq +u_s, \beta \geq -u_s$.

The response of the system, $f(t)$, to an input, $u(t)$, is the sum of contributions of each elementary loop weighed

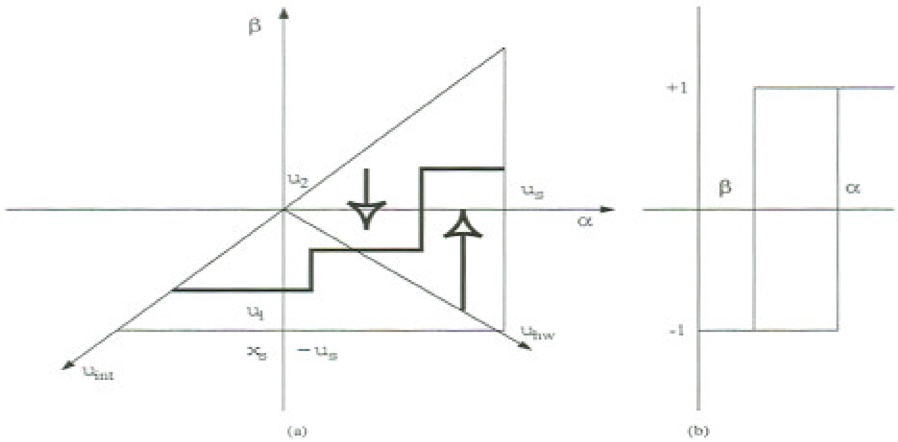

Figure 2: (a) The Preisach plane with the staircase boundary, (b) The scalar operator.

by the probability density function $\rho(\alpha,\beta)$:

$$f(t) = \iint_{\alpha\geq\beta} \rho(\alpha,\beta)\gamma_{\alpha,\beta}u(t)d\alpha d\beta. \tag{2}$$

The identification of such a model consists in the determination of the characteristic density $\rho(\alpha,\beta)$. It has been shown [6, 7] that the density in the scalar case can be measured. This method has also been used in non-linear control applications where one needs to invert the model and determine the input value $u(t)$ is needed to obtain an output $f(t)$ [8]. The density can also be constructed as a product of two independent probability density functions when appropriate: $\rho(\alpha,\beta) = \rho(u_{\rm hw}, u_{\rm int}) = \rho(u_{\rm hw})\rho(u_{\rm int})$.

Letapply an input sequence u_0, u_1, u_2, with $t_0 < t_1 < t_2$: If $u(t_0) = u_0 < -u_s$, $f(t_0) = -f_s$, and $\forall \alpha, \beta$, $\gamma_{\alpha,\beta} = -1$, i.e. the system is in the negative saturation state:

$$f(t_0) = \int_{-u_s}^{+u_s}\int_{-u_s}^{\beta} \rho(\alpha,\beta)(-1)u(t_0)d\alpha d\beta = -f_s. \tag{3}$$

For $u(t_1) = u_1 > -u_s > u_0$,

$$\text{if} - u_s < \alpha < u_1, \gamma_{\alpha\beta} = +1 \tag{4}$$

$$\text{else}\gamma_{\alpha\beta} = -1 \tag{5}$$

i.e., for $\cdot u > 0$ a horizontal boundary separating the regions of +1 - and 1 - states is established at $\alpha = u_1$.

For $u(t_2) = u_2 < u_1$,

$$\text{if}\beta > u_2, \gamma_{\alpha\beta} = -1 \tag{6}$$

$$\text{else}\gamma_{\alpha\beta} = +1 \tag{7}$$

i.e., for $\cdot u < 0$ a perpendicular boundary segment at $\beta = u_2$ appears.

Continuing in this manner, at the end of an input sequence a staircase boundary $B(u(t))$, serving as the memory of the system, is established between areas of positive and negative state (Fig. 2a).

Notice that if $u(t) > u_s$ or $u(t) < -u_s$ the boundary is "wiped-out". This is a result of the *wipe-out property* of the model according to which the model retains only the local extrema of the input:

Let the boundary $B(u(t))$ be the result of an input sequence $u(t) = \{u_1, u_2, \ldots, u_i, \ldots, u_n\}$. Then $B(u(t)) = 0$ for $u_{n+1} > \max\{|u(t)|\}$. In general, the wipe-out, or return-point memory, property is also a property of the physical systems exhibiting hysteresis.

Another property of the Preisach model is congruency: For an input sequence $u(t) = \{u_{i-1}, u_i, u_{i+1}\}$ with $u_{i-1} = u_{i+1}$, the change in output is always $\Delta f(t) = 2\int_{u_{i-1}}^{u_i}\int_{u_{i-1}}^{\beta}\rho(\alpha,\beta)\gamma_{\alpha\beta}d\alpha d\beta$ regardless of the previous input sequence. This property has as a result congruent minor loops in an input/output plot. In general, congruent minor loops are

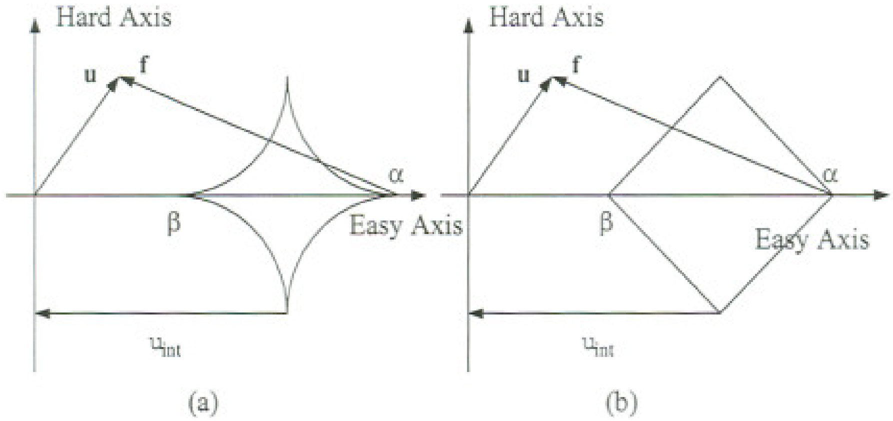

Figure 3: Vector operators: (a) The SW astroid, and (b) The diamond.

not a property of physical systems with hysteresis. It has been shown that wipe-out and congruency are two properties of the original Preisach model which are necessary and sufficient in order for a system to be modeled by it [4].

Note that the treatment of hysteresis in the original model is quasistatic: the rate of change of the inputs is low enough to allow for the transients to die out. The original model cannot treat dynamic phenomena but modifications have been proposed in order to accommodate them [9].

3 The Two - Dimensional Model

Another characteristic of the traditional model is that because of the elementary loop structure, it allows only for irreversible processes to take places and the reversible part has to be added on. It has been found that substituting the scalar operator $\gamma_{\alpha\beta}$ by a vector one $\Gamma_{\alpha\beta}$ not only adds vector properties to a traditionally scalar model but accommodates reversible processes as well [10].

The modified 2D Preisach model for a perfectly oriented anisotropic system is then given by:

(8) $$\mathbf{f}(t) = \iint_{\alpha\geq\beta} \rho(\alpha,\beta)\Gamma_{\alpha\beta}\mathbf{u}(t)d\alpha d\beta.$$

The operator $\Gamma_{\alpha\beta}$ acts on the vector input $\mathbf{u}(t)$, the result is weighed by the density function $\rho(\alpha,\beta)$ and integrated over the Preisach plane yielding the vector output $\mathbf{f}(t)$.

For imperfectly oriented systems, angular dispersion around the easy axis is included by superimposing the effect of several such models normally distributed around the preferred axis of orientation which to the limit reads as:

(9) $$\mathbf{f}(t) = \int_{\frac{-\pi}{2}} \frac{\pi}{2} \iint_{\alpha\geq\beta} \rho(\phi)\rho(\alpha,\beta)\Gamma_{\alpha\beta}\mathbf{u}(t)d\alpha d\beta d\phi,$$

where ϕ is the angle each models forms with the preferred orientation axis and $\rho(\phi)$ is the corresponding probability density.

One difficulty with this formulation is the lack of operators for the various classes of systems $\gamma_{\alpha\beta}$ or of a generalized vector operator. In magnetics, such an operator is the well known Stoner-Wohlfarth (SW) astroid (Fig. 3a) [10]. It is not an abstract mathematical structure like its scalar counterpart but rather a physical model resulting from the minimization of an energy equation.

The use of the SW astroid in the 2D Preisach model has yielded satisfactory results in several applications in magnetics [11, 12]. The 2D formulation maintains the simplicity of the original formalism and the resulting algorithms. Alternatively, the diamond-shaped operator (Fig. 3b) given by the first-order approximation of the SW astroid can be used.

Finally, the model can be modified to apply to inhomogeneous systems [12] consisting of more than one phases or components with different hysteresis properties. The density function of the halfwidths, $\rho(u_{\mathrm{hw}})$ is taken as the

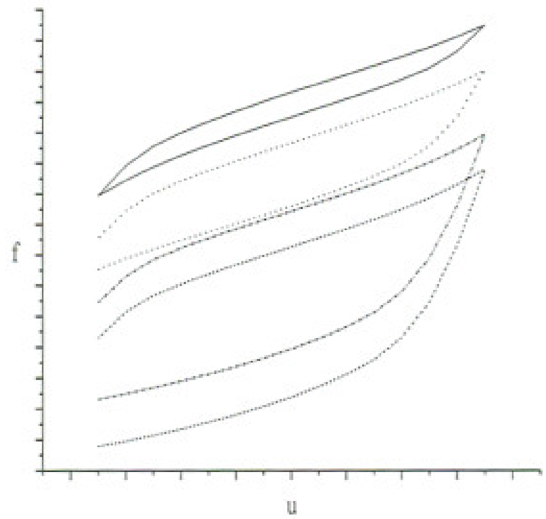

Figure 4: Noncongruent minor loops.

weighted average of two or more density functions, one for each phase, e.g. in the case of a two-phase system $\rho(u_{\text{hw}}) = w \times (u_{\text{hw},1}) + (1 - w) \times \rho(u_{\text{hw},2})$ where w is the % content of one phase in the system.

In the vector model, the identification problem is not as straightforward as in the scalar case. The density cannot be measured as before because of the reversible processes predicted by the vector operator. One way to get around this, is the factorization of the density on the assumption, valid in ferromagnets, that the halfwidths of the elementary operators, u_{hw}, and the interactions, u_{int}, experienced by them are independent variables.

Then $\rho(u_{\text{hw}})$ and $\rho(u_{\text{int}})$ are modeled by gaussians or other probability density functions and their product yields the characteristic density (,). The identification problem then reduces into determining their parameters based on macroscopic experimental measurements. In the inhomogeneous case, the identification consists of determining the parameters of the density functions $\rho(u_{\text{hw},1})$, $\rho(u_{\text{hw},2})$, and $\rho(u_{\text{int}})$. To the above, the parameters of the angular dispersion, $\rho(\phi)$, should also be added.

In order to illustrate the performance of 2D Preisach-type models, the SW astroid and normal distributions $N(\mu, \sigma^2)$ are used for all four probability density functions needed for the identification of a two-phase system. A more detailed study of the effect of the identification parameters on the major loop characteristic is presented in [13]. Fig. 1 shows the trajectory obtained for a cycling input of decreasing magnitude. The system, initially saturated, reaches the state [0,0]. Fig. 4 shows noncongruent loops predicted by the model. Both of these curves are in qualitative agreement with observed behavior in actual systems because of the use of the vector operator that accounts automatically for reversible processes as well. On the other, the model has the wipe-out property which applies in actual systems as well. Finally the good vector properties of the model are evident in Fig. 5 showing a rotational hysteresis loss curve calculated by the model. The points on the curve are obtained by applying rotating fields of increasing magnitude and calculating the torque for a full revolution of the input: $W_r = \int_0^{2\pi} (\mathbf{u} \times \mathbf{f}) d\theta$. For very small or large input values, the lag of the output is negligible and the torque is practically zero.

4 Concluding Remarks

The 2D Preisach model is a quasistatic, macroscopic model maintaining the major advantages of simple and fast algorithms of the classical model. This is especially important when the model is used in long calculations or simulations. The substitution of the original scalar operator by vector ones gives the model good vector properties and allows it to reproduce both reversible and irreversible properties and predict realistic minor loops. More vector operators, based on the physics of the actual systems, need to be developed and tried. The identification procedure needs to become more systematic and accurate in order to make the model more user-friendly.

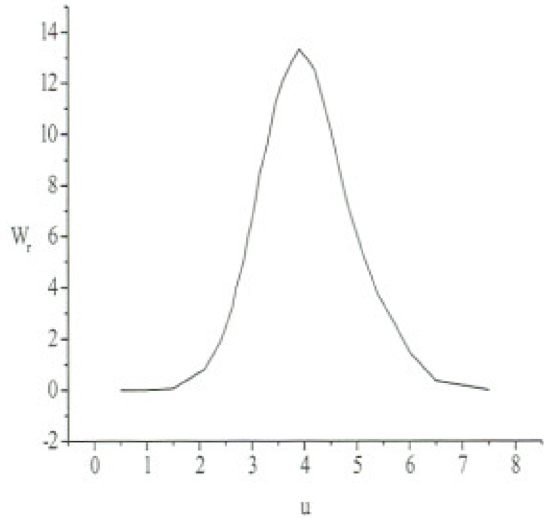

Figure 5: Rotational hysteresis curve.

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