NONUNIFORM MAGNETIZATION REVERSAL IN STRESSED THIN FERROMAGNETIC FILMS

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Nonuniform Magnetization Reversal in Stressed Thin Ferromagnetic Films

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Abstract

The effect of applied mechanical stresses on the magnetization reversal, well known as inverse magnetostricton effect, is studied for thin ferromagnetic films. The model used, is a micromagnetic one proposed in Ref. [1]. Numerical nonuniform (NU) solutions for the Brown' s magnetoelastic equations are presented and compared with uniform Stoner-Wolfarth (SW) ones. We study only the case were the applied stresses are oriented parallel to the field' s direction (Case 1 of Ref. [1]). The dependence of coercivity and remanence on applied stress and thin film thickness is discussed. The framework for stability analysis is developed, but it is applied only to the saturation solutions of the NU modes, which are proved to be unstable. Energy considerations confirm that the NU modes are unfavorable ones throughout the magnetization reversal.

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1 INTRODUCTION

Magnetostrictive materials belong to the class of "smart material" since its hysteresis is controlled by mechanical stresses and its mechanical deformations by magnetic fields [2]. Thus they are very attractive as actuator and sensor devices, such as robotics, micromotors, etc. [3]. Also magnetomechanical effects in thin film recording media can cause undesirable anisotropy during fabrication processes, which result in "noisy" characteristics of the final recording head products during operation [4]. Therefore tightly control of magnetostriction effects, either intrinsic (internal stresses) or external (applied mechanical stresses), is needed. The effect of stress on magnetic properties of ferromagnetic materials (well known as inverse magnetostriction effect) is one of the favorable research topics [2, 5, 6]. A rigorous phenomenological theory of magnetoelastic interactions has been proposed [7], but due to its complexity only elementary solutions have been obtained [8]. Recently, an energetic approach was introduced to explain the large magnetostriction observed in a class of ferromagnetic materials [9]. This theory is applied to large enough materials, where the division of the crystal into domains is preferable, and thus is not capable of describing accurately the underlying microstructure. Domain rotational models, that included stress effects on ferromagnetic hysteresis have been developed [10], but they ignored NU modes of magnetization reversal. Such NU modes have been investigated in some special cases of pure magnetostrictive domain wall oscillations [11], or torsional effects in amorphous ribbons [12]. A one-dimensional micromagnetic model for studying the inverse magnetostriction effect in thin

ferromagnetic films, that embodies the non-uniformity in the magnetization distribution in its basic postulates and thus is self-consistent, was proposed in a previous work [1]. Only the uniform SW solution were presented, as well as the nucleation modes for NU magnetization reversal.

The aim of this paper is to discuss possible NU solutions to the previously proposed micromagnetic model that accounts for the inverse magnetostriction effect in thin ferromagnetic films, and examine their relation with the applied stresses. Brown' s micromagnetic equilibrium equations are solved numerically by the Galerkin Finite Element Method (GFEM). We investigate here only the case were the in-plane applied magnetic field is perpendicular to the applied stress direction. We looked for typical Bloch wall solution to the problem under discussion [13]. The general framework for studying stability of solutions, was developed, but it was applied only to the special case of the saturation states. The coercivity-stress $(h_c(\sigma))$ and remanece-stress $(m_r(\sigma))$ laws were obtained and compared with analytical [1] and experimental results [6]. Size effects were also discussed.

2 Brown' s Micromagnetic Equations

We rescale the micromagnetic equations (20) of Ref. [1], according to

$$\bar{S} = S\sqrt{\frac{|h_k|}{2}}. \tag{1}$$

Since we limit our discussion to materials with negative magnetocrystalline anisotropy and negative magnetostriction, like Ni, $h_k = -|h_k|$, and $h_{me} \geq 0$,

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thus Eqs. (20) of Ref. [1] reduce to:

$$\frac{d^2\theta}{d\bar{x}^2} + \left(\frac{\sin 4\theta}{4} - \bar{h}_{me}e\sin 2\theta - \bar{h}\sin\theta\right)\bar{S}^2 = 0,$$

$$\left.\frac{d\theta}{d\bar{x}}\right|_{\bar{x}=\pm\bar{S}} = 0, \qquad\qquad (2)$$

$$\sigma = h_e\, e.$$

with

$$\bar{h}_{me} = \frac{h_{me}}{|h_k|} = \frac{B_1}{2\,|K_1|}, \quad \bar{h} = \frac{h}{|h_k|} = \frac{H_x^o}{|H_K|}, \qquad (3)$$

and H_K is the anisotropy field ($H_K = 2K_1/\mu_o M_s$). Due to the definition (1):

$$\bar{S} = \frac{a}{\delta}, \qquad\qquad (4)$$

with a being the half thin films thickness and δ the domain wall width:

$$\delta = \sqrt{\frac{C}{2\,|K_1|}}. \qquad\qquad (5)$$

The rescaled nucleation field (Eq. (60) of Ref. [1]) is:

$$\bar{h}_n = 1 - 2\bar{h}_{me}\, e. \qquad\qquad (6)$$

For convenience in the following we will ignore the bars in the definitions (3-4,6). For each solution to the boundary value problem (BVP) (2) the reduced magnetization $m = M/M_s$ is computed using the relation

$$m \equiv \frac{1}{2S}\int_{-S}^{+S}\cos\theta\, dx. \qquad\qquad (7)$$

Before proceeding to the numerical solution of the BVP (2) we have to mention, that like in all one dimensional micromagnetic problems, a first integral of (2.1)

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can be obtained [14], and thus the whole solution procedure results in the evaluation of an elliptic integral, with an unknown constant of integration. Since this integral cannot be computed analytically, a numerical solution is applied.

3   Numerical Solution

Nowadays, there is a variety of numerical schemes (NS) for treating micromagnetic problems. All can be classified in three major categories: (1) Energy minimization algorithms [15] (2) Landau-Lifshitz equation solvers [16] and (3) hybrid methods [17]. A complete and critical review on the subject is given in Chapt. 11 of Ref. [18]. In NS(1) the discretization of the domain of solution is followed by numerical minimization of the energy functional. For this reason the finite element method is used, since it allows to handle irregular geometries. In the NS(2), after using calculus of variation to obtain the Euler-Lagrange equations (well known for the present problem as Landau-Lifshitz equations), numerical integration is performed. Finally, in NS(3), a combination of analytical and numerical techniques is followed, guided by the geometry of the problem and the proper treatment of the magnetostatic self energy, which is the most time consuming part of all three approaches.

In the present work we followed the second approach but instead of using already proposed algorithms [16], for numerical integration of Brown' s micromagnetic equations (2), we preferred to solve them with the GFEM. In this method the solution is expanded in quadratic elements

$$\theta = \sum_{i=1}^{3} \theta_i \Phi_i(\xi), \tag{8}$$

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where Φ_i is the quadratic basis function and θ_i is the unknown at the ith node of the element. The GFEM calls for the weighted residuals R_i to vanish at each nodal position i :

$$R_i = \int_D L(\theta)det(\boldsymbol{J})d\xi, \tag{9}$$

where L is the nonlinear operator that acts on θ (see Eq. (1)) and $\boldsymbol{J}$ is the Jacobian of the isoparametric mapping, with $D = [-S, S]$ the domain of solution. For the present problem

$$R_i = -\int_0^1 \left[\frac{\theta'\Phi_i'}{(\Delta x)^2} - \left(\frac{\sin 4\theta}{4} - h_{me}e\sin 2\theta - h\sin\theta \right) S^2\Phi_i \right] \Delta x \, d\xi, \tag{10}$$

with $\Delta x \equiv 2S/N_e$, where N_e is the total number of elements used. All the numerical results obtained correspond to material constants of Ni [1]. In general each solution m obtained, represents a point in the three dimensional space designated by (h, σ, S). In the computations we varied all three parameters, but the results presented here are for varying h, σ and $S = 1.5$ For the material constants of Ni this corresponds to a film thickness $84.75nm$.

In all our numerical computations, the degeneration of the solution to the SW limit represents more than a problem. This is because the nucleation mode for the presented problem is a coherent one [1]. Like in all nonlinear BVPs, and the present one (Eqs. (2)) possesses multiple solutions. Since the search for solutions in a nonlinear BVP depends strongly on the initial guess, we searched for typical Bloch domain walls excitations [13]. All the obtained solutions are summarized in the three modes shown in Fig. 1, for the same set of parameters: a compressive stress $\sigma = -5 \times 10^3$ and a reduced thickness $2S = 3$. Close

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to jumping fields in the hysteresis curve the numerical approach fails, since it is designed to find static equilibrium solutions, while the jumping field is a non-equilibrium (saddle) point [19]. Nevertheless, for very small systems with only few discretization points, the jumping fields can be computed with high accuracy [19, 20], by refining the continuation step in the initial and the final solution branch. Magnetization profiles at various stages of the reversal are also included in Fig. 1. The departure from saturation for mode A is at $h_s = 0.65 < h_n = 1.175$, and corresponds to typical perturbation of the SW mode B. At $h = -0.75$ the magnetization profile of mode A has the classical Bloch wall structure, while at $h = -1.5$ a complete Bloch wall has been formed. For the mode C, that is a physically non acceptable, since it corresponds to negative susceptibilities and passes from the origin ($m(h = 0) = 0$), continuation for $h_c = -0.204 \le h \le -0.16$ resulted into degeneration to the SW mode B. The nature of the singularity of this point, for mode C, cannot be determined and it is under investigation. We proved that it is a higher order singularity, since it cannot be treated as a turning point. All the magnetization profiles in mode C are perturbations of the Bloch wall solutions. Notice that mode A corresponds to higher coercivity and remanence with respect to the SW mode B. Typical solutions of the BVP are presented in Fig. 2 for varying h. Fig. 2a corresponds to mode A, while Fig. 2b to mode C. The source of the non uniformity of the solutions of mode A is the fact that $\theta(x = 0) = 0 \ \forall \ h$. In mode C the solution for $h = -0.16$ is quit close to the SW limit.

In Appendix A we prove that the saturation states of mode A are unstable

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ones. It can also be verified, by the following argument that they are and energetically unfavorable ones. The reduced free energy per unit length ($g = G/2|K_1|a$) is:

$$g(h, e) = \int_{-S}^{S} \left[\frac{1}{2S^2} \left(\frac{d\theta}{dx} \right)^2 - \frac{\sin^2 2\theta}{8} - h_{me} e \cos^2 \theta + h_e e^2 - h \cos \theta - \sigma e \right] dx, \tag{11}$$

For the saturation states $\theta = 0, \pi$, along the positive or negative field directions h_s, this reduces to:

$$g(h_s, e) = 2S(f(e) \mp h_s), \quad f(e) = h_e e^2 - h_{me} e - \sigma e. \tag{12}$$

Thus, for example, for the initial saturation state with $h_s < h_n \Rightarrow g(h_s, e) > g(h_n, e)$. It is also obvious from Fig. 3 that the NU modes are energetically unfavorable ones throughout the magnetization reversal, were we plot the total energy for the three modes of solution as a function of the applied field h. We also included in the plot the exchange energy (dashed curves) for comparison. Notice the metastable states $-h_c \leq h \leq 0$, in mode B that lead to the irreversible rotation to the magnetization at $h = h_c$. We have thus derived that the mode A deviates unstably from saturation and is an energetically unfavorable one throughout the magnetization reversal. These arguments alone, do not prove that this is an unstable mode of magnetization reversal. The stability must be examined solving the eigenvalue problem (A.3-A.4), of Appendix A, for every value of h, which is a subject of a future communication.

According to a theorem proved by Brown and Shtrikman [14]: *all one dimensional non uniform equilibrium solutions to the micromagnetic problem are*

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unstable for a homogeneous, rigid, ferromagnetic crystal in a uniform external field. For magnetically inhomogeneous materials, this is not always valid [14, 21, 22]. Magnetostrictive deformations (intrinsic, applied field or mechanical stress, in origin) might change the elastic *but not* the magnetic homogeneity of the material. Without being able to prove it, it is believed that the above mentioned theorem will held and for a ferromagnet that deforms uniformly, and has no volumetric changes (zero dilatation $e_{ii} = 0$), like the problem studied here. This is because the only thing that changes due to magnetostriction in such a deformation mode, is the orientation of the magnetocrystalline easy axis, with respect to the crystallographic axes, and not its distribution within the material.

Typical magnetization curves for the mode A, for varying tensile and compressive stresses are plotted in Fig. 4. Though the remanence of mode A seems to decrease with applied stress, as it is the case for SW reversal [1], with a bit higher values, the coercivity has a minimum for tensile stresses and then increases, like the SW mode B. The stress dependence of coercivity and remanence is given in Fig. 5a and Fig. 5b, respectively. For the particular case of Ni, the coercivities in Fig. 5a are as high as $H_c = 341\,kA/m$ for stresses $T_x = \pm 250\,MPa$. At least in the stress range we performed our simulations, there are no critical stresses for eliminating hysteresis in the NU mode A. The minimum in the $h_c(\sigma)$ curves is analogous to related experiments on amorphous ribbons and wires [23] and thus further examination is needed to verify if it can be applied to these problems. Thus the NU solutions of mode A are proved

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to be worst candidates for explaining the observed behavior in related experiments [6], than the SW coherent solutions of mode B [1]. Simulations have also been performed for varying size parameter S and all resulted in a decrease of the coercivity and remanence of mode A, as it is the case for experiments [6].

4 Conclusions

The already proposed models for estimating the effect of applied stresses on the magnetization reversal in ferromagnetic materials are domain wall rotation in character [10], or they invoke the domain structure as a prerequisite [9]. Thus they are not capable to describe accurately the underlying microstructure. The one-dimensional micromagnetic model presented in this investigation, that accounts for stress effects on the magnetization reversal in thin ferromagnetic films and embodies the non-uniformity in the magnetization distribution in its basic postulates, was solved numerically by the GFEM. The material constants are taken for Ni, but the analysis is quit general and applies to materials with negative magnetostriction and magnetocrystalline anisotropy constants. Three branches in the magnetization curve were obtained: two NU modes that correspond to positive (mode A) and negative (mode C) susceptibilities, respectively, and the Stoner-Wolfarth (SW) mode (mode B) studied previously [1]. Mode A deviates from the saturation state in an unstable way and is energetically unfavorable throughout the magnetization reversal, compared to the SW coherent rotation. Mode C is physically non permissible one, since it corresponds to negative susceptibilities, and degenerates for large negative fields to the SW mode

B. The coercivity and remanence of mode A are higher than those of the SW mode B and thus fail to explain the related experiments [6]. Thus, the NU magnetization distribution along the thin films thickness, is not capable to explain experimental results. Modification of the model after taking into account shearing and NU strains might be a step towards quantitative agreement with experiment. The minima in the $h_c(\sigma)$ curves are qualitative the same with related experiments on amorphous wires [23], and further investigation is needed to examine if the present model applies to such materials. The general stability is not studied, though the possibility of existence of stable NU solution branches is discussed, based on a theorem for one dimensional NU magnetization reversal for rigid ferromagnet [14].

A   Stability Analysis

The numerical approach is capable to obtain only equilibrium solutions. To test the static stability of these solutions, we introduce the dynamic criterion proposed by Brown [24]. According to this, let θ_0 be an equilibrium i.e. a solution of (2) and let $\theta = \theta_0 + \epsilon$. If (2) is regarded as an equilibrium of forces, the equation of motion in the vicinity of θ_0 is obtained by adding a dissipative term:

$$\frac{\partial^2(\theta_0 + \epsilon)}{\partial x^2} + \left(\frac{\sin 4(\theta_0 + \epsilon)}{4} - h_{me} e \sin 2(\theta_0 + \epsilon) - h \sin(\theta_0 + \epsilon) \right) S^2 = \alpha \frac{\partial \epsilon}{\partial t},$$

$$\text{A.1}$$

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with t we denote time and α is a positive constant. To test the stability only first order terms need to be considered with

$$\epsilon = \epsilon_0(x) \exp(-t/\tau). \qquad \text{A.2}$$

Thus Eq. (A.1) reduces to $(()' \equiv d/dx)$:

$$\epsilon_0'' + \left((\cos 4\theta_0 - 2h_{me} \cos 2\theta_0 - h \cos \theta_0)S^2 + \frac{\alpha}{\tau} \right) \epsilon_0 = 0. \qquad \text{A.3}$$

If and only if all eigenvalues a/τ of (A.3) with

$$\epsilon_0'(\pm S) = 0, \qquad \text{A.4}$$

are positive $(\tau > 0)$, the equilibrium θ_0 is stable. We check here just the stability of the saturation solutions $\theta_i = 0$ and $\theta_r = \pi$, for large positive and negative applied fields h_s, respectively. In this case (A.1) reduces to:

$$\epsilon_0'' + k_\pm^2 \epsilon_0 = 0, \qquad k_\pm \equiv \sqrt{(h_n \mp h_s)S^2 + \frac{\alpha}{\tau}}. \qquad \text{A.5}$$

The general solution of (A.5) is $\epsilon_0(x) = C_1 \exp(k_\pm x) + C_2 \exp(-k_\pm x)$ and satisfaction of the BCs (A.4) result in $k_\pm = 0$ or:

$$\frac{\alpha}{\tau} = (h_s \pm h_n)S^2. \qquad \text{A.6}$$

Stable and unstable saturation states are summarized in in Table 1. Since for

Saturation State	Stable	Unstable
$\theta_0 = 0$	$h_s > h_n$	$h_s < h_n$
$\theta_0 = \pi$	$h_s < h_n$	$h_s > h_n$

Table 1: Stability of saturation states.

mode A the saturation state correspond to smaller saturation fields compared to the nucleation field, due to Table 1 all the numerical solutions, of this mode at $h = h_s$, obtained, are unstable.

References

[1] P. A. Voltairas, D. I. Fotiadis and C. V. Massalas, Int. J. Eng. Sci. (accepted).

[2] R. M. Bozorth, Ferromagnetism (D. van Nostrand Company Inc., Princeton NY 1951).

[3] K. I. Arai and T. Honda, Robotica 14 (1996) 477; M. Takezawa et al., H. Appl. Phys. 83 (1998) 7303-7305.

[4] J. R. Cullen, K. B. Hathaway, A. E. Clark, J. Appl. Phys. 81 (1997) 5417-5419.

[5] A. J. Moses, IEEE Trans. Mag. 15(6) (1979) 1575-1579; R. Langman, IEEE Trans. Mag. 21(4) (1985) 1314-1320; J. M. Makar and B. K. Tanner, I. Mag. Mag. Mat. 184 (1998) 193-208.

[6] De - Hua Han et al. , Appl. Phys. Lett., 70 (1997) 664; De - Hua Han et al. , J. Appl. Phys., 81 (1997) 4519; L. Callegaro et al. , Appl. Phys. Lett., 68 (1996) 1279; J. Nowak et al. , J. Appl. Phys.79 (1996) 3175; E. Puppin et al. , IEEE Trans. Mag. 32 (1996) 281; L. Callegaro et al. , IEEE Trans. Mag. 33 (1997) 1007.

[7] W. F. Brown, Magnetoelastic Interactions (Springer tract in natural Philosophy Vol. 9, Ed. by C. Truesdell, Berlin 1966); A. C. Maugin and G. A. Maugin, Electrodynamics of Continua I-II (Spinger-Verlag, NY, 1990);

G. A. Maugin, Continuum Mechanics of Electromagnetic Solids (North-Holland, Amsterdam, 1988).

[8] F. C. Moon, Magnetosolid Mechanics (John-Wiley and Sons, NY, 1984).

[9] R. D. James and D. Kinderleher, Cont. Mech. and Therm. 2 (1990) 215; Phil. Mag. B 68 (1993) 237; A. DeSimone, Int. J.. Mat. Syst and Str. 5 (1994) 787; A. DeSimone, Script. Metal. Mater., 33 (1995) 1869; A. DeSimone and R. D. James, J. Appl. Phys. 81 (1997) 5706.

[10] D.C. Jiles and D. L. Atherton, J. Phys. D: Appl. Phys., 17 (1984) 1265-1281; M. J. Sablik et al., J. Appl. Phys. 61(8) (1987) 3799-3801; M. J. Sablik and D. C. Jiles, IEEE Trans. Mag., 29(3) (1993) 2113-2123; M. J. Sablik et al, J. Appl. Phys., 74(1) (1993) 480-488; D. C. Jiles and M. K. Devine, J. Appl. Phys., 76(10) (1994) 7015-7017; T. G. Jeong and D. B. Bogy, IEEE Trans Mag., IEEE Trans. Mag., 31(2) (1995) 1007-1012; A. Bergqvist and G. Engdahl, J. Appl. Phys. 79(8) (1996) 6476-6478.

[11] M. Kleman, Phil. Mag., 19 (1969) 285-302; How et al., Phys. Rev. B, 40 (1989) 4808-4817.

[12] A. Hernando and J. M. Barandiaran, Phys.Rev. B, 22 (1980) 2445-2449.

[13] A. Hubert and R. Schäfer, Magnetic Domains (The Analysis of Magnetic Microstructures), (Springer, Berlin, 1998); A. Hubert, Theorie der Domänenwände in geordneten medien (Springer-Verlag, Berlin/Heidelberg 1974).

[14] W. Fr. Brown and S. Shtrikman, Phys. Rev. 125 (1962) 825-828.

[15] D. R. Fredkin, IEEE Trans. Magn. 23(5) (1987) 3385-3387; W. Williams and D. J. Dunlop, Nature 337(16) (1989) 634-637; D. V. Berkov, K. Rämstock and A. Hubert, Phys. Stat. Sol. (a) 137 (1993) 207-225; T. Schrefl, J. Fidler and H. Kronmüller, Phys. Rev. B 49(9) (1994) 6100-6110;

[16] Y. Nakatani, Y. Uesaka and N. Hayashi, Jap. J. Appl. Phys. 28(12) (1989) 2485-2507; M. Mansuripur, R Giles, Comp. Phys. May/June (1990) 291-302; R. Giles, P. S. Alexopoulos and M. Mansuripur, Compt. Phys. Jan/Feb (1992) 53-70; M. E. Schabes, J. Mag. Mag. Mat. 95 (1991) 249-288.

[17] A. E. Labonte, J. Appl. Phys., 40(6) (1969) 2450-2458; M. E. Schabes and A. Aharoni, IEEE Trans. Mag. 23(6) (1987) 3882-3888.

[18] A. Aharoni, Introduction to the Theory of Ferromagnetism (Oxford University Press., NY 1996).

[19] A. Hubert and W. Rave, Phys. Stat. Sol.(b) 211 (1999) 815-829.

[20] J. Ehlert, F. K. Hübner and W. Sperber, Phys. Stat. Sol. (a) 106 (1988) 239-248; J. Ehlert, F. K. Hübner and W. Sperber, Phys. Stat. Sol. (a) 116 (1989) 389-397.

[21] A. Aharoni, Phys. Rev. 119 (1960) 127-131; C. Abraham and A. Aharoni, Phys. Rev. 120 (1960) 1676-1579; A. Aharoni, J. Appl. Phys. 32 (1961) 245S-246S.

[22] Xiao Hu, Phys. Rev. B 55 (1997) 8382-8388;T. Leineweber and H. Kronmüller, Phys. Stat. Sol. (b) 201 (1997) 291-301; T. Leineweber and H. Kronmüller, J. Mag. Mag. Mat. 176 (1997) 145-154; R. A. Hyman and A. Zangwill, Phys. Rev. B 58 (1998) 9276-9286;

[23] R.Grossinger et al. , J. Mag. Mag. Mat. 133 (1994) 20; R. Sato Turtelli et al., J. Appl. Phys. 78 (1995) 2590; A. M. Severino et al., J. Mag. Mag. Mat. 103 (1992) 117; P. T. Squire et al., 132 (1994) 10.

[24] A. Aharoni, E. H. Frei and S. Shtrikman, J. Appl. Phys. 30 (1959) 1956-1961.

Figure 1: Magnetization curves for $S = 1.5$ and $\sigma = -5 \times 10^3$.

Figure 2: (a) θ vs x for varying $h \in [-0.7, 0.65]$ with step 0.05 and $h \in [-7.75, -0.75]$ with step 1.0. (b) θ vs x for varying $h \in [-0.16, 0]$ with step 0.05.

Figure 3: Total reduced energy g vs applied field h, for $S = 1.5$, $\sigma = -5 \times 10^3$.

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Figure 4: Magnetization curves for $S = 1.5$ and varying σ. The numbering from 1 to 7 corresponds to $\sigma = -1.25 \times 10^5$, -5×10^4, -2.5×10^4, -5×10^3, 2×10^4, 5×10^4, and 9×10^4, respectivelly .

Figure 5: (a) h_c and (b) m_r vs. σ for $S = 1.5$ (the SW solution [1] are plotted for comparison).

$2S$

A

B

C

$S = 1.5$
$\sigma = -5000$

1

0.5

0

-0.5

-1

-1.5   -1.0   -0.5   0   0.5   1.0   1.5

m

h

(a) Mode A, (b) Mode C

Mode A