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INHOMOGENEOUS FERROMAGNETIC MATERIALS**

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## **COMPARISON OF NUCLEATION MODELS FOR INHOMOGENEOUS FERROMAGNETIC MATERIALS**

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### **1. SUMMARY**

A detailed study of micromagnetic models of the nucleation mechanism in permanent magnets is presented. All approaches are based on the minimization of the linearized free energy equation for a hard magnetic medium with a soft spherical inclusion. A comparison between the nucleation fields calculated by the various expressions is carried out. The material parameters used are typical of SmFeN. The different expressions for the nucleation field obtained by the two approaches are presented. The limitations of the existing models are discussed and a new approach is suggested.

### **2. INTRODUCTION**

The reversal of magnetization in ferromagnetic materials is a highly non-linear process due to the hysteresis of the magnetization response (output) to the applied magnetic field (input). It involves both rotation (a reversible process) and switching (an irreversible process) of the magnetization of the particles or grains in the medium. Both processes depend on the microstructure of the material, its intrinsic properties as well as the complex network of interactions developed within the material. The mechanism controlling the magnetization reversal is either due to domain nucleation or to pinning of domain walls. Modeling of either process is not a simple task and, for that, a topic which attracts the attention of many researchers.

The models attempting to describe the magnetization process fall under two general categories. The most popular category, the so-called micromagnetic approach, includes the models which attempt to describe the (de)magnetization process in terms of thermomagnetic equilibrium. The starting point is the free (Gibbs) energy equation of the system (magnet), the minimization of which, with respect to the magnetization direction, yields the stable states (field, magnetization) of the system. Because of the structure and "nature" of the energy equation, this approach allows for a detailed description of the microstructure of the materials and the nature of interactions that develop. The trade-off for this convenience is the high complexity of the minimized energy equation, worthy of the high complexity of the magnetic materials.



In the quest for solutions of the minimized energy equation, the researchers have either obtained analytical solutions for convenient geometries and microstructures [1,2] or have used finite element methods to obtain solutions for more complicated systems [3,4]. In both cases, the goal is to obtain a realistic expression or value for the coercive field, *i.e.*, the field at which the bulk magnetization of the material reverses.

The other category, involves models that view the material as a black-box with known inputs (applied fields) and outputs (macroscopic properties such as the bulk magnetization). This systemic approach is based on the Preisach formalism [5] and has been quite successful, especially with media that allow one-dimensional treatment of the process. The fast algorithms led themselves to be efficient tools for media design but they are still quite “primitive” for the more “sophisticated” modern materials where some of the simplifying assumptions of these models collapse since the ‘black-box’ approach does not easily allow for a detailed description of the material microstructure.

In this work, we present a comparison between analytical solutions to the micromagnetic equation obtained by two different research groups. The solutions discussed and compared are all implicit or explicit expressions of the nucleation field in permanent magnets when nucleation is assumed to be the governing mechanism of the magnetization reversal.

When a magnetic material is assumed to be homogeneous, perfectly aligned, made of single domain particles, the magnetizations of which rotate coherently, the expression obtained for the nucleation field,  $H_n$ , is the well-known  $H_n = 2K_1/M_s - NM_s$ , where  $K_1$  and  $M_s$  are the anisotropy constant and saturation magnetization of the material, respectively, and the second term describes the effect of self-demagnetizing fields[6]. This expression is usually referred to as the *ideal nucleation field* because first, the actual coercive fields measured in the laboratory are lower by orders of magnitude and second, no real magnet has a microstructure allowing for the assumptions underlying the expression.

The models presented in this work are based on rare-earth intermetallics (RE-3d alloys) taking over the traditional hard ferrites because of their high coercivity which is due to the exchange coupling between the RE and 3d atoms. Obviously these are inhomogeneous, two-phase materials. The material used for testing the models in this work is  $\text{Sm}_2\text{Fe}_{17}\text{N}_x$ , a relatively new permanent magnet with an unusually high energy product,  $(BH)_{\text{max}}$ .

In the following, we give an overview of the micromagnetic equation and the solutions obtained by Skomski and Coey [2,7], on the one hand, and Kronmüller and coworkers [1,8-11], on the other. Then, all models are tested for the material parameters of  $\text{Sm}_2\text{Fe}_{17}\text{N}_x$ , as described in Table 1, and their results are compared. Finally, we suggest that a Preisach-type model for inhomogeneous magnetic materials be constructed in an attempt to waive the weaknesses or inadequacies of the existing models.

### 3. THE MODELS

Both the model of Skomski and Coey and the model of Kronmüller are based on the minimization of the free energy equation,  $E$ :

$$E = \int \{E_{\text{ex}}(\mathbf{r}) + E_{\text{k}}(\mathbf{r}) + E_{\text{a}}(\mathbf{r}) + E_{\text{m}}(\mathbf{r})\} d\mathbf{r} \quad (1)$$



where  $E_{ex}$  is the exchange energy density,  $E_k$  is the anisotropy energy density,  $E_a$  is the energy density of interaction with the applied field (Zeeman energy density), and  $E_m$  is the self-magnetostatic energy density; all are implicit functions of position,  $\mathbf{r}$ :

$$E_{ex}(\mathbf{r}) = A(\mathbf{r}) [\nabla \mathbf{m}(\mathbf{r})]^2$$

$$E_k(\mathbf{r}) = - (K_1(\mathbf{r}) (\mathbf{n} \cdot \mathbf{m}(\mathbf{r}))^2 + K_2(\mathbf{r}) (\mathbf{n} \cdot \mathbf{m}(\mathbf{r}))^4 + \dots)$$

$$E_a(\mathbf{r}) = - \mathbf{m}(\mathbf{r}) \cdot \mathbf{H}$$

$$E_m(\mathbf{r}) = - 1/2 \mathbf{M}(\mathbf{r}) \cdot \mathbf{H}_d(\mathbf{r}),$$

where  $A(\mathbf{r})$  is the exchange constant,  $\mathbf{m}(\mathbf{r})$  is the normalized magnetization vector,  $K_1$  and  $K_2$  are the first and second order anisotropy constants,  $\mathbf{n}$  is the unit outward vector normal to the surface,  $\mathbf{H}$  is the applied field vector,  $\mathbf{H}_d$  is the demagnetization field vector and  $\mathbf{M}(\mathbf{r})$  is the magnetization vector.

#### *The model of Skomski and Coey*

Skomski and Coey [7] have treated the problem of a soft spherical inclusion of diameter  $D$  in a hard magnetic matrix. Under the assumption of perfect alignment the energy equation reads:

$$E = \int \{ A(\mathbf{r})[\nabla \mathbf{m}]^2 + K_1(\mathbf{r}) \mathbf{m}^2 + 1/2 \mu_0 M_0(\mathbf{r}) \mathbf{H} \mathbf{m}^2 \} d\mathbf{r} \quad (2)$$

where  $\mathbf{H}$  is the field combining the effect of both the applied and demagnetizing fields and  $M_0(\mathbf{r})$  is the saturation magnetic moment.

For small perturbations, the linearized form of the above equation is minimized and the following *Schrödinger-type* equation of state is obtained:

$$-[2A(\mathbf{r})/\mu_0 M_0(\mathbf{r})]\nabla^2 \Psi + [2K_1(\mathbf{r})/\mu_0 M_0(\mathbf{r})]\Psi = H_n \Psi \quad (3)$$

where  $\Psi = \mathbf{m}_x \cdot \mathbf{e}_x + \mathbf{m}_y \cdot \mathbf{e}_y$

The interface boundary condition is:

$$A_s(\mathbf{e} \cdot \nabla) \Psi_s = A_h(\mathbf{e} \cdot \nabla) \Psi_h,$$

where  $\mathbf{e}$  is the unit vector normal to the interface and  $A_s, A_h$  refer to the exchange constants of the soft and hard phases, respectively.

For an ideally soft inclusion ( $K_s = 0$ ) of radius  $R$ , the following implicit equation for  $H_n$  is obtained:

$$A_s/A_h [R(M_s H_n / 2A_s)^{1/2} \cot[R(M_s H_n / 2A_s)^{1/2}] - 1] + 1 + R \{ [(2K_h - M_h H_n) / 2A_h]^{1/2} \} = 0 \quad (4)$$

where  $A_s, M_s, K_s$ , and  $A_h, M_h, K_h$  are the material parameters of the soft and the hard phase, respectively.

#### *The model of Kronmüller et. al.*

Kronmüller and his group have studied for years the properties of sinter magnets like  $\text{Nd}_2\text{Fe}_{14}\text{B}$  and have offered solutions for a variety of configurations. More recently, Kou *et. al.* [8,9] have studied  $\text{SmFeN}$ -type magnets and modeled their experimental results using Kronmüller's nucleation model.

The basic model introduced by Kronmüller assumes a two-phase sinter magnet with grains perfectly aligned along the  $z$ -axis which also coincide with their crystalline easy axis, as in the Skomski and Coey model. The long-range magnetostatic interactions are treated separately: they consist of the vector sum of two fields,  $\mathbf{H}_d$ , an external field to the nucleus due to misoriented and nonmagnetic grains and the surface charges of the magnet and  $\mathbf{H}_{nuc}$ , due to rotation inside the nucleus. Then, the energy equation per unit length of the nucleus writes:

$$E = \int \{ A(z) [ (d\theta/dx)^2 + (d\theta/dy)^2 ] + K_1(z) \sin^2 \theta + K_2(z) \sin^4 \theta + (H_a - H_d) M_s(z) \cos \theta - 1/2 H_{nuc}(r) M_s(r) \} dx dz \quad (5)$$

where  $\theta$  is the angle of the magnetization with respect to the z-axis. The nucleus is assumed to be infinite in the y-direction, this way reducing the dimensionality of the problem. For small deviations of  $\theta$ , the energy equation can be linearized:

$$2A(z) \{-d^2\theta/dx^2 + d^2\theta/dz^2\} - \{2K_1(z) - (H_a - H_{d,z})M_s\}\theta - \frac{1}{2} \{[H_{nuc,z} - \partial H_{nuc,z}/\partial\theta]_{\theta=0}\}M_s\theta = 0 \quad (6)$$

Notice the analogy of the above equation to the equation of state obtained by Skomski and Coey.

The general form of the solution of equation (6) is:

$$H_n = \alpha(\Delta K, r_0) [2K_1/M_s] - N_{eff}M_s \quad (7)$$

where  $\alpha$  and  $N_{eff}$  are the parameters describing the microstructure of the material being modeled.  $\alpha(\Delta K, r_0)$  is the factor by which the ideal nucleation field is reduced to inhomogeneities of anisotropy  $\Delta K$  and width  $2r_0$ .

In the following section, we present the expressions obtained by Kronmüller and his coworkers [3,8,10-11] for  $\alpha(\Delta K, r_0)$  for one-, and two- dimensional rotation of the magnetization of the nucleated region, and for both harmonic and quasiharmonic diffusion profiles of anisotropy.

#### One-dimensional rotation

The linearized energy equation for one-dimensional rotation writes:

$$2A(z) \{d^2\theta/dz^2\} - \{2K_1(z) - M_s(H_a - H_d + 2\pi M_s)\}\theta = 0 \quad (8)$$

The exchange stiffness  $A(z)$  is assumed to be constant throughout the sample,  $A(z)=A$ , but the diffusion profile of  $K_1(z)$  can be adjusted in two ways:

- the harmonic case :

$$K_1(z) = K_s + \Delta K(1 - \exp(-z^2/r_0^2))$$

Where  $K_s$  is the anisotropy constant at the center of the inhomogeneity and  $K_1(\infty) = K_s + \Delta K$  is the anisotropy constant of the 'homogeneous' material.

The expression for  $\alpha$  is:

$$\alpha = \delta_B/(\pi r_0) + (1 - \Delta K/K_1) \quad (9)$$

where  $\delta_B = \pi\sqrt{(A/\Delta K)}$  corresponds to the Bloch-type wall width of the material assuming that  $K_s \ll K_1(\infty)$ .

- the quasiharmonic case, i.e., the inhomogeneity has finite boundary values:

$$K_1(z) = K_s + \Delta K/[ch^2(z^2/r_0^2)]$$

For this case,

$$\alpha = 1 - (\Delta K/K_1)[\delta_B/2\pi r_0]^2[-1 + (1 + 4\Delta K r_0^2/A)^{1/2}]^2 \quad (10)$$

Notice that:

For  $r_0 \rightarrow 0$ , i.e., the material is homogeneous and the ideal nucleation field is obtained because

$$H_n = 2K_1(\infty)/M_s + H_d - 2\pi M_s; \quad \alpha = 1$$

For  $2r_0 \gg [A/\Delta K]^{1/2}$ , i.e., the inhomogeneity is larger than the domain wall,  $\delta_B$ ,

$$H_n = 2K_s/M_s + H_d - 2\pi M_s; \quad \alpha = 1 - \Delta K/K_1$$

For  $2\pi r_0 \cong \delta_B$ , i.e., average thickness inhomogeneities,



$$H_n = (2K_1(\infty)/M_s)(\delta_B / \pi r_0) + 2K_s/M_s + H_d - 2\pi M_s; \quad \alpha = (1-\Delta K/K_1) + (\delta_B / \pi r_0)$$

The reader can easily see the first differences in the assumptions made by the group of Kronmüller and that of Skomski and Coey. The former assume constant exchange stiffness throughout the material and an anisotropy profile which is a function of  $z$  in contrast with the latter who assumes zero anisotropy throughout the soft inclusion and constant anisotropy throughout the hard phase but different exchange constants for the two phases.

#### Two-dimensional rotation

In the case of two-dimensional rotation (the inhomogeneity is again infinite in the  $y$ -direction but of length  $L$  in the  $x$ -direction), the stray field due to the nucleus,  $H_{nuc}=2\pi M_s$ , may be neglected in the first approximation, and the two-dimensional energy equation becomes:

$$2A(z)\{d^2\theta/dx^2 + d^2\theta/dz^2\} - \{2K_1(z) - (H_a - H_{d,z})M_s\}\theta = 0$$

Then, for the *harmonic*  $K_1$ -profile

$$\alpha = (\delta_B / \pi r_0) + (\delta_B / L)^2 + (1-\Delta K/K_1) \quad (11)$$

and for the *quasiharmonic* case:

$$\alpha = 1 - (\Delta K/K_1)[\delta_B/2\pi r_0]^2[-1+(1+4\Delta K r_0^2/A)^{1/2}]^2 + (\delta_B / L)^2 \quad (12)$$

A solution for three-dimensional rotation has also been presented but it has been criticized as rather unrealistic [11] and it is not being shown here.

#### 4. THE MATERIAL

The material parameters of SmFeN alloys reported in the literature are summarized in Table 1. Sm<sub>2</sub>Fe<sub>17</sub>N<sub>3</sub> can be isotropic or anisotropic. The second column of the table contains the material parameters used by Skomski and Coey [2] in their model. The third column contains the material parameters reported by Kou *et. al.* [8,9] for both isotropic and Zn-bonded anisotropic Sm<sub>2</sub>Fe<sub>17</sub>N<sub>3</sub> samples. As expected, the anisotropic sample exhibits higher remanence but lower coercivity due to the strong exchange coupling between the two phases. Overall, the anisotropic Sm<sub>2</sub>Fe<sub>17</sub>N<sub>3</sub> has superior magnetic properties but is expensive to prepare. Tests have been run for both sets of parameters. Notice that no value is being reported by Skomski and Coey for the second anisotropy constant  $K_2$  since it is not being used in the model.

Table 1: Material Properties

Properties	Sm <sub>2</sub> Fe <sub>17</sub> N <sub>3</sub> [1]	Sm <sub>2</sub> Fe <sub>17</sub> N <sub>3</sub> [2]
(BH) <sub>max</sub>	880 KJ/m <sup>3</sup>	
$\mu_0 M_s$	1.55 T	1.52 T
$\mu_0 M_r$	-	0.90T (anisotropic) 0.65T (isotropic)
H <sub>c</sub>	-	1.20 MA/m (anis.) 2.40 MA/m (isot.)
K <sub>1</sub>	12 MJ/m <sup>3</sup>	4.53 MJ/m <sup>3</sup>
K <sub>2</sub>	-	1.79 MJ/m <sup>3</sup>
A	10.7×10 <sup>-12</sup> J/m	-
H <sub>k</sub>	-	11 MA/m
$\delta_B$	3×10 <sup>-9</sup> m	4.3×10 <sup>-9</sup> m



#### 4. DISCUSSION

As we have already pointed out, there are differences between the models which do not allow for a direct comparison and the parameters need be adjusted appropriately in order for the comparison to be meaningful:

- In the Skomski and Coey model,  $A_s$  can be different than  $A_h$ : ( $A_s=16.7\times 10^{-12}$  J/m and  $A_h=10.7\times 10^{-12}$  J/m) while Kronmüller's model can accept only a constant A which is set equal to  $A_h$ . The ratio  $A_s/A_h$ , in the Skomski and Coey model, is therefore set to 1.0 for the sake of comparison with Kronmüller's results.
- In Kronmüller's model, two types of anisotropy profiles (harmonic and quasiharmonic) are used while in the Skomski and Coey model  $K_1$  is assumed to be constant. This suggests that the value of  $\Delta K$  in Kronmüller's equations must be set equal to  $1.0K_1$  in order to compare them to the Skomski and Coey solution.

The results using the parameters suggested by Skomski and Coey are presented in Figures 1-6. In all of the figures, we plot the percentage of the ideal nucleation field versus the diameter of the inhomogeneity.

The material parameters used in Figs. 1-6 are those used by Skomski and Coey and can be found in the second column of Table 1. In Fig. 1, we plot the nucleation field, as calculated from the solution of Skomski and Coey for one spherical inclusion of diameter  $D$ , versus  $D$ . The ratio  $A_s/A_h$  is varied: 1.0, 1.5, 2.0. According to the Skomski and Coey model, the Bloch wall width of the hard phase,  $\delta_B = \pi\sqrt{A_h/K_h} = 3\text{nm}$ , is responsible for the "plateau" region: for soft inclusions of diameter less than  $\delta_B$ , the nucleation field is equal to the ideal field [7] because of the exchange interactions. For an inclusion of diameter twice the wall width, the nucleation field decreases down to 40% of the ideal one. When we vary the exchange constant of the hard phase while keeping that of the soft phase constant we notice that a higher ratio of  $A_s/A_h$  yields slightly higher nucleation fields.

Plotting Kronmüller's solutions for one- and two-dimensional rotation with a harmonic anisotropy profile (figure 2) we notice that the "plateau" region breaks down at diameters smaller than the wall width of the hard phase. For the case of one-dimensional rotation, the "break-down" value of the diameter is equal to 1.5nm and for the case of two-dimensional rotation it is 2 nm. In this latter case, the size of the "wall-width" is given as approximately  $2\sqrt{A_h/K_h}$  [1]. We also notice that one-dimensional rotation predicts lower nucleation fields by 10-15%. The nucleation fields predicted by the 2-D rotation are higher for lower  $L$  (length of inhomogeneity in the x-direction) which is consistent with the results about 1-D rotation, *i.e.*, as  $L$  tends to infinity,  $(\delta_B/L)^2$  tends to 0, and the nucleation field decreases down to the value predicted for 1-D rotation.

Fig. 3 shows the decrease of the nucleation field as predicted by Kronmüller's model using a quasiharmonic anisotropy profile. Again, 1-D rotation yields lower nucleation fields than 2-D and so does a higher value of  $L$  for 2-D rotation. The quasiharmonic profile also predicts slightly lower nucleation fields than the harmonic one. The big difference however between the two anisotropy profiles is that in the quasiharmonic case there is no "plateau" region, *i.e.*, the nucleation field predicted by this model is lower than the ideal nucleation field even for very small inhomogeneities, e.g.  $\delta_B/2r_0 < 1$ . However, when the inhomogeneity's width is smaller than the domain wall width the exchange energy is big enough to prevent deterioration of the



nucleation field. According to our calculations, this model predicts that more than 40% of the ideal nucleation field is lost for diameters up to  $\delta_B$ .

In Fig. 4 we compare the Skomski and Coey solution against Kronmüller's solutions for 1-D rotation and both types of anisotropy profiles. We should point out once more that  $\Delta K=1.0K_1$  (we accept Skomski and Coey's assumption for zero anisotropy in the soft inclusion) and  $A_s/A_h=1.0$  (we accept Kronmüller's assumption for constant exchange throughout the material). For diameters of inclusions higher than  $3\delta_B = 9\text{nm}$  all three solutions predict a decrease in the nucleation field down to approximately 10-15% of the ideal one. The discrepancy is big for smaller diameters with the Skomski and Coey model yielding the highest nucleation fields and Kronmüller's quasiharmonic case the lowest ones.

Given that the diameters of inhomogeneities are more likely to be quite larger than the wall-width parameter and the experimental evidence that the coercivity can be up to 10 times less than the ideal nucleation field, we can assume that the solution for 1-D rotation is more likely.

We have mentioned time and again that in the above figures we have set  $\Delta K=1.0 K_1$  in order to make the comparison between the two models more meaningful. However, Kronmüller's model allows for a nonzero anisotropy constant ( $K_s = K_1 - \Delta K$ ) in the soft (or inhomogeneous) region which is a more realistic approach. So, in figs 5-6,  $\Delta K$  is varied ( $\Delta K/K_1 = 1.0, 0.9, 0.8, 0.7$ ) in Kronmüller's solutions and the results are compared against the Skomski and Coey model. As  $\Delta K$  decreases, the nucleation fields, predicted by Kronmüller's model, increase. The best agreement with the Skomski and Coey model is in the case of  $\Delta K/K_1 = 1.0$ , as expected. We also notice that the "plateau" region in Kronmüller's solution (fig.5) has now increased; this is because the "wall-width" parameter in Kronmüller's model is now given by  $\delta_B = 2\sqrt{A/\Delta K}$  and as  $\Delta K$  decreases the wall width increases. In the case of a quasiharmonic anisotropy profile (fig. 6) the nucleation field predicted by Kronmüller's model deteriorates fast but follows the Skomski and Coey model for diameters greater than  $3\delta_B$ .

Kou *et al.* [9] prepared and studied a sample of Zn-bonded anisotropic SmFeN, the magnetic properties of which are shown in the third column of Table 1. The magnetization reversal in the sample is assumed to follow the nucleation mechanism which is modeled according to Kronmüller's model for 1-D rotation. The assumption of perfect grain alignment is relaxed in the modeling of this sample. Martinek and Kronmüller [10] have obtained a solution for the nucleation field when the grains are not perfectly aligned. In this case, nucleation starts in misaligned grains and on the grain surface where the demagnetizing field  $N_{eff}M_s$  is bigger and the second anisotropy constant,  $K_2$ , must be taken into account. For imperfectly oriented materials, parameter  $\alpha$  in Kronmüller's model can be viewed as the product  $\alpha_\kappa\alpha_\phi$  where  $\alpha_\kappa$  corresponds to the effect of the crystallographic defects on the grain surfaces and  $\alpha_\phi$  corresponds to the effect of misaligned grains. Then,  $\alpha_\kappa$  is given by the model described in equation 10:

$$\alpha_\kappa = 1 - (\Delta K/K_1)[\delta_B/2\pi r_0]^2[-1+(1+4\Delta K r_0^2/A)^{1/2}]^2.$$

We used this expression to compare Kronmüller's model with the Skomski and Coey approach using the material parameters reported by Kou *et al.*[9]. Taking into account the effect of  $K_2$ , the wall width is now  $\delta_B = \pi\sqrt{A/(K_1 + K_2)} = 4.3 \text{ nm}$ .  $\Delta K$  is set equal to  $0.9K_1$  (Fig.7) and  $1.0K_1$  (Fig. 8). In Figs. 7-8, we have plotted the Skomski and Coey solution and Kronmüller's solution for 1-D rotation for both anisotropy profiles. As expected from the



previous results, the agreement is better for diameters larger than  $3\delta_B$ . Notice that the model suggested in Ref. [9] for this sample is that of 1-D rotation with a quasiharmonic anisotropy profile and  $\Delta K = 0.9K_1$  (Fig.7).

We have been unable to reproduce some of the results concerning the identification of the model for the particular sample of Ref. [9]. The parameters  $\alpha_K$  and  $N_{eff}$  are determined from temperature measurements of the coercive field, and reported as:  $\alpha_K = 0.91$  and  $N_{eff} = 2.01$ .

Using equation (10), the nucleus diameter is found:  $2r_0 = 1.58\text{nm}$ . According to the authors, this value lies within one unit cell (lattice constants:  $a = 0.87\text{nm}$ ,  $c = 1.27\text{nm}$ ) which is interpreted as an indication that the grain surfaces are free from crystallographic effects (an assumption which needs to be verified in the laboratory). This diameter is three times smaller than the wall width  $\delta_B$ . For diameters of that size, the Skomski and Coey model predicts that the coercive field is equal to the ideal nucleation field.

Kou and coworkers use the Martinek and Kronmüller nucleation model for imperfectly aligned grains in order to determine the effect of  $\alpha_\phi$  on the nucleation field, taking into account the second anisotropy constant,  $K_2$ , as well:

$$H_n = [1/2\sqrt{2\mu_0 M_s}][K_1 + (K_2/4)(W - K_1/K_2 + 3)] \times \{[W(K_1/K_2 + 1) - (K_1/K_2)^2 - 2K_1/K_2 + 3]\}^{1/2} \quad (13)$$

where  $W = [(1 + K_1/K_2)^2 + 8]^{1/2}$ .

Kou *et al.* report that fitting this model to temperature measurements of coercivity the following parameters are obtained:  $\alpha_K = 0.71$  and  $N_{eff} = 1.50$ .

These values are obtained through plotting  $H_c/M_s$  vs.  $H_n/M_s$  for temperatures from 5 to 494 K, which is a linear relationship. However, the slope of the line is then  $\alpha = \alpha_K \alpha_\phi$  and not  $\alpha_K$ . The other point which is unclear in this analysis is the value obtained for the nucleus diameter. Using equation 10, for 1D rotation, a diameter of  $2r_0 = 1.58\text{nm}$  is reported. Substituting this value into equation 10, we obtain  $\alpha_K = 0.88$ , which is quite different than the value obtained from the plot ( $\alpha_K = 0.71$ ). Finally, if we actually use  $\alpha_K = 0.71$  in equation (10) we get a nucleus diameter of  $2r_0 = 4.18\text{ nm}$  which compares with the wall width.

## 5. A NEW MODEL: THE PREISACH APPROACH

So far the two approaches in the modeling of magnetization reversal mechanisms have been the finite element method and the analytical solution of the micromagnetic equation. An approach that hasn't been tried so far in inhomogeneous materials is the Preisach modeling. Preisach models are phenomenological hysteresis models inherently one-dimensional. However, vector Preisach models have already been applied successfully [12] to homogeneous magnetic materials. The Preisach models use distributions of anisotropies and interparticle interaction fields in order to predict the levels of magnetization for any sequence of applied fields. They can therefore reproduce the major loop parameters with the appropriate tuning in of the parameters of the distributions. These parameters depend on the microstructure but it is not clear how they can be mapped onto microstructural parameters: for example, the distribution of interactions doesn't distinguish between exchange and magnetostatic interactions. One possible approach to tackling this problem is the distinction between the reversible and irreversible component of the magnetization [13] after having determined how these are affected by the presence of inhomogeneities.

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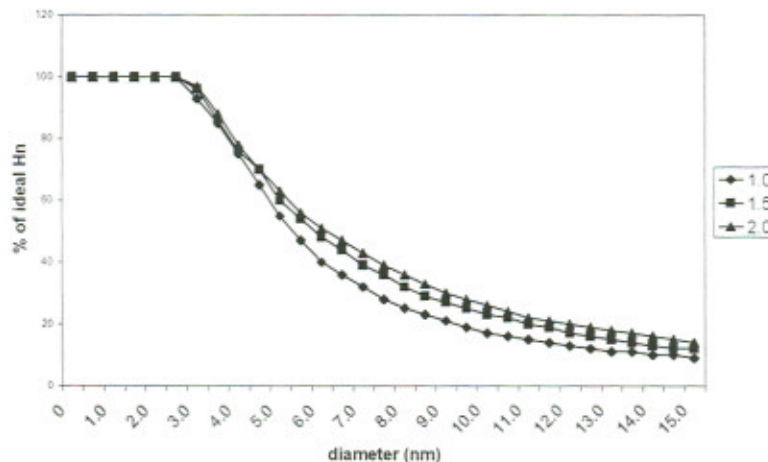


Figure 1: The reduced nucleation field vs. the diameter of the inhomogeneity. The Skomski and Coey model for  $A_s/A_h=1.0, 1.5, 2.0$ .



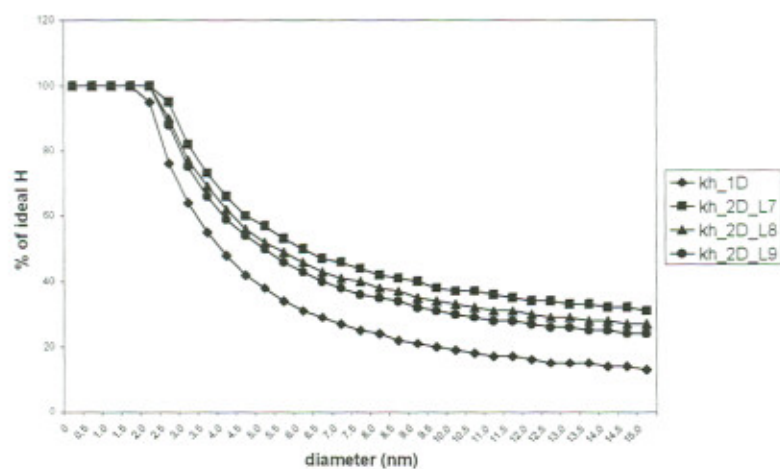


Figure 2: The reduced nucleation field vs. the diameter of the inhomogeneity. (Kronmüller's model for 1-D, 2-D rotation –  $L=7,8,9$  nm; harmonic anisotropy profile;  $\Delta K=1.0K_1$ ).

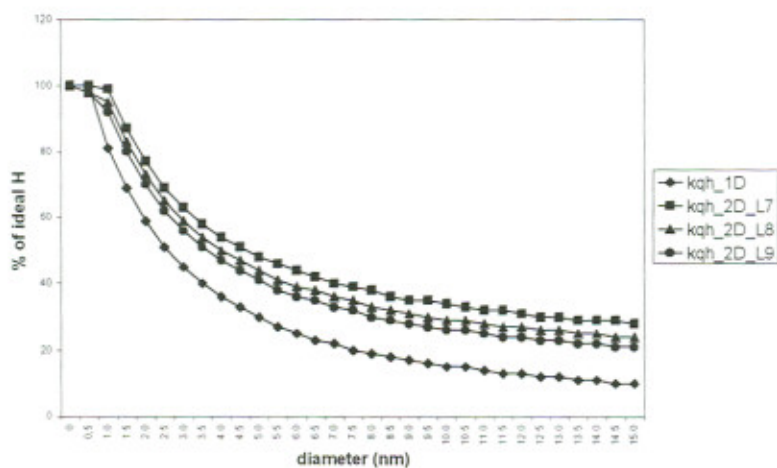


Figure 3: The reduced nucleation field vs. the diameter of the inhomogeneity. (Kronmüller's model for 1-D, 2-D rotation –  $L=7,8,9$  nm; harmonic anisotropy profile;  $\Delta K=1.0K_1$ ).

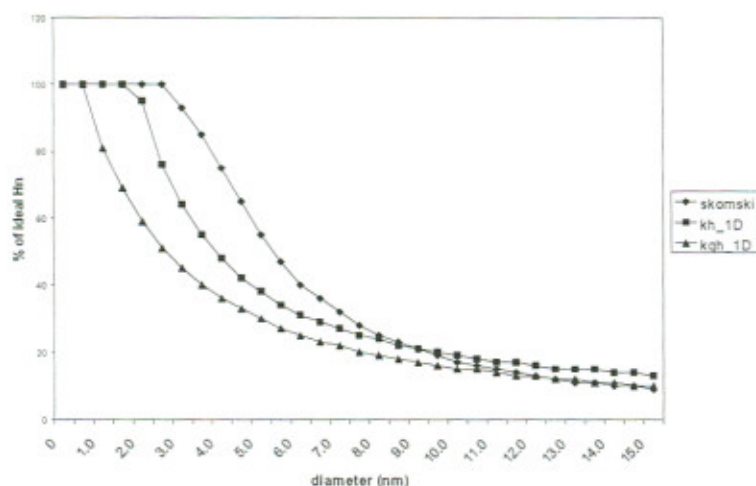


Figure 4: The reduced nucleation field vs. the diameter of the inhomogeneity. Comparison between the Skomski and Coey model and Kronmüller model for 1-D rotation (both harmonic and quasi-harmonic anisotropy profiles);  $\Delta K=1.0K_1$ .

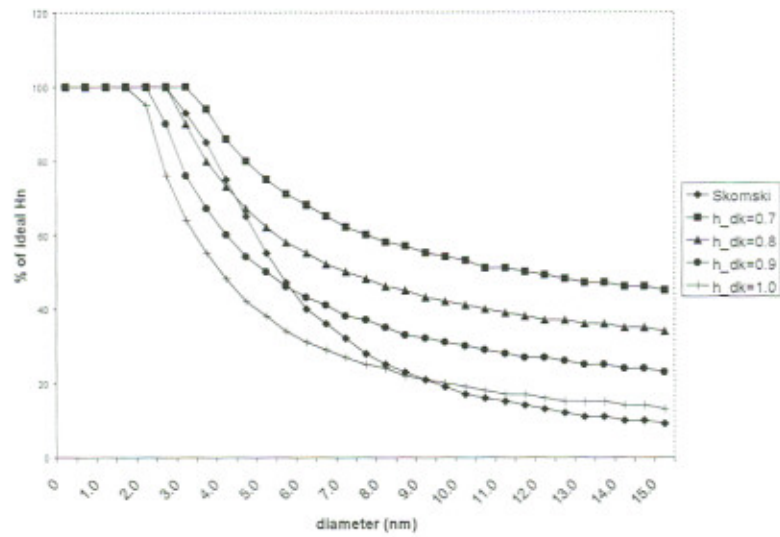


Figure 5: The reduced nucleation field vs. the diameter of the inhomogeneity. Comparison between the Skomski and Coey model and Kronmüller model for 1-D rotation (harmonic anisotropy profile);  $\Delta K=0.70K_1$ ,  $0.80K_1$ ,  $0.90K_1$ ,  $1.0K_1$ .

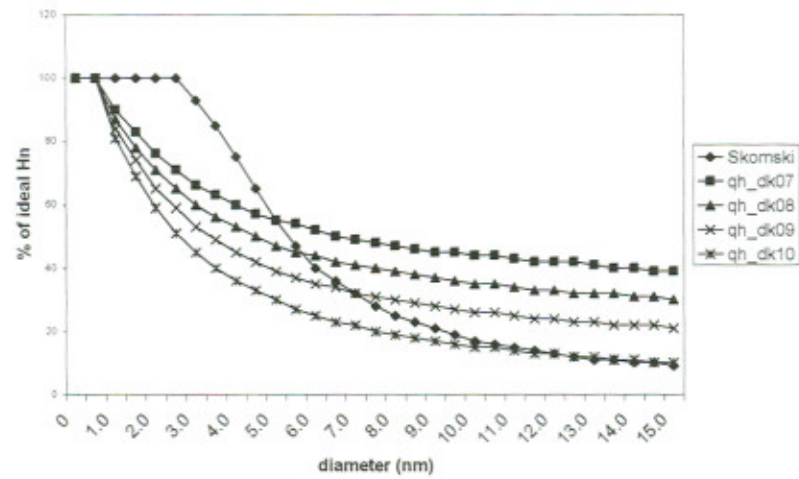


Figure 6: The reduced nucleation field vs. the diameter of the inhomogeneity. Comparison between the Skomski and Coey model and Kronmüller model for 1-D rotation (quasiharmonic anisotropy profile);  $\Delta K=0.70K_1$ ,  $0.80K_1$ ,  $0.90K_1$ ,  $1.0K_1$ .



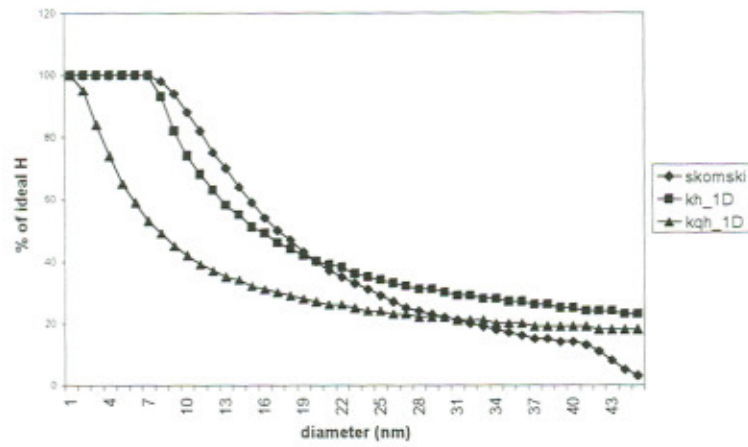


Figure 7: The reduced nucleation field vs. the diameter of the inhomogeneity. Comparison between the Skomski and Coey model and Kronmüller model for 1-D rotation (harmonic and quasiharmonic anisotropy profile);  $\Delta K=0.90K_1$ ; Zn – bonded anisotropic SmFeN parameters are used.

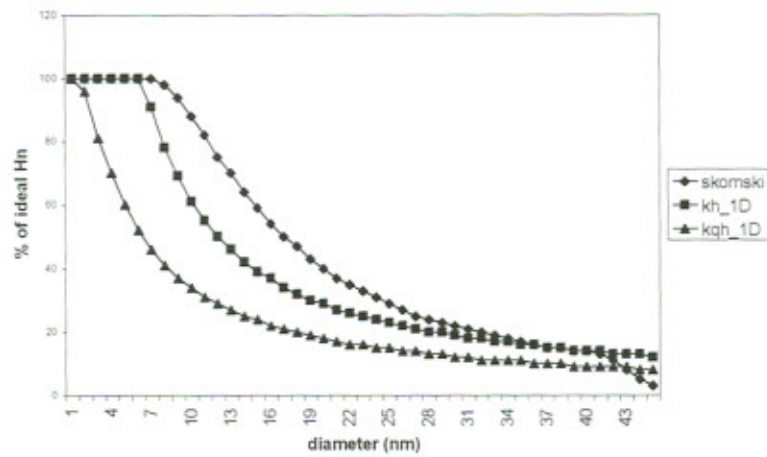


Figure 8: The reduced nucleation field vs. the diameter of the inhomogeneity. Comparison between the Skomski and Coey model and Kronmüller model for 1-D rotation (harmonic and quasiharmonic anisotropy profile);  $\Delta K=1.00K_1$ ; Zn – bonded anisotropic SmFeN parameters are used.