BIOMECHANICS OF THE HUMAN CRANIAL SYSTEM

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1. Introduction

Several researchers have proposed models to study brain injuries due to an external cause such as those resulting from car accidents. The human head - neck system is very complicated structure and various geometrical and material behavior approximations have been used in the modeling studies.

Various approaches are given to the problem both from the engineering and from the medical point of view [1-4]. Among the most important contributions Landkof and Goldsmith [5] performed an analytical and experimental study involving non-destructive, axisymmetric impact on a fluid - filled shell constrained by a viscoelastic artificial neck, and Hickling and Wenner [6] developed a mathematical model using three - dimensional equations of linear viscoelasticity for the brain and the skull to predict the response of a human head to axisymmetric impact.

In previous communications we have presented an extensive investigation of the dynamic characteristics of the human head - neck system [7-9]. The mathematical analysis was based on the expansion of the solution in terms of the Navier eigenvectors and determination of the eigenfrequencies from the frequency equation defined by the existing boundary conditions. In some cases the solution of the problem required the use of arguments from complex analysis and complicated numerical schemes.

In this work we present the analysis concerning the mathematical formulation of the general problem, in which the system under consideration is assumed to constitute a stratified spherical medium. In particular we study the elastic, isotropic and homogeneous human skull under an external stimulus. Our results can be extended to other systems as they are presented in [10]. The methodology we follow includes determination of the eigenvectors of the homogeneous system and computation of the displacement fields when a Dirac force in space and time is applied on the external surface of the human skull.

2. Problem Formulation

The system under consideration is shown in Fig. 1. There exist n elastic spherical layers simulating the several regions of the human skull. Every spherical layer of the model is assumed to be filled with an isotropic, homogeneous, elastic material. The well - posedness of the problem requires the satisfaction of suitable boundary conditions on the discontinuity surfaces of the system.

Our goal is the determination of the response of the system when a Dirac force, in space and time, is applied on it. More precisely, we suppose that the system is subjected to an external force $\boldsymbol{F}(\boldsymbol{r},t)=\boldsymbol{F}_o\delta(\boldsymbol{r}-\boldsymbol{r}_f)\delta(t)$, per unit mass, where $\boldsymbol{F}_o$ is an arbitrary vector and $\boldsymbol{r}_f$ is an arbitrary point of region V_j. It is noticed here that response of the system to more general external forces can be deduced easily from the response to the Dirac force, which constitutes the Green function of the stimulus procedure. In addition, the position of $\boldsymbol{r}_f$ can be arbitrary but its selection depends on the nature of the external force. For the case of an external stimulus, $\boldsymbol{r}_f$ is placed very close to the external skull surface.

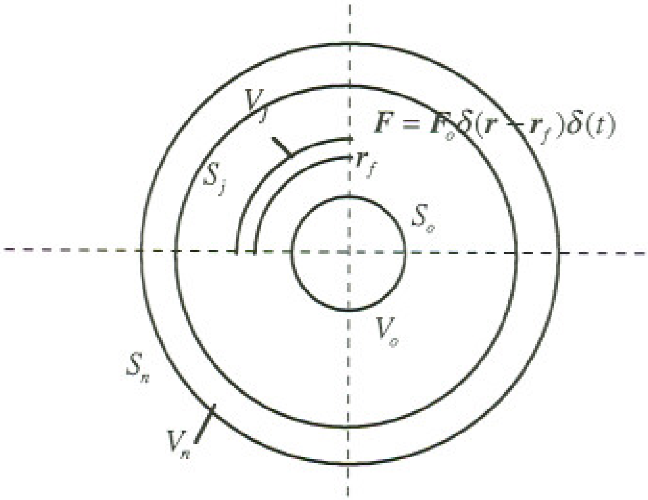

Figure 1: Geometry of the Human Elastic Skull

In order to study the kinematic behavior of the system, we will determine the displacement field of every particular region. Since the region V_i is an elastic one then the motion of this region is characterized by the displacement field $\boldsymbol{u}_i(\boldsymbol{r},t)$ satisfying the time - dependent non - homogeneous equation of elasticity.

$$\mu_i\nabla^2\boldsymbol{u}_i(\boldsymbol{r},t)+(\lambda_i+\mu_i)\nabla(\nabla\cdot\boldsymbol{u}_i(\boldsymbol{r},t))+\rho_i\delta_i^{(j)}\boldsymbol{F}_o\delta(\boldsymbol{r}-\boldsymbol{r}_f)\delta(t)=\rho_i\frac{\partial^2\boldsymbol{u}_i(\boldsymbol{r},t)}{\partial t^2} \qquad (1)$$

where μ_i, λ_i are Lamè's constants, ρ_i is the density of region V_i, ∇ is the del operator, $\delta_i^{(j)}=1$ only if $i=j$ which means that $\boldsymbol{r}_f$ belongs to the V_i region, and repetition of a subscript does not mean addition with respect to it

We apply Fourier transform analysis to the problem defining

$$\hat{\boldsymbol{u}}_i(\boldsymbol{r},\omega)=\int_{-\infty}^{+\infty}\boldsymbol{u}_i(\boldsymbol{r},t)e^{i\omega t}dt. \qquad (2)$$

We suppress the dependence of previous transformed functions on their argument ω and taking advantage of Fourier transform properties we get equations involving the transformed functions from the equations governing the initial fields.

For every region V_i we obtain

$$\mu_i\nabla^2\hat{\boldsymbol{u}}_i(\boldsymbol{r})+(\lambda_i+\mu_i)\nabla(\nabla\cdot\hat{\boldsymbol{u}}_i(\boldsymbol{r}))+\rho_i\omega^2\hat{\boldsymbol{u}}_i(\boldsymbol{r})=-\rho_i\delta_i^{(j)}\boldsymbol{F}_o\delta(\boldsymbol{r}-\boldsymbol{r}_f). \qquad (3)$$

Introducing the velocities $c_{p,i}=\sqrt{\dfrac{\lambda_i+2\mu_i}{\rho_i}}$, $c_{s,i}=\sqrt{\dfrac{\mu_i}{\rho_i}}$ which characterize completely the elastic properties of elastic medium V_i, they give place to the dimensionless quantities $c'_{p,i}=\dfrac{c_{p,i}}{c_{p,n}}$, $c'_{s,i}=\dfrac{c_{s,i}}{c_{p,n}}$. The density ρ_i in every region V_i is replaced by $\rho'_i=\dfrac{\rho_i}{\rho_n}$, and the following dimensionless quantities are defined

$$\boldsymbol{r}'=\frac{\boldsymbol{r}}{\alpha},\ \ \Omega=\frac{\omega\alpha}{c_{p,n}},\ \ \nabla'=\alpha\nabla,\ \ (\alpha=r_n).$$

The differential equations governing materials motion take the dimensionless form

$$c'^2_{s,i}\nabla'^2\hat{\boldsymbol{u}}_i(\boldsymbol{r}')+(c'^2_{p,i}-c'^2_{s,i})\nabla'(\nabla'\cdot\hat{\boldsymbol{u}}_i(\boldsymbol{r}'))+\Omega^2\hat{\boldsymbol{u}}_i(\boldsymbol{r}')=-\delta_i^{(j)}\frac{\boldsymbol{F}_o\alpha}{c^2_{p,n}}\delta(\boldsymbol{r}'-\boldsymbol{r}'_f). \qquad (4)$$

$\forall\ \ i=0,1,2,...,n.$

The boundary conditions for equations (4) are: the exterior surface S_n is stress free, $(\boldsymbol{T}\hat{\boldsymbol{u}}_n(\boldsymbol{r}')\big|_{r'=\alpha}=\boldsymbol{0}$, where $\boldsymbol{T}$ is the surface traction operator [7]). Every surface S_i separating two elastic media must support equal displacement and stress fields from the two sides. Denoting uniformly

$$\left.\begin{array}{l}\hat{\boldsymbol{u}}(\boldsymbol{r}')=\hat{\boldsymbol{u}}_i(\boldsymbol{r}'),\ \text{for}\ \boldsymbol{r}'\in V_i\\ c'_s(\boldsymbol{r}')=c'_{s,i},\ \text{for}\ \boldsymbol{r}'\in V_i\\ c'_p(\boldsymbol{r}')=c'_{p,i},\ \text{for}\ \boldsymbol{r}'\in V_i\end{array}\right\} \qquad (5)$$

the equation governing the displacement fields can be written through only one equation with non-constant coefficients

$$c'_s(\boldsymbol{r}')^2\nabla'^2\hat{\boldsymbol{u}}(\boldsymbol{r}')+\left[c'_p(\boldsymbol{r}')^2-c'_s(\boldsymbol{r}')^2\right]\nabla'(\nabla'\cdot\hat{\boldsymbol{u}}(\boldsymbol{r}'))+\Omega^2\hat{\boldsymbol{u}}(\boldsymbol{r}')=-\frac{\boldsymbol{F}_o\alpha}{c_{p,n}^2}\delta(\boldsymbol{r}'-\boldsymbol{r}'_f),\quad \boldsymbol{r}'\in V. \tag{6}$$

Equation (6) with the above mentioned boundary conditions is a well -posed non-homogeneous boundary value problem.

Its solvability reduces to the corresponding homogeneous problem. More precisely, let us consider the boundary value problem consisting of the equation

$$c'_s(\boldsymbol{r}')^2\nabla'^2\hat{\boldsymbol{u}}(\boldsymbol{r}')+\left[c'_p(\boldsymbol{r}')^2-c'_s(\boldsymbol{r}')^2\right]\nabla'(\nabla'\cdot\hat{\boldsymbol{u}}(\boldsymbol{r}'))+\lambda^2\hat{\boldsymbol{u}}(\boldsymbol{r}')=0,\quad \boldsymbol{r}'\in V \tag{7}$$

and the same set of the boundary conditions satisfied by the solution of the inhomogeneous problem. But this is exactly the problem studied in [7]. The solution of this problem is based on the representation of the displacement field of the skull in terms of the Navier eigenvectors [11]

$$\hat{\boldsymbol{u}}(\boldsymbol{r}')=\sum_{l=1}^{2}\left\{\alpha_n^{m,l}\boldsymbol{L}_n^{m,l}(\boldsymbol{r}')+\beta_n^{m,l}\boldsymbol{M}_n^{m,l}(\boldsymbol{r}')+\gamma_n^{m,l}\boldsymbol{N}_n^{m,l}(\boldsymbol{r}')\right\} \tag{8}$$

Where $\boldsymbol{L},\boldsymbol{M},\boldsymbol{N}$ stand for the Navier eigenvectors, given in [7].

The frequency equation is constructed by imposing the satisfaction of the boundary conditions. This, in matrix form, is written as

$$\boldsymbol{Dx}=\boldsymbol{0} \tag{9}$$

or

$$\begin{bmatrix} A_n^1(r'_1) & A_n^2(r'_1) & 0 & 0 & D_n^1(r'_1) & D_n^2(r'_1) \\ B_n^1(r'_1) & B_n^2(r'_1) & 0 & 0 & E_n^1(r'_1) & E_n^2(r'_1) \\ 0 & 0 & C_n^1(r'_1) & C_n^2(r'_1) & 0 & 0 \\ A_n^1(r'_0) & A_n^2(r'_0) & 0 & 0 & D_n^1(r'_0) & D_n^2(r'_0) \\ B_n^1(r'_0) & B_n^2(r'_0) & 0 & 0 & E_n^1(r'_0) & E_n^2(r'_0) \\ 0 & 0 & C_n^1(r'_0) & C_n^2(r'_0) & 0 & 0 \end{bmatrix}\begin{bmatrix}\alpha_n^{m,1}\\ \alpha_n^{m,2}\\ \beta_n^{m,1}\\ \beta_n^{m,2}\\ \gamma_n^{m,1}\\ \gamma_n^{m,2}\end{bmatrix}=\boldsymbol{0}. \tag{10}$$

The elements of the above matrix are given in [7]. The existence of a non trivial solution for (10) imposes that

$$\det(\boldsymbol{D})=0. \tag{11}$$

The problem (11) can be solved numerically and this leads to a sequence of eigenvalues λ (Ω_n^k, $n = 1,2,3,...;$ $k = 1,2,3,...$) and a sequence of the corresponding eigenvectors $\hat{\boldsymbol{u}}_n^{m,k}(r')$, $\boldsymbol{r}' \in V$, $n = 1,2,...;$ $|\mathrm{m}| \le n;$ $k = 1,2,3,...$

Generalized Sturm - Liouville theory guarantees that the set of functions $\hat{\boldsymbol{u}}_n^m(\boldsymbol{r}')$ constitutes a complete orthogonal set of functions in the space of square integrable functions in V. Orthogonality can be deduced by suitable application of Green's type theorem in space V using Betti's formulae and exploiting the boundary conditions satisfied by these solutions.

Orthogonality is expressed through the equation

$$\int \hat{\boldsymbol{u}}_n^{m,k}(\boldsymbol{r}') \cdot \hat{\boldsymbol{u}}_{n'}^{m',k'^*}(\boldsymbol{r}')d\boldsymbol{r}' = 0, \ \ n \ne n' \ \text{ or } m \ne m' \ \text{ or } k \ne k'. \tag{12}$$

Completeness of $\hat{\boldsymbol{u}}_n^{m,k}(\boldsymbol{r}')$ permits the following representation of the solution $\hat{\boldsymbol{u}}(\boldsymbol{r}')$ of the non-homogeneous problem

$$\hat{\boldsymbol{u}}(\boldsymbol{r}') = \sum_{n,m,k} \delta_n^{m,k}(\boldsymbol{r}'_f)\hat{\boldsymbol{u}}_n^{m,k}(\boldsymbol{r}') \tag{13}$$

where summation extends over all possible values of indices n,m,k, $\delta_n^{m,k}(\boldsymbol{r}'_f)$ are the coefficients to be determined in order for $\hat{\boldsymbol{u}}(\boldsymbol{r}')$ to be found which depend on the location of the impact.

Introducing (13) in (8), we find that

$$\sum_{n,m,k} \delta_n^{m,k}(\boldsymbol{r}'_f)(\Omega^2 - \Omega_n^{k^2})\hat{\boldsymbol{u}}_n^{m,k}(\boldsymbol{r}') = -\frac{\boldsymbol{F}_o\alpha}{c_{p,n}^2}\delta(\boldsymbol{r}' - \boldsymbol{r}'_f). \tag{14}$$

Using the orthogonality of $\hat{\boldsymbol{u}}_n^{m,k}(\boldsymbol{r}')$ we obtain after projecting (14) on $\hat{\boldsymbol{u}}_n^{m,k}(\boldsymbol{r}')$ and integrating over V

$$\delta_n^{m,k}(\boldsymbol{r}'_f) = -\frac{\alpha\boldsymbol{F}_o \cdot \hat{\boldsymbol{u}}_n^{m,k^*}(\boldsymbol{r}'_f)}{c_{p,n}^2(\Omega^2 - \Omega_n^{k^2})} \frac{1}{\int_V |\hat{\boldsymbol{u}}_n^{m,k}(\boldsymbol{r}')|^2 d\boldsymbol{r}'}. \tag{15}$$

Denoting $\|\hat{\boldsymbol{u}}_n^{m,k}\| = \left(\int_V |\boldsymbol{u}_n^{m,k}(\boldsymbol{r}')|^2 d\boldsymbol{r}'\right)^{1/2}$, the norm of $\hat{\boldsymbol{u}}_n^{m,k}(\boldsymbol{r}')$ in the space of square integrable functions, equation (13) can be written as

$$\hat{\boldsymbol{u}}(\boldsymbol{r}') = -\frac{\alpha\boldsymbol{F}_o}{c_{p,n}^2} \cdot \sum_{n,m,k} \frac{\hat{\boldsymbol{u}}_n^{m,k}(\boldsymbol{r}'_f)\hat{\boldsymbol{u}}_n^{m,k}(\boldsymbol{r}')}{(\Omega^2 - \Omega_n^{k^2})\|\hat{\boldsymbol{u}}_n^{m,k}\|^2}. \tag{16}$$

We can now determine $\boldsymbol{u}(\boldsymbol{r}',t)$ by taking the inverse Fourier transform of (16).

Consequently

$$\boldsymbol{u}(\boldsymbol{r}',t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{\boldsymbol{u}}(\boldsymbol{r}',\omega)e^{-i\omega t}d\omega=$$
$$=-\frac{1}{2\pi}\boldsymbol{F}_o\cdot\sum_{n,m,k}\hat{\boldsymbol{u}}_n^{m,k^*}(\boldsymbol{r}'_f)\hat{\boldsymbol{u}}_n^{m,k}(\boldsymbol{r}')\frac{1}{\left\|\hat{\boldsymbol{u}}_n^{m,k}\right\|^2}\int_{-\infty}^{+\infty}\frac{e^{-i\omega t}}{\omega^2-\omega_n^{k^2}}d\omega \tag{17}$$

where $\omega_n^k=\dfrac{c_{p,n}}{\alpha}\Omega_n^k$.

Using complex analysis integration arguments, we finally obtain

$$\boldsymbol{u}(\boldsymbol{r}',t)=-\frac{i}{2}\boldsymbol{F}_o\cdot\sum_{n,m,k}\hat{\boldsymbol{u}}_n^{m,k^*}(\boldsymbol{r}'_f)\hat{\boldsymbol{u}}_n^{m,k}(\boldsymbol{r}')\frac{e^{i\omega_n^k t}}{\left\|\hat{\boldsymbol{u}}_n^{m,k}\right\|^2\omega_n^k}. \tag{18}$$

The explicit form of $\hat{\boldsymbol{u}}_n^{m,k}(\boldsymbol{r}')$ in equation (18) is given by the solution of (10). As a matter of fact

$$\hat{\boldsymbol{u}}_n^{m,k}(\boldsymbol{r}')=\hat{\boldsymbol{u}}_{n,i}^{m,k}(\boldsymbol{r}')\ \text{ for }\ \boldsymbol{r}'\in V_i \tag{19}$$

with

$$\hat{\boldsymbol{u}}_{n,i}^{m,k}(\boldsymbol{r}')=\sum_{l=1}^{2}\left\{\alpha_{n,i}^{m,l}\boldsymbol{L}_{n,i}^{m,l}(\boldsymbol{r}')+\beta_{n,i}^{m,l}\boldsymbol{M}_{n,i}^{m,l}(\boldsymbol{r}')+\gamma_{n,i}^{m,l}\boldsymbol{N}_{n,i}^{m,l}(\boldsymbol{r}')\right\}\Big|_{\Omega=\Omega_n^k}.$$

The norm $\left\|\hat{\boldsymbol{u}}_n^{m,k}\right\|$ appearing in (18) can be calculated as it is shown in [10]

$$\int_{V_i}\hat{\boldsymbol{u}}_{n,i}^{m,k}(\boldsymbol{r}')\cdot\hat{\boldsymbol{u}}_{n,i}^{m,k^*}(\boldsymbol{r}')d\boldsymbol{r}'=\sum_{l=1}^{2}\left\{\begin{array}{l}\left|\alpha_{n,i}^{m,l}\right|^2\left\|\boldsymbol{L}_{n,i}^{m,l}(\boldsymbol{r}')\right\|^2+\left|\beta_{n,i}^{m,l}\right|^2\left\|\boldsymbol{M}_{n,i}^{m,l}(\boldsymbol{r}')\right\|^2+\left|\gamma_{n,i}^{m,l}\right|^2\left\|\boldsymbol{N}_{n,i}^{m,l}(\boldsymbol{r}')\right\|^2\\+2\,\mathrm{Re}\left\{\alpha_{n,i}^{m,l}\gamma_{n,i}^{m,l^*}\int_{V_i}\boldsymbol{L}_{n,i}^{m,l}(\boldsymbol{r}')\cdot\boldsymbol{N}_{n,i}^{m,l^*}(\boldsymbol{r}')d\boldsymbol{r}'\right\}\end{array}\right\}\Bigg|_{\Omega=\Omega_n^k} \tag{33}$$

where $\left\|\boldsymbol{L}_{n,i}^{m,l}(\boldsymbol{r}')\right\|$, $\left\|\boldsymbol{M}_{n,i}^{m,l}(\boldsymbol{r}')\right\|$, $\left\|\boldsymbol{N}_{n,i}^{m,l}(\boldsymbol{r}')\right\|$ stand for the L^2 - norms of the corresponding functions in space V_i. Details on the expressions in (20) are given in [10].

3. Numerical Results

The homogeneous problem is solved and the eigenfrequencies Ω_n^k and the corresponding vectors of coefficients $\boldsymbol{x}$ from the frequency equation can be found. Then the eigenvector can be easily determined from the expansion in terms of Navier eigenvectors. A quicksort [12] algorithm is used to order Ω_n^k in ascending order. Equation (18) is used to compute the displacement fields under the state of an external Dirac force $\boldsymbol{F}_0$.

The parameters for the human skull used in our model are

$E = 1.379\times10^9 N/m^2,\ \nu = 0.25,\ \rho = 2.132\times10^3 kg/m^3$

$r_1 = 0.082m,\ r_0 = 0.076m$,

and the parameters for the external force are

$\boldsymbol{r}_f = (r_1, 0, 0),\ \ \boldsymbol{F}_0 = (1,0,0)$.

The results obtained for the displacement fields are shown in Fig. 2 and 3 as a function of time for $\theta = 0$ and $\theta = \pi/2$ respectively. The time scale displayed corresponds to $0 \le t \le 1000\ \mu$sec and in the enclosed framed figure this is extended up to 500 μsec. The corresponding displacement fields as function of $0 \le \varphi \le 2\pi$ for discrete time steps are shown in Fig. 4. The number of eigenfrequencies (k') used in (18) strongly influences the convergence of the computed displacement fields as it is shown in Fig. 5. In our computations we have used the first 44 eigenfrequencies and we have computed 45 eigenfrequencies for $0 \le n \le 20$ and $0.0 \le \Omega_n^k \le 10.0$.

4. Conclusions

We have presented a method for the computation of the response of the human skull to an external stimulus. The method is based on the fact that the elastic medium is stratified and can be extended to any morphology of the human head - neck system. However, the complexity of its geometry and material behavior lead to much more difficult mathematical and numerical manipulations which are cited in [10].

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Figure 2: Displacements u_r, u_φ, u_θ as a function of time
$(r = r_1,\ \varphi = \pi/2,\ \theta = 0)$.

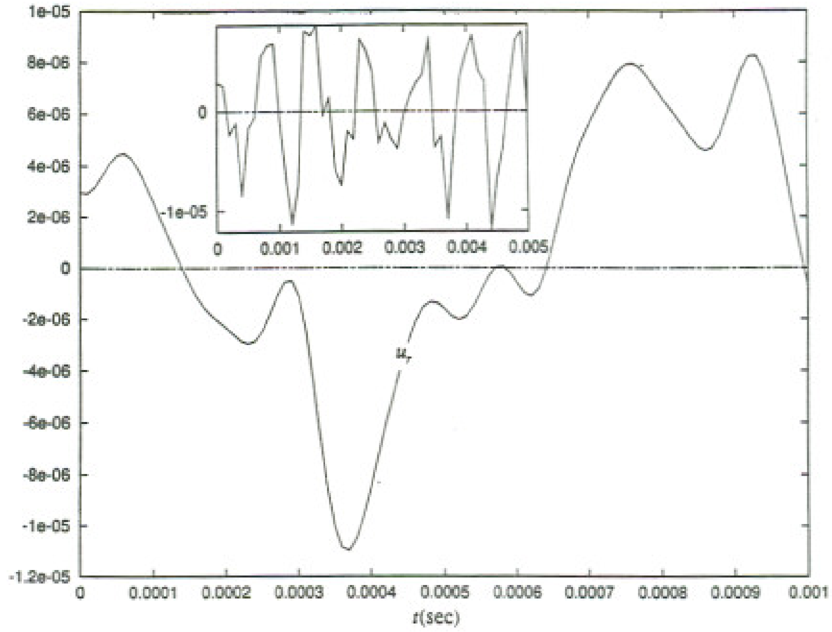


Figure 3: Displacements u_r, u_φ, u_θ as a function of time
$(r = r_1,\ \varphi = \pi/2,\ \theta = \pi/2)$.

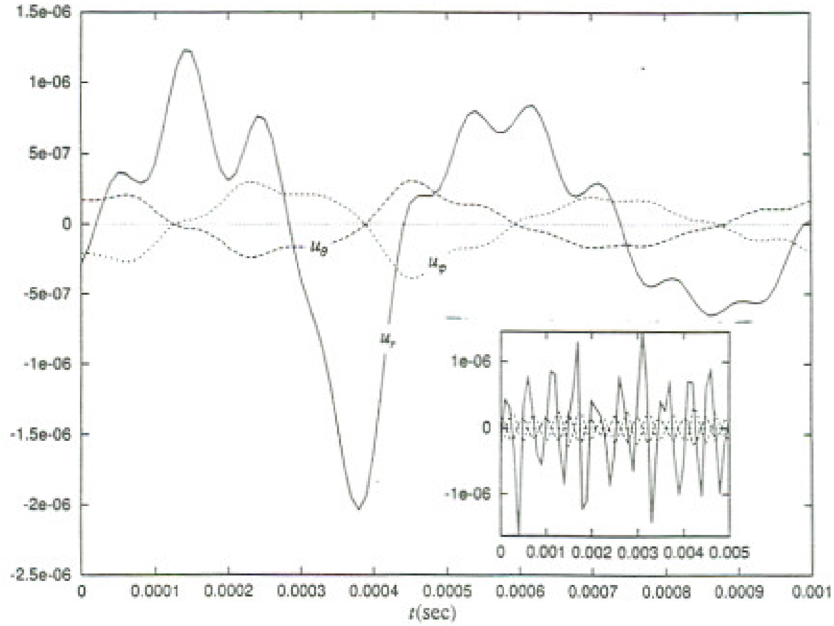


Figure 4: Displacements u_r, u_φ, u_θ for $r = r_1$
(A: $\theta = \pi$, $t = 0.0004$sec, B: $\theta = \pi/2$, $t = 0.0003$sec)

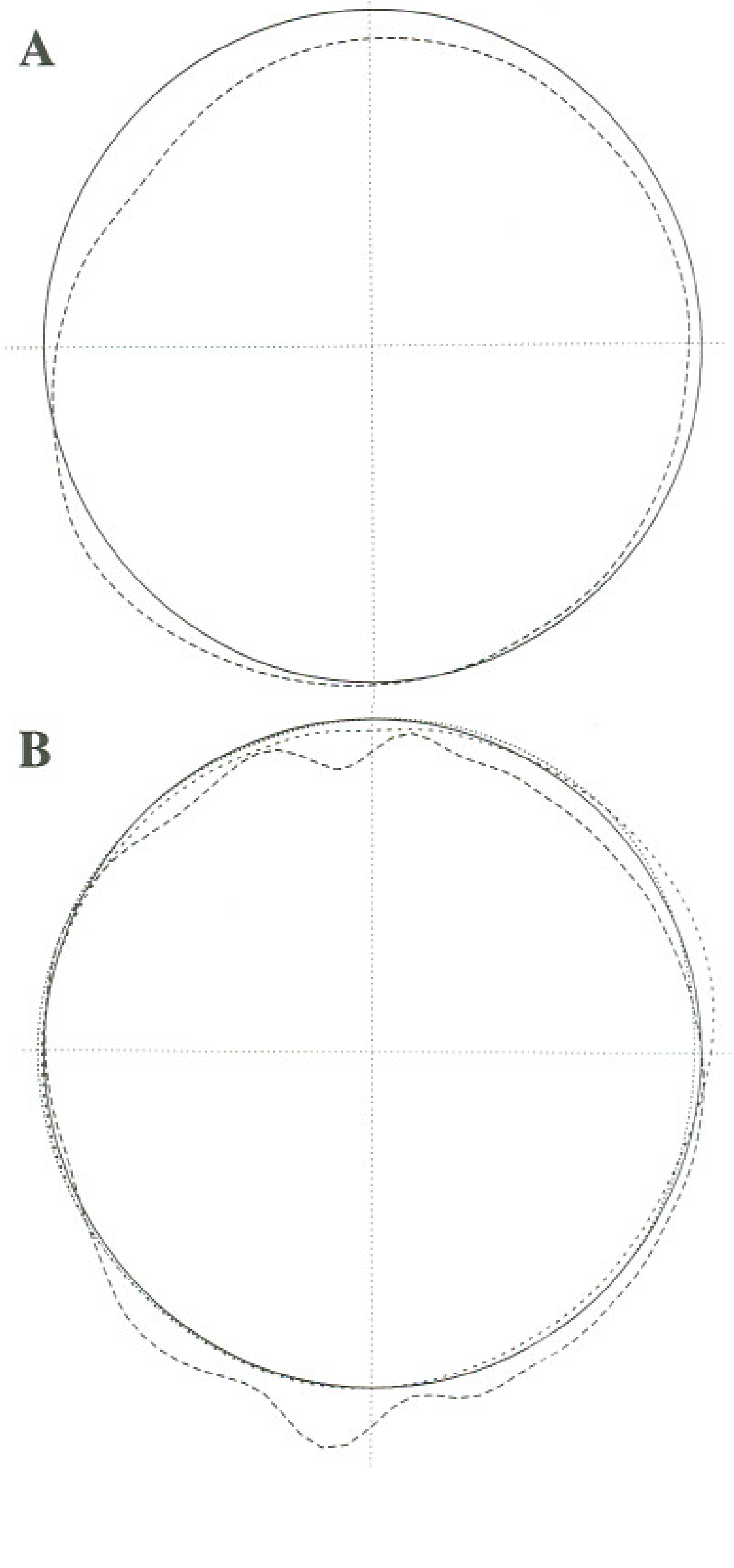
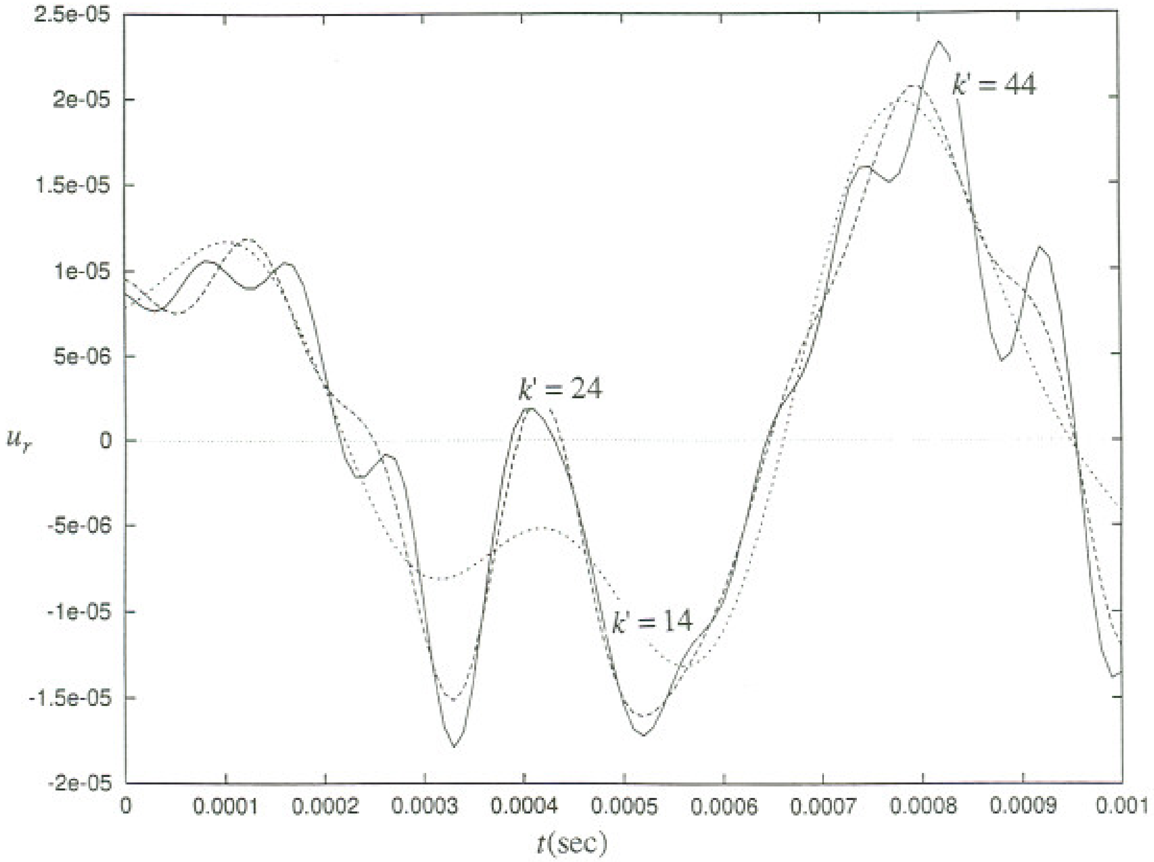


Figure 5: Convergence of u_r as a function of time t for $k' = 14, 24, 44$.
$(r = r_1,\ \theta = \pi/2,\ \varphi = \pi/2)$



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