Localization principles in set theory

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Abstract

ZFC is an absolutistic theory in the sense that its axioms are asserted to hold in the (absolute) universe V. This calls for absolutistic solutions to equations like $\mathcal{P}(\omega) = x$ and $|\mathcal{P}(\omega)| = x$, a demand that looks however hopeless if we judge by the experience gained up to now. A relativistic/localistic variant on the other hand would consist in claiming that the axioms of ZFC make sense only locally, just as in modern physics measurements of fundamental magnitudes, like time, mass and length, make sense only with respect to local reference frames. This leads to the intuition that the strong axioms of Powerset and Replacement may not make sense in V itself, but only in "reference frames". These reference frames are just transitive set-models of ZFC, which in compensation are supposed to exist everywhere across the universe. That is, we are led to the localization principle *Loc*(ZFC):= "every set belongs to a transitive model of ZFC". *Loc*(ZFC) coupled with a number of elementary facts like pair, union, infinity, etc., form the "local variant of ZFC" or LZFC.

In this talk I will discuss some consequences, prospects and extensions of LZFC, as well as of ZFC+LZFC, since *Loc*(ZFC), despite its motivation, remains compatible with ZFC, with low consistency strength.

In LZFC the emphasis is shifted from (large) cardinals to transitive models (as analogues of inaccessible cardinals) and their classification according to properties that imitate other large cardinal properties. E.g. by iterating *Loc*(ZFC) we define α-Mahlo models. Further we define the stronger and stronger classes of Π^1_1-indescribable models, extendible models, embeddable models, critical and strongly critical ones. We examine the consistency strength of their existence. We examine also the compatibility of strongly critical models with $V = L$. Finally we consider Vopěnka's Principle (VP) in the context of LZFC and prove that LZFC+VP restores ZFC.

In the talk I shall survey published work, work which is still under review, as well as work in progress.