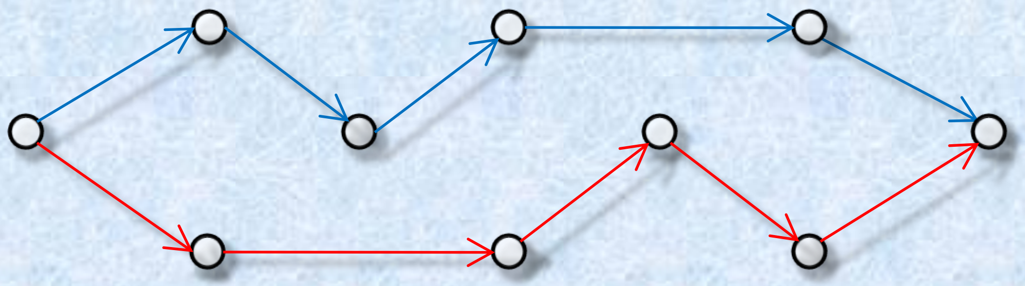
Efficient Algorithms for Reachability and Path-Selection Problems

http://www.icte.uowm.gr/lgeorg/RPS/



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Public Benefit Foundation
Research Projects 2010

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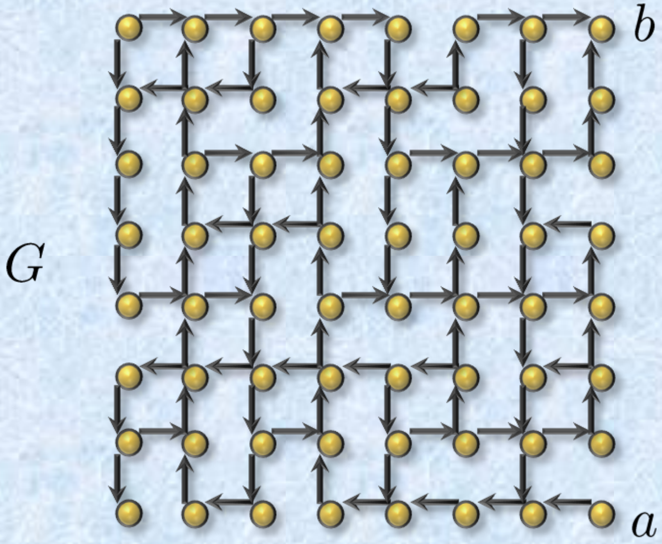
Reachability



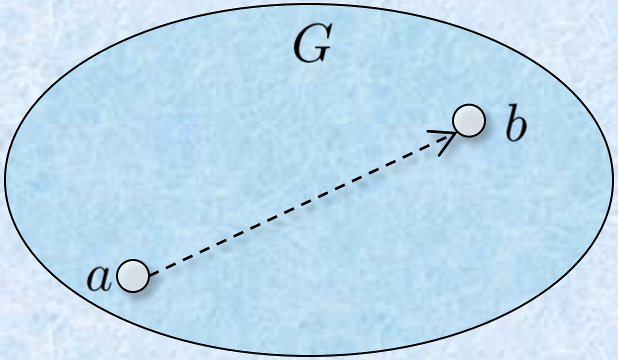
Reachability Query :

Is vertex b reachable from vertex a ?

(Is there a path in G from a to b ?)

Goal: Construct a Data Structure that answers reachability queries **efficiently**

Reachability



Reachability Query :

Is vertex b reachable from vertex a ?

(Is there a path in G from a to b ?)

Goal: Construct a Data Structure that answers reachability queries **efficiently**

Efficiency of a Data Structure: $\langle s(n), q(n) \rangle$

$s(n)$ storage space

$q(n)$ query time

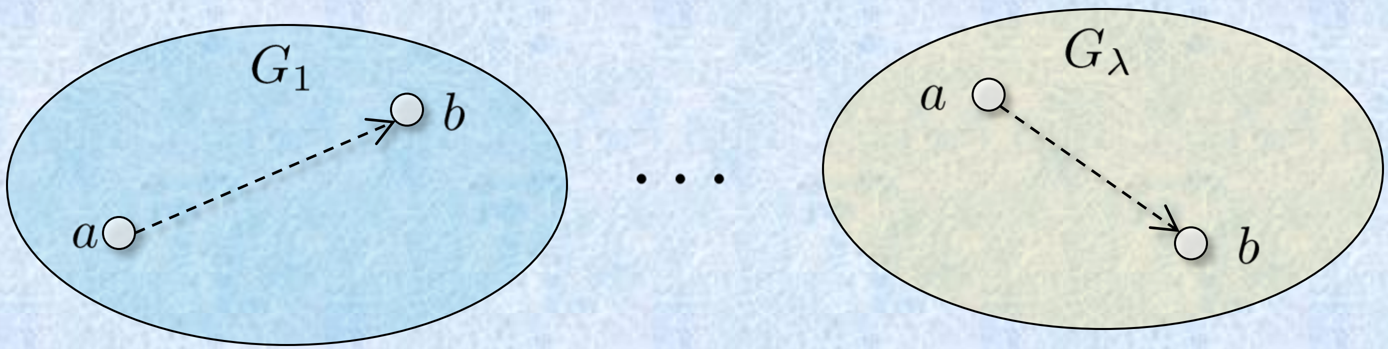
Easy : Efficiency $\langle n^2, 1 \rangle$ or $\langle m+n, m+n \rangle$

Hard : Efficiency close to $\langle m+n, 1 \rangle$

So far achieved only for restricted graph classes (e.g., planar graphs)

Join-Reachability

Collection of graphs $\mathcal{G} = \{G_1, G_2, \ldots, G_\lambda\}$



Join-Reachability Query :

Report all vertices that reach b in all graphs $G_i \in \mathcal{G}$

(Vertices a such that there is a $a \rightsquigarrow b$ path in all $G_i \in \mathcal{G}$)

Efficiency of a Data Structure: $\langle s(n), q(n,k) \rangle$

$s(n)$ storage space

$q(n,k)$ time to report k vertices

Join-Reachability

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Join-Reachability Query :

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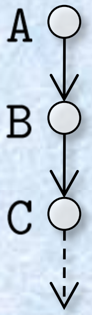
Applications: Graph Algorithms, Data Bases

Example: Rank Aggregation



Ranking 1

1. Item A
2. Item B
3. Item C

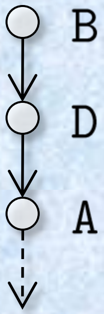
...

. . .

Ranking λ

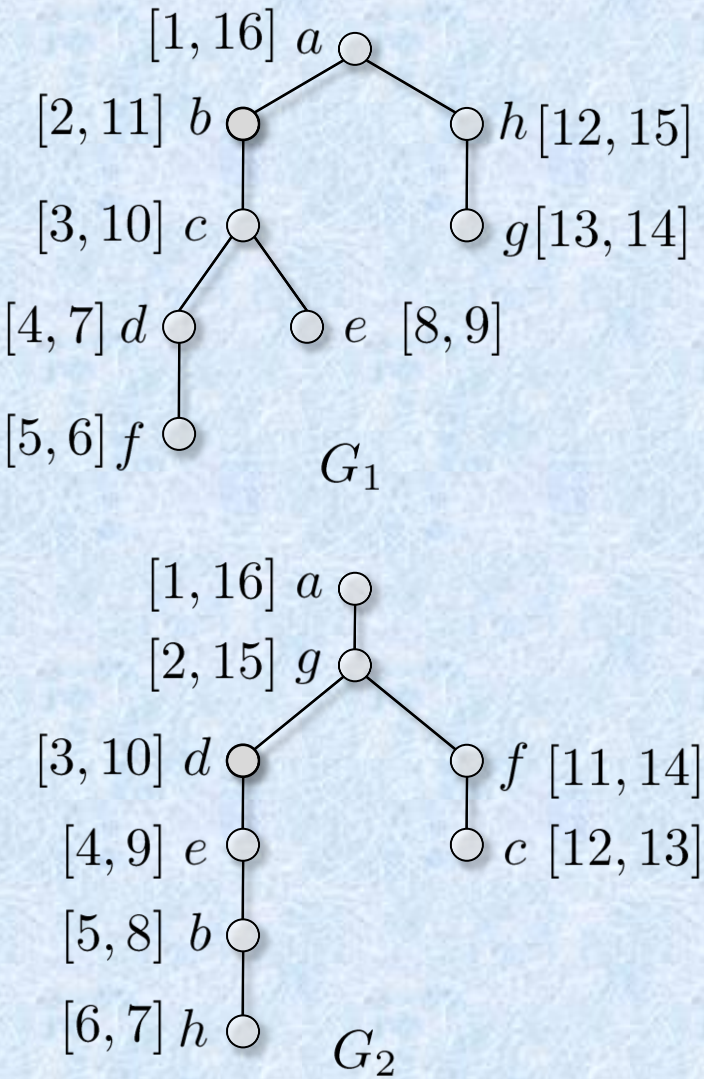
1. Item B
2. Item D
3. Item A

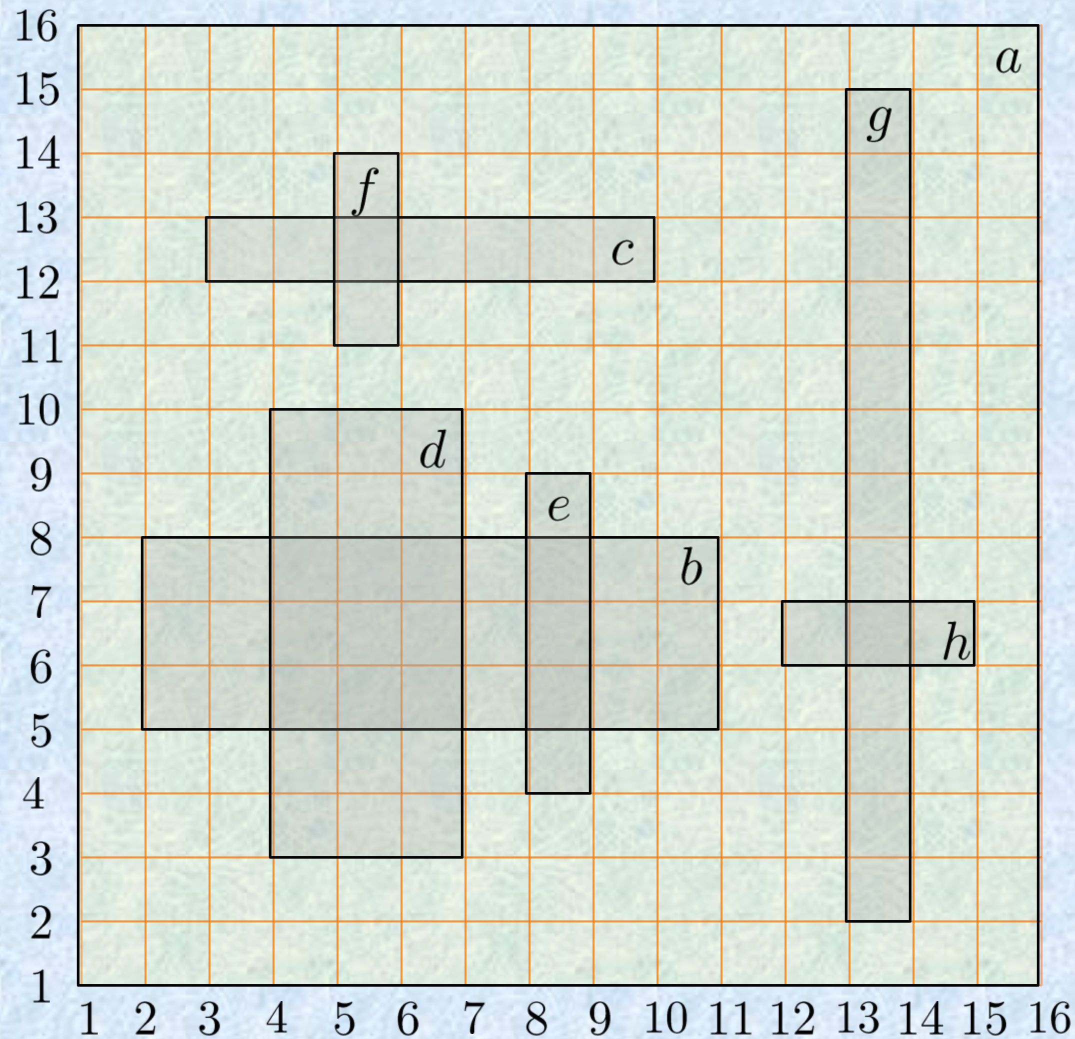
...



Given a collection of rankings of some items, we would like to report fast all items ranked higher than a query item in all rankings.

Join-Reachability





Main Idea: Geometric mapping of simple graphs

Join-Reachability

Given two digraphs G_1 and G_2 with n vertices we can construct join-reachability data structures with the following efficiency:

(a) $\langle n, k \rangle$ when G_1 is an unoriented tree and G_2 is an unoriented dipath.

(b) $\langle n, \log n + k \rangle$ when G_1 is an out-tree and G_2 is an unoriented tree.

(c) $\langle n \log^{\varepsilon} n, \log\log n + k \rangle$ (for any constant $\varepsilon > 0$), when G_1 and G_2 are unoriented trees.

(d) $\langle n \log n, k \log n \rangle$ when G_1 is planar digraph and G_2 is an unoriented tree.

(e) $\langle n \log^2 n, k \log^2 n \rangle$ when both G_1 and G_2 are planar digraphs.

(f) $\langle n\kappa_1, k \rangle$ when G_1 is a general digraph that can be covered with κ_1 vertex-disjoint dipaths and G_2 is an unoriented tree.

(g) $\langle n(\kappa_1 + \log n), k\kappa_1 \log n \rangle$ or $\langle n\kappa_1 \log n, k \log n \rangle$ when G_1 is a general digraph that can be covered with κ_1 vertex-disjoint dipaths and G_2 is planar digraph.

(h) $\langle n(\kappa_1 + \kappa_2), \kappa_1\kappa_2 + k \rangle$ or $\langle n\kappa_1\kappa_2, k \rangle$ when each G_i, $i = 1, 2$, is a digraph that can be covered with κ_i vertex-disjoint dipaths.

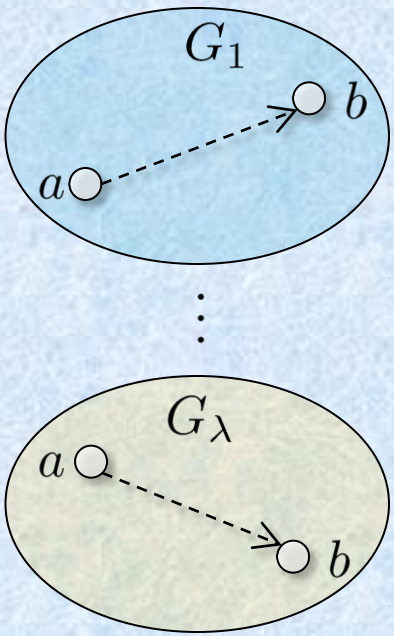
Join-Reachability

Collection of graphs $\mathcal{G} = \{G_1, G_2, \ldots, G_\lambda\}$

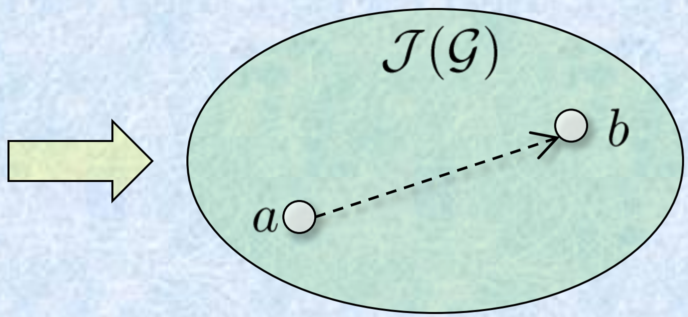
Join-Reachability Query :

Report all vertices that reach b in all graphs $G_i \in \mathcal{G}$

(Vertices a such that there is a $a \rightsquigarrow b$ path in all $G_i \in \mathcal{G}$)



Join-Reachability Graph



$a \rightsquigarrow b$ in $\mathcal{J}(\mathcal{G})$

$\Updownarrow$

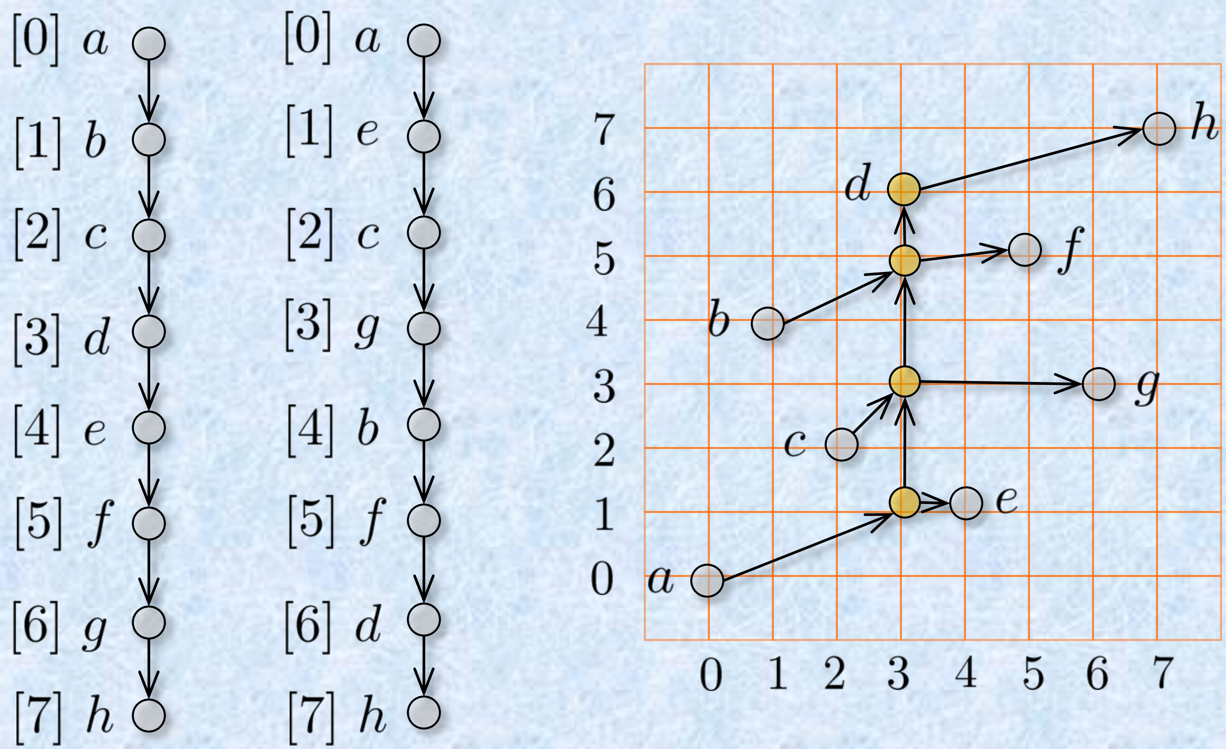
$a \rightsquigarrow b$ in all $G_i \in \mathcal{G}$

Computing the smallest $\mathcal{J}(\mathcal{G})$ (in terms of the number of arcs plus vertices) is **NP-hard**

Join-Reachability

Collection of graphs $\mathcal{G} = \{G_1, G_2, \ldots, G_\lambda\}$

Construction of a compact join-reachability graph $\mathcal{J}(\mathcal{G})$



Join-Reachability

Given two digraphs G_1 and G_2 with n vertices, the following bounds on the size of the join-reachability graph $\mathcal{J}(\{G_1, G_2\})$ hold:

(a) $\Theta(n \log n)$ in the worst case when G_1 is an unoriented tree and G_2 is an unoriented dipath.

(b) $O(n \log^2 n)$ when both G_1 and G_2 are unoriented trees.

(c) $O(n \log^2 n)$ when G_1 is a planar digraph and G_2 is an unoriented dipath.

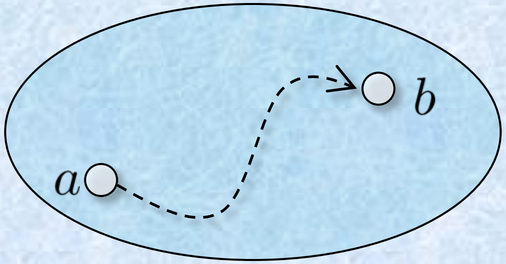
(d) $O(n \log^3 n)$ when both G_1 and G_2 are planar digraphs.

(e) $O(\kappa_1 n \log n)$ when G_1 is a digraph that can be covered with κ_1 vertex-disjoint dipaths and G_2 is an unoriented dipath.

(f) $O(\kappa_1 n \log^2 n)$ when G_1 is a digraph that can be covered with κ_1 vertex-disjoint dipaths and G_2 is a planar graph.

(g) $O(\kappa_1 \kappa_2 n \log n)$ when each G_i, $i = 1, 2$, is a digraph that can be covered with κ_i vertex-disjoint dipaths.

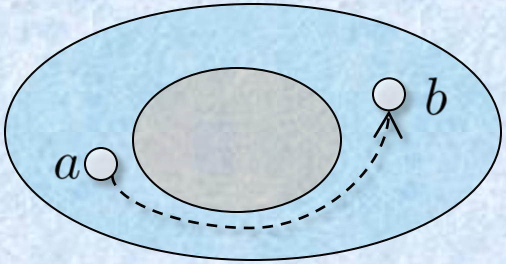
Path-Selection



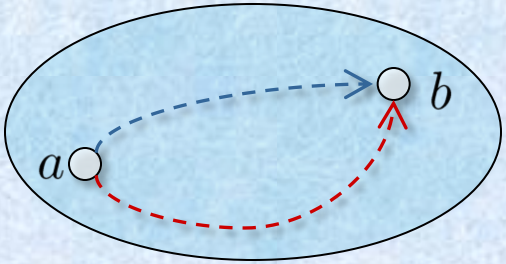
Compute paths in a graph G so that certain requirements are satisfied

E.g.

Avoid a forbidden part of G



Disjoint paths

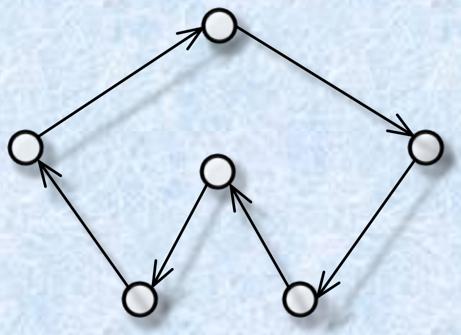


Applications: Communications, Scheduling, VLSI design

Vertex Connectivity

Strongly connected digraph $G = (V, E)$

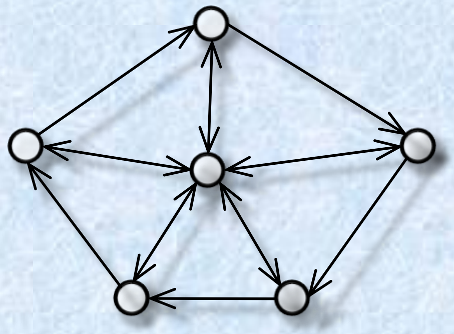
contains an $s \rightsquigarrow t$ path for any pair $s, t \in V$



k-vertex connected digraph $G = (V, E)$

the removal of any subset $X \subseteq V, |X| \leq k - 1$

leaves the graph strongly connected



Basic problems :

- Compute vertex connectivity (largest k such that G is k-vertex connected)
- Test if the given digraph is k-vertex connected

Vertex Connectivity

Basic problems :

- Compute vertex connectivity $\kappa =$ largest k such that G is k-vertex connected

  $O((n + \min\{\kappa^{5/2}, \kappa n^{3/4}\})m)$ [Gabow 2006]

- Test if the given digraph is k-vertex connected

  $O(\min\{k^3 + n, kn\}m)$
  $O(mn)$ with error probability $1/2$
  [Henzinger, Rao and Gabow 2000]

  $O((M(n) + nM(k)) \log n)$ with error probability $1/n$
  $O((M(n) + nM(k))k)$ expected
  [Cheriyan and Reif 1994]

$n = |V|, m = |A|$

$M(n)$ = matrix multiplication time $(= O(n^{2.376}))$

Vertex Connectivity

Undirected graphs: $O(m+n)$ algorithms for testing

$k=2$ [Tarjan 1972]

$k=3$ [Hopcroft and Tarjan 1973]

Directed graphs: $O(m+n)$ algorithm for testing $k=2$?

$n=|V|, m=|A|$

Results

$O(m+n)$-time algorithm for testing 2-vertex connectivity

$\Downarrow$

$O(n)$-space data structure :

compute two vertex-disjoint s-t paths in $O(\log^2 n)$ time

report the two paths, P and Q, in $O(|P|+|Q|)$ time

$n = |V|, m = |A|$

Vertex Connectivity

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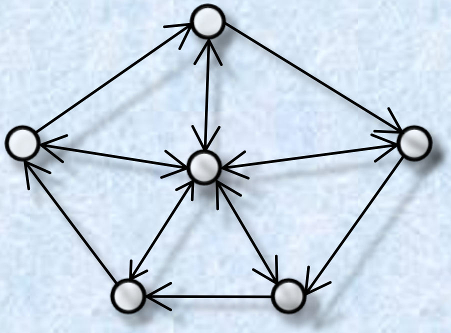


From Menger's theorem :

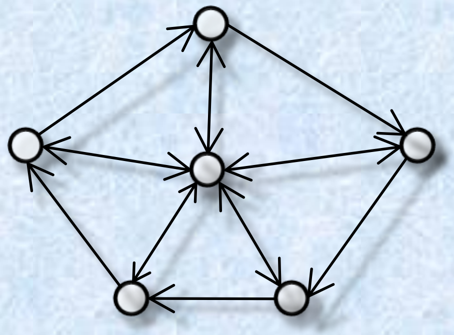
G is k-vertex connected $\Longleftrightarrow$ G contains k vertex-disjoint s-t paths for any $s, t \in V$

2-Vertex Connectivity

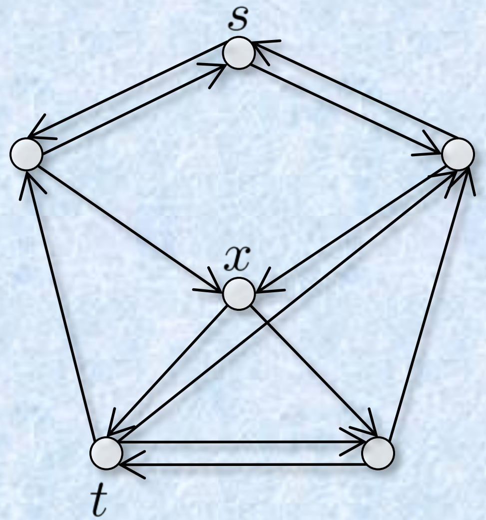
2-vertex connected digraph $G = (V, E)$

the removal of at most one vertex

leaves the graph strongly connected



If G is strongly connected but not 2-vertex connected :

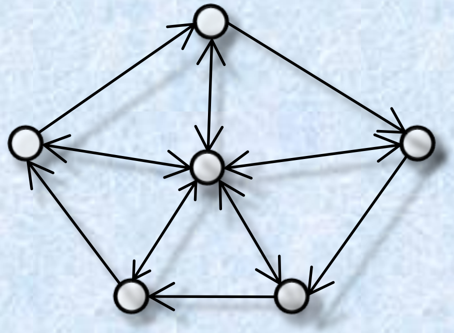


There are $s, t \in V$ such that all

s-t paths contain a common vertex

$x \neq s, t$

2-Vertex Connectivity



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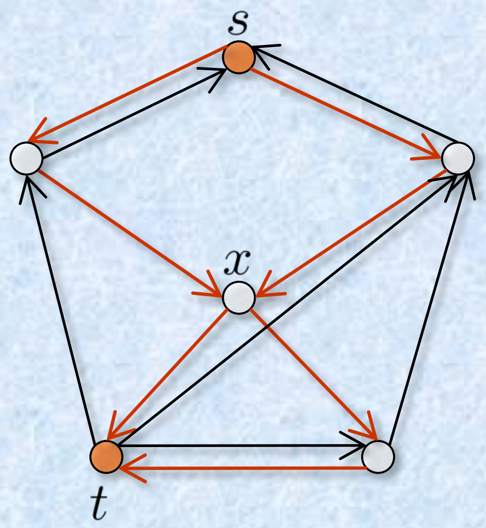

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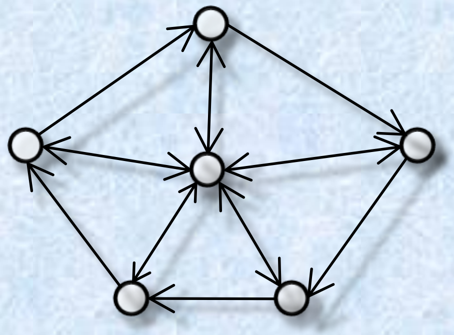

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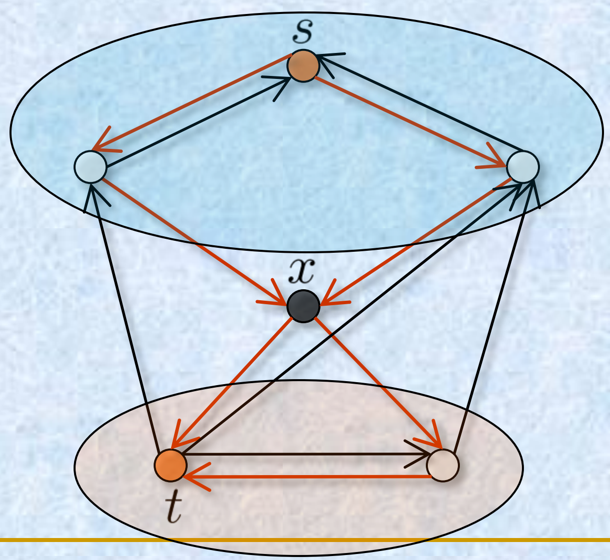

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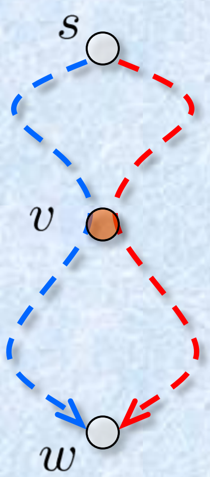
$x \neq s, t$

Flowgraphs and Dominators

Flowgraph $G(s) = (V, E, s)$: all vertices are reachable from start vertex s

v dominates w if every path from s to w includes v



$dom(w)$: set of vertices that dominate w

Trivial dominators : $s, w \in dom(w)$

Application areas : Program optimization, VLSI testing, theoretical biology, distributed systems, constraint programming

Flowgraphs and Dominators

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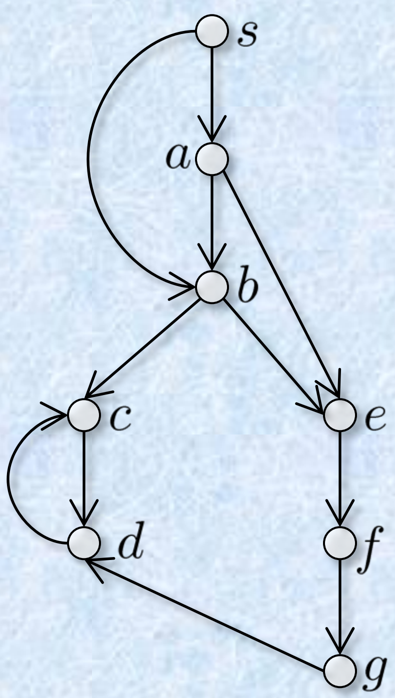
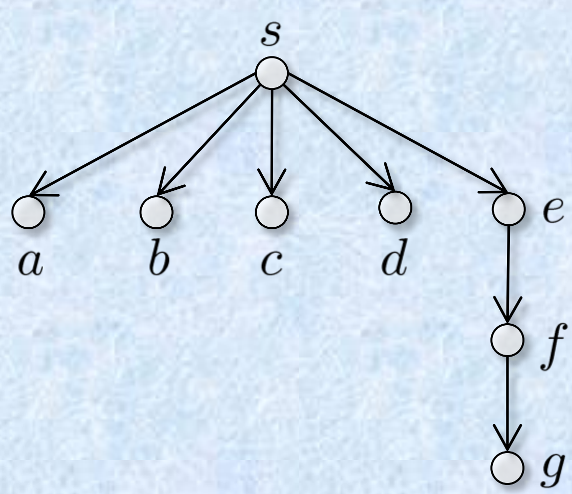
$G(s)$



dominator tree of $G(s)$



$O(m\alpha(m, n))$ algorithm: [Lengauer and Tarjan '79]

$O(m + n)$ algorithms:

[Alstrup, Harel, Lauridsen, and Thorup '97]

[Buchsbaum, Kaplan, Rogers, and Westbrook '04]

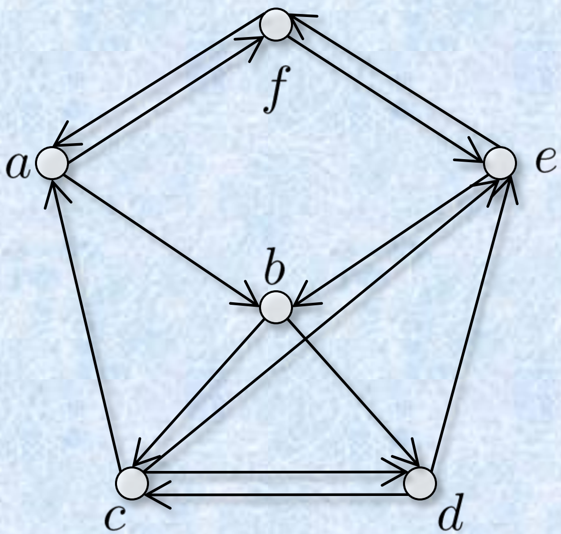
[G., and Tarjan '04]

2-Vertex Connectivity

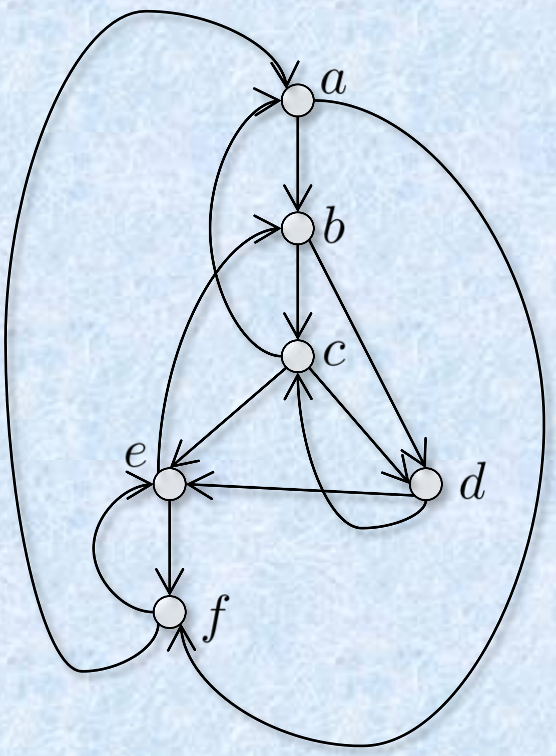
Main Idea : Compute dominators in $G(s)$ and $G^r(s)$ for arbitrary $s \in V$

G^r : has reversed arcs

G



$G(a)$



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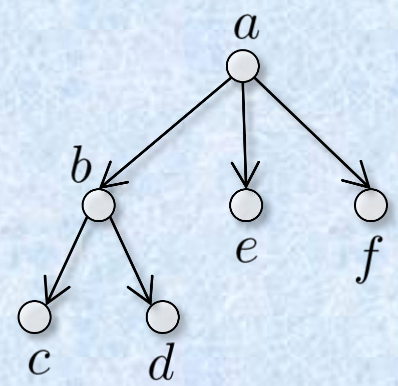

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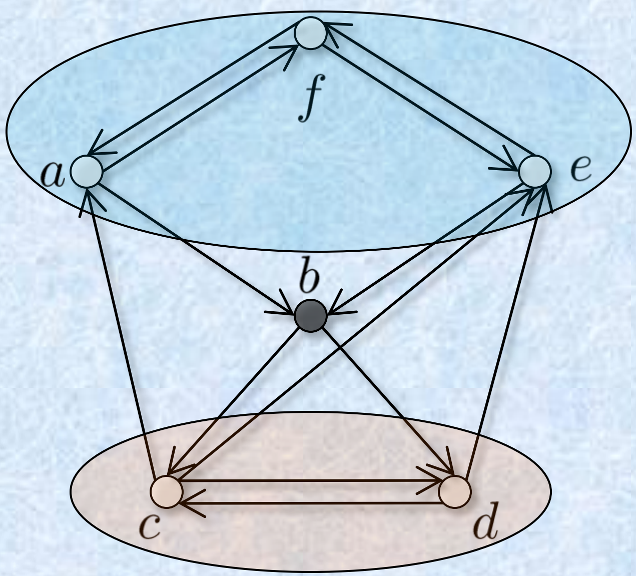

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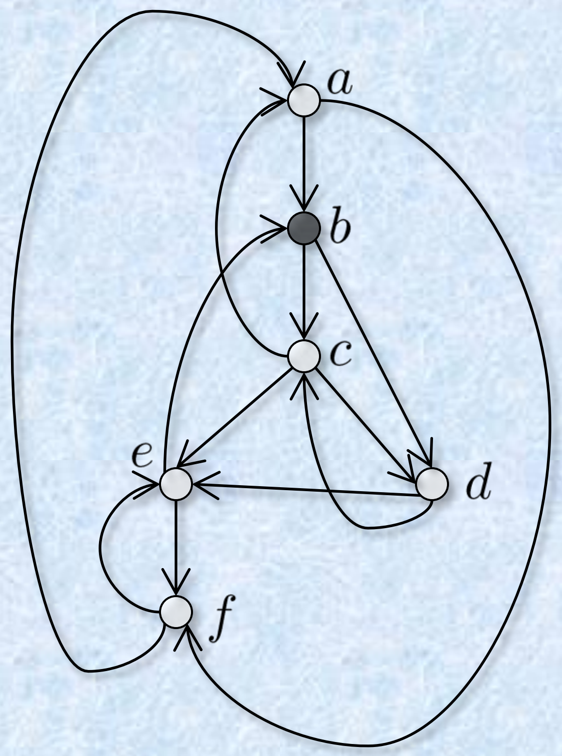

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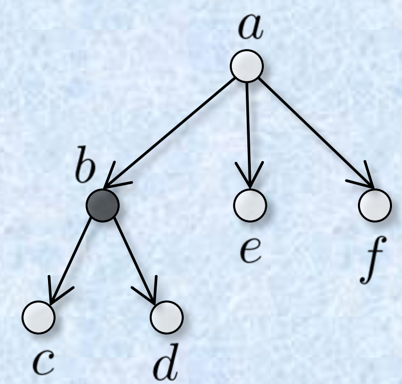

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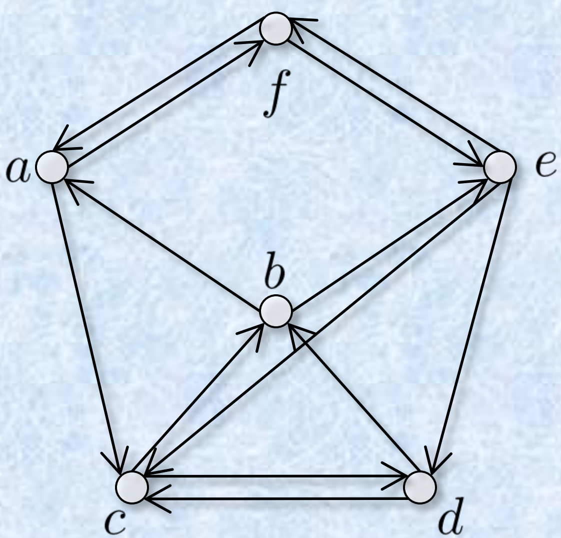

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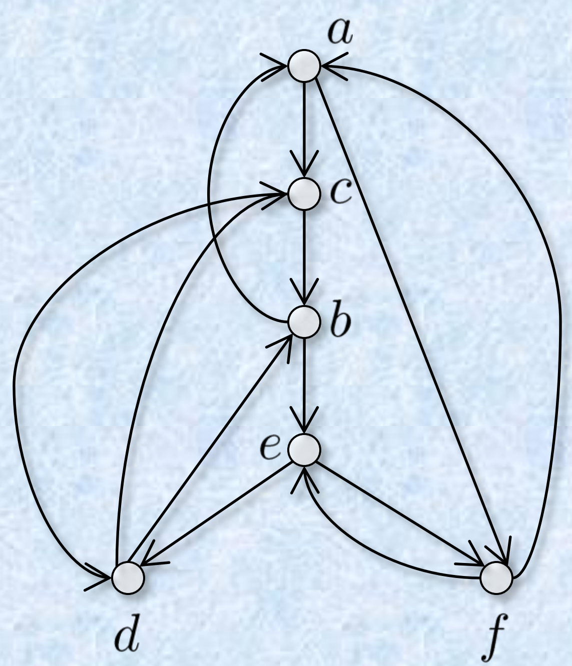

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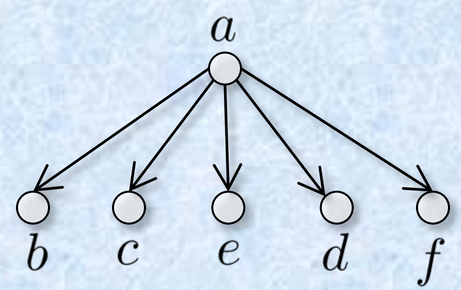

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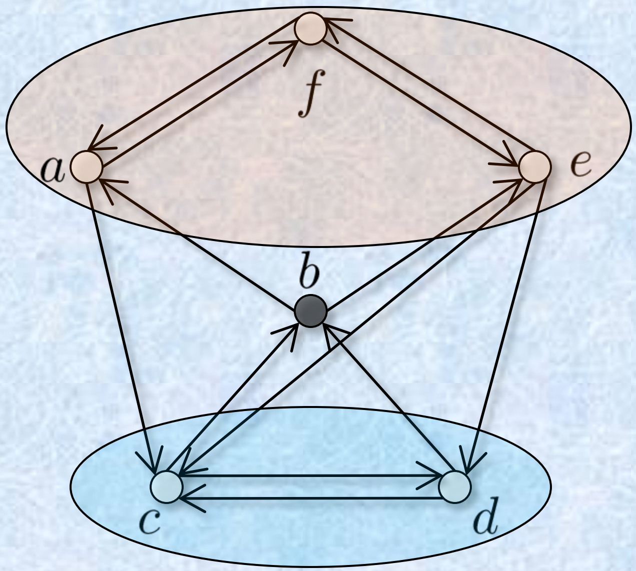

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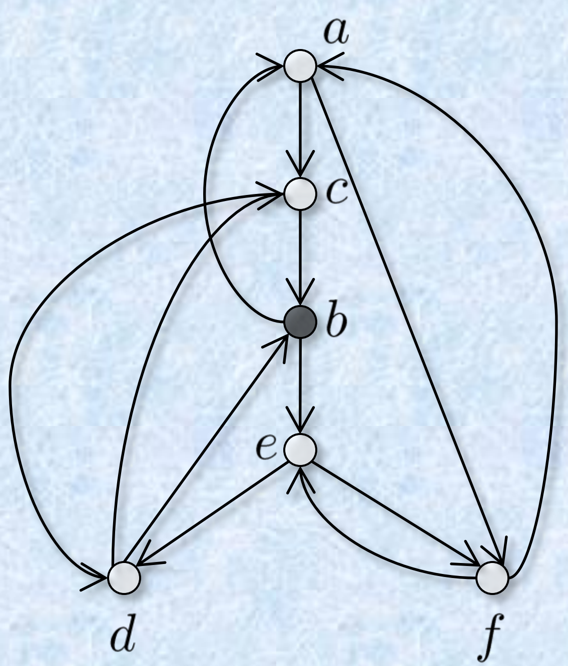

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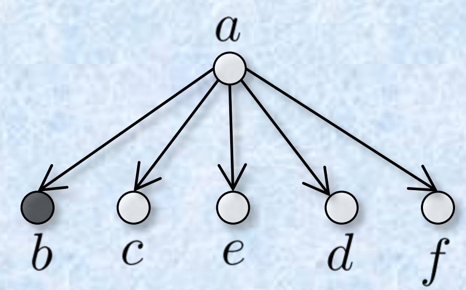

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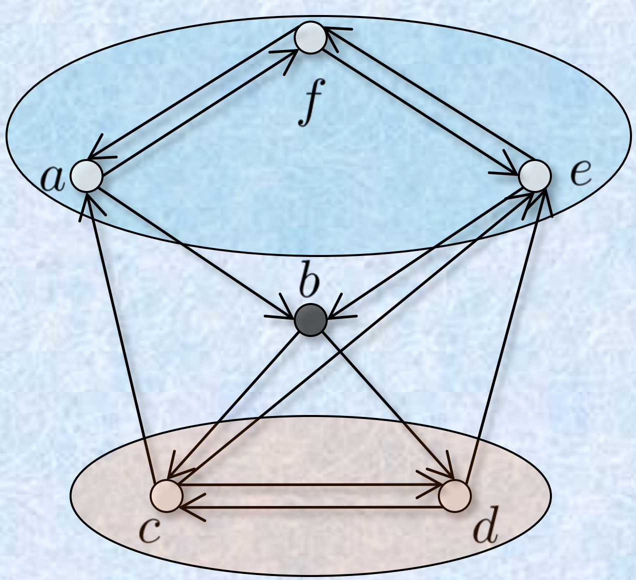
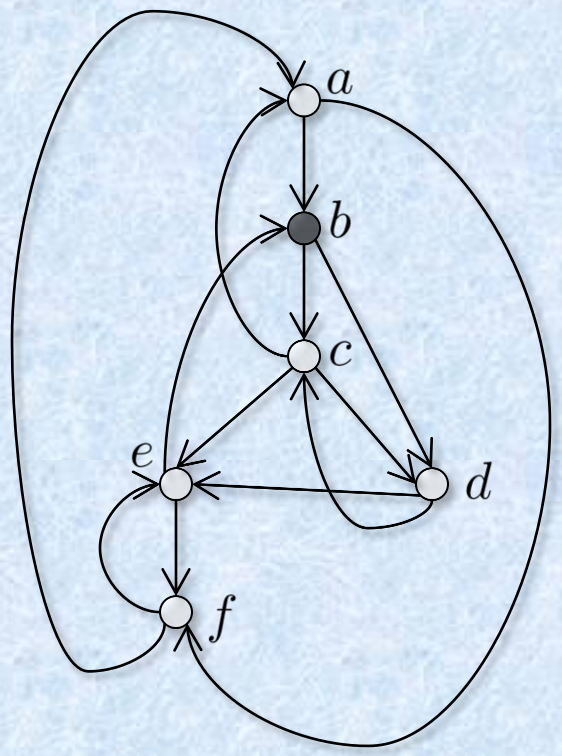
G^r



$G^r(a)$



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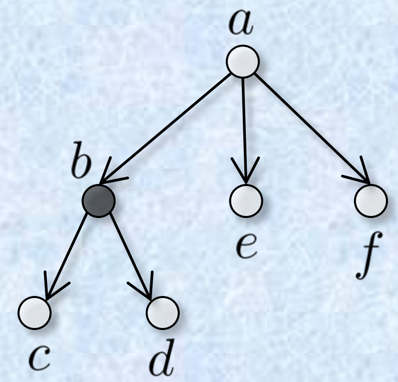


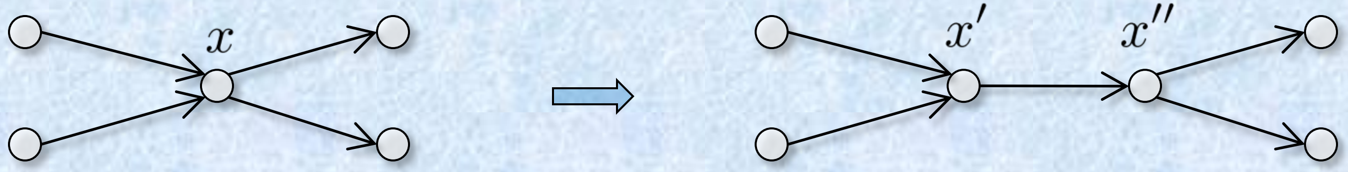
Vertex-Disjoint s-t Paths

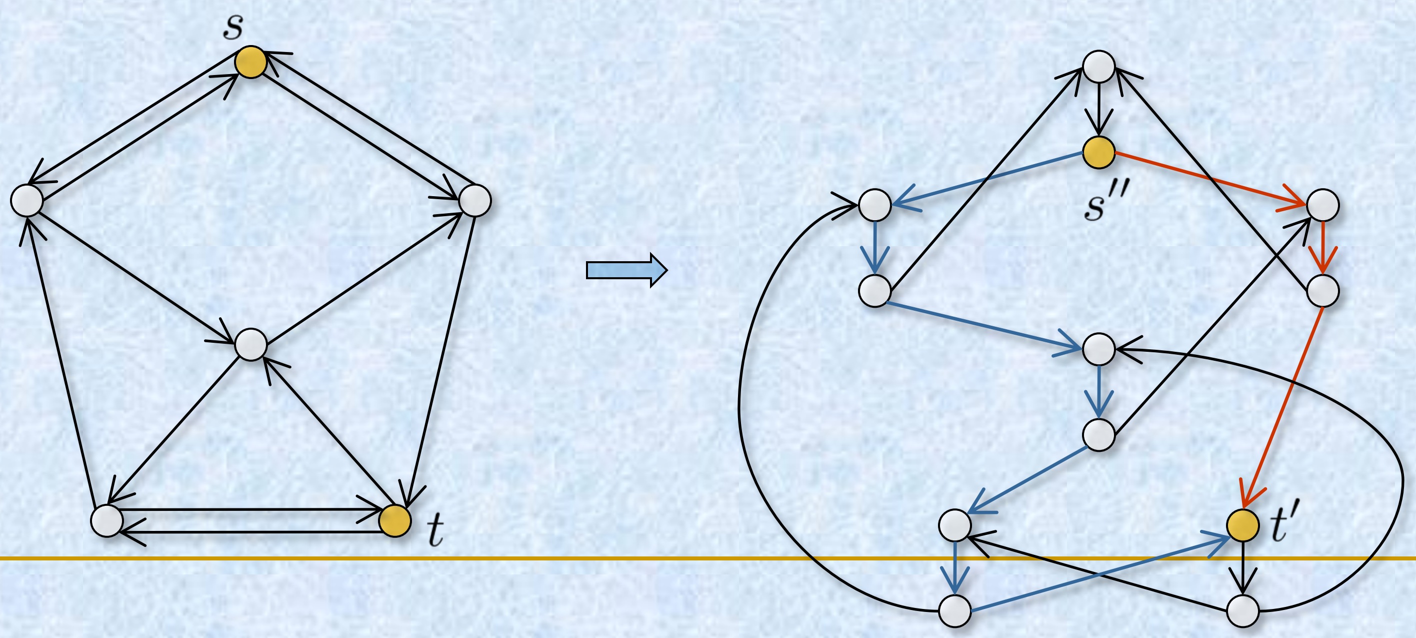
Given a digraph $G = (V, E)$ how fast can we compute a pair of vertex-disjoint s-t paths?

Vertex-Disjoint s-t Paths

Given a digraph $G = (V, E)$ how fast can we compute a pair of vertex-disjoint s-t paths?

$O(m+n)$ time : “vertex-splitting” + “flow augmentation”





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We can get a more efficient solution when G is 2-vertex connected

- Use a 2-vertex connected spanning subgraph of G with $O(n)$ arcs

  [Cheriyan and Thurimella 2000] : $1 + 1/k$ approximation of the minimum k-vertex connected spanning subgraph in $O(km^2)$ time

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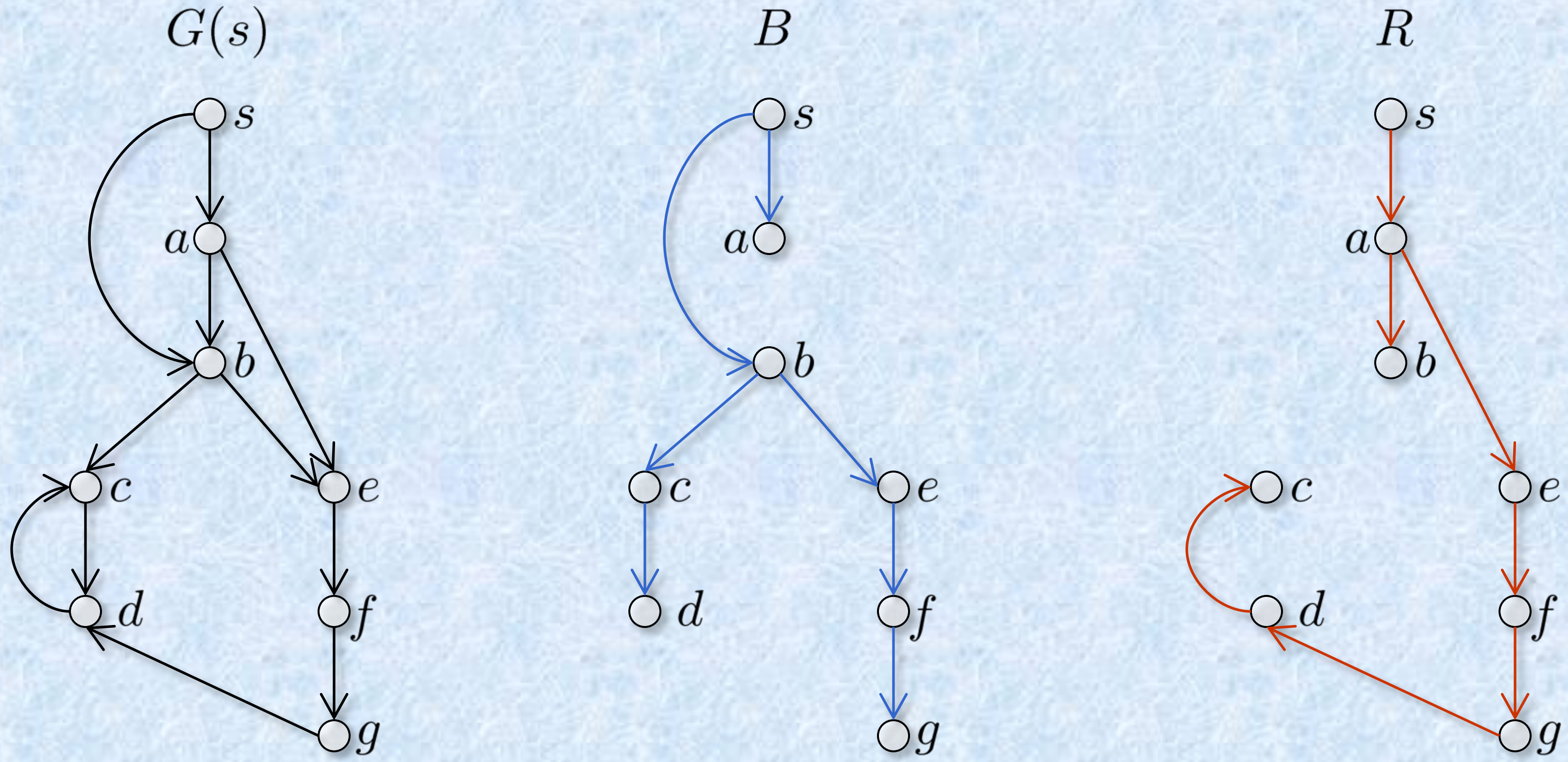
- Use pairs of independent trees

Vertex-Disjoint s-t Paths

Any flowgraph $G(s) = (V, A, s)$ has two spanning trees, B and R, such that for any $v \in V$

$$B[s,v] \cap R[s,v] = dom(v)$$

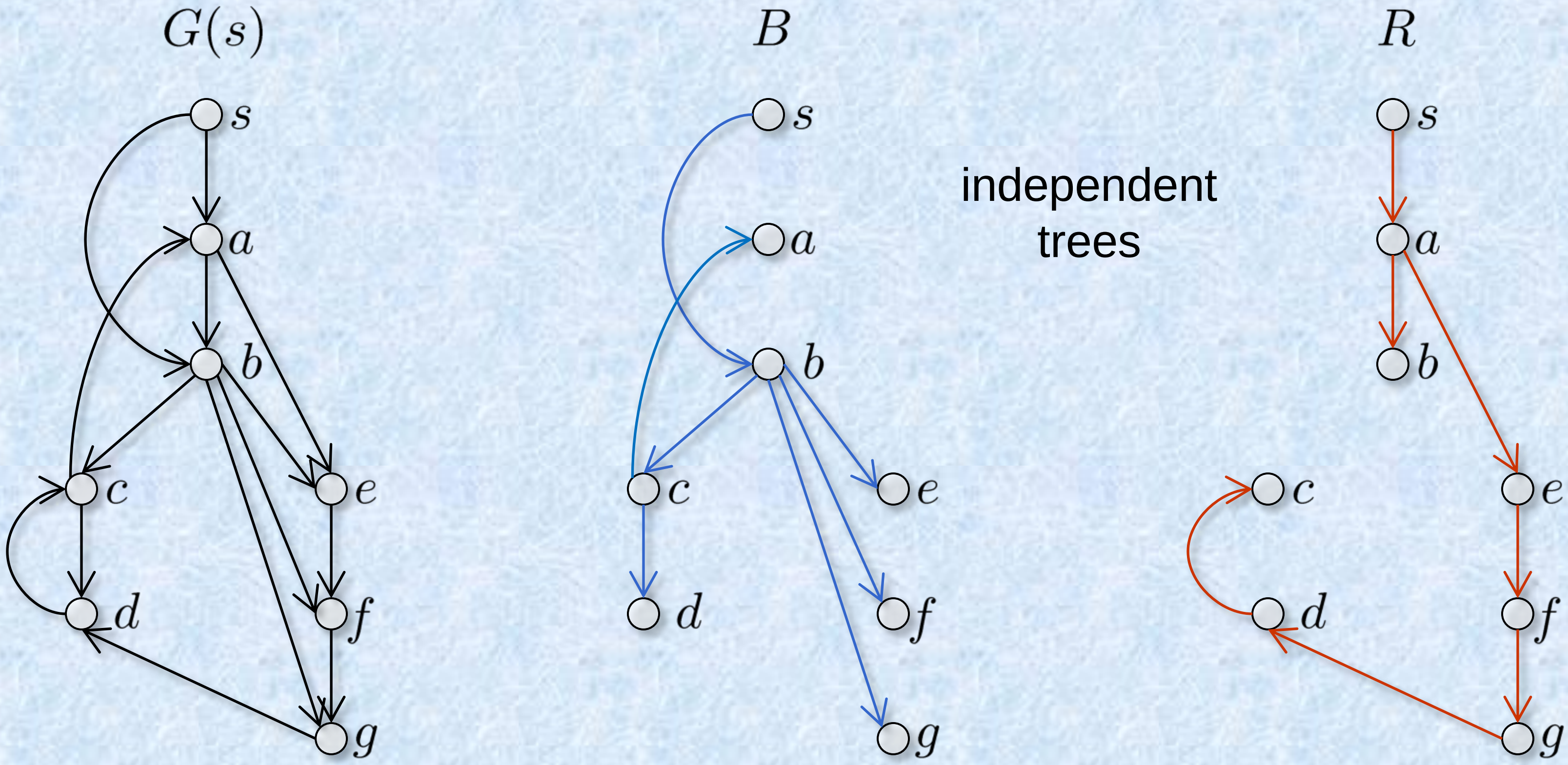


the two trees can be computed in linear time

Vertex-Disjoint s-t Paths

Corollary : If $G(s)$ has trivial dominators only then for any $v \in V$

$$B(s,v) \cap R(s,v) = \emptyset$$



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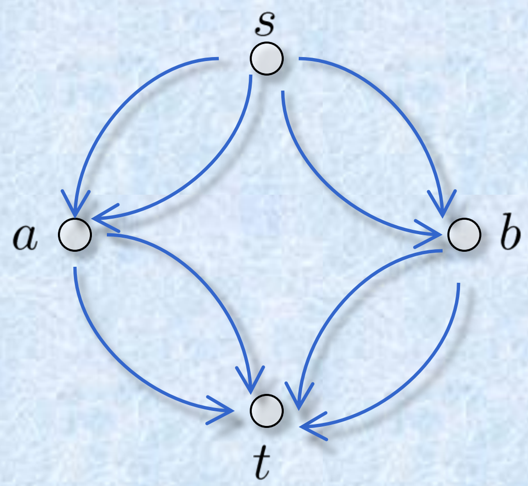
Vertex-Disjoint s-t Paths

Corollary : A digraph $G = (V, A)$ is 2-vertex connected if and only if for two arbitrary vertices $a, b \in V$ $(a \neq b)$ the flowgraphs $G(a), G^r(a), G(b)$ and $G^r(b)$ have trivial dominators only.

We use a pair of independent spanning trees for each of the flowgraphs

$$G(a), G^r(a), G(b), G^r(b)$$



2-Vertex Connectivity

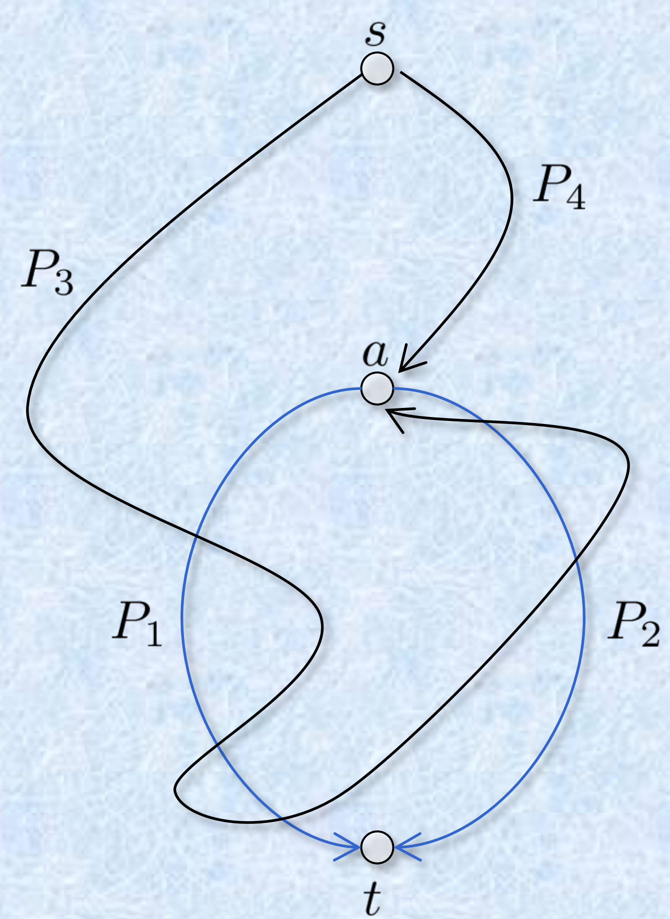
P_1, P_2 : vertex-disjoint a-t paths

P_3, P_4 : vertex-disjoint s-a paths

Suppose

$$P_3[s,a) \cap \big(P_1(a,t] \cup P_2(a,t]\big) \neq \emptyset$$

$$P_4(s,a) \cap \big(P_1(a,t) \cup P_2(a,t)\big) = \emptyset$$



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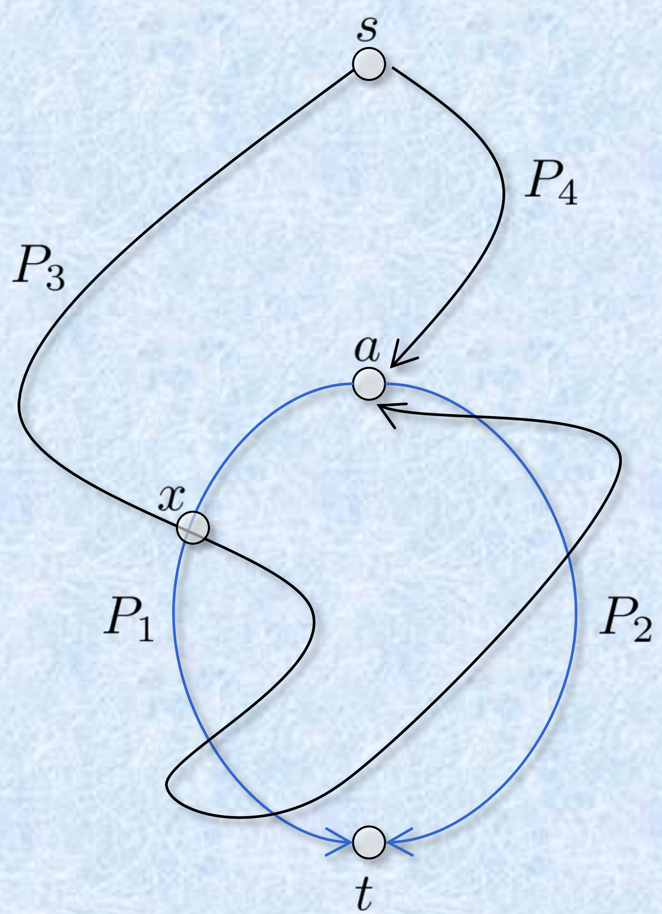
Let x be the first vertex on $P_3[s,a]$ such that $x \in \big(P_1(a,t] \cup P_2(a,t]\big)$.



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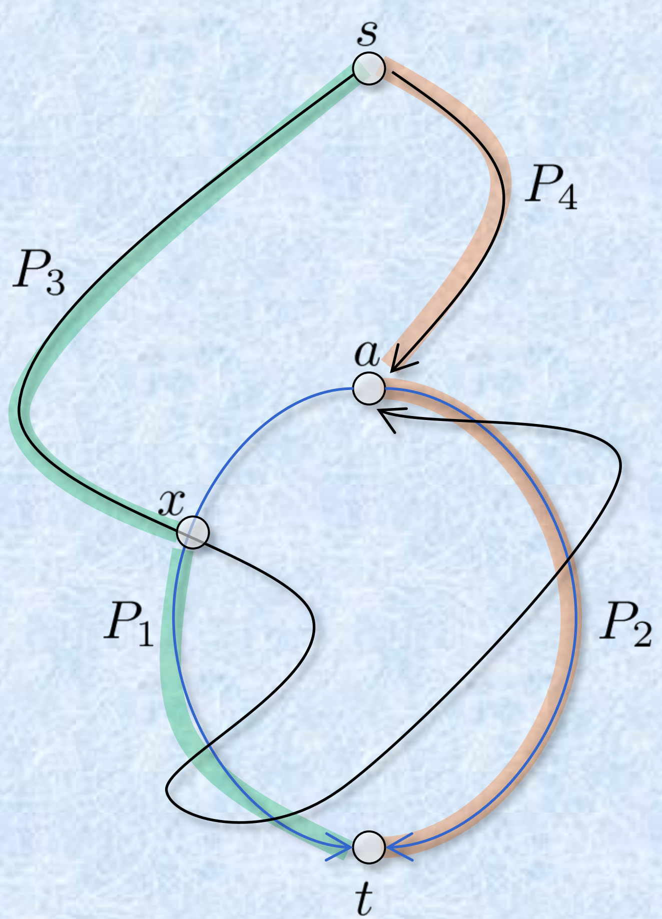
Consider $x \in P_1(a,t] \quad \Rightarrow$

$P_3[s,x] \cdot P_1[x,t]$ and $P_4[s,a] \cdot P_2[a,t]$ are vertex-disjoint s-t paths



2-Vertex Connectivity

Data Structure : Given rooted trees S_1 and S_2 on the same nodes support the operations:

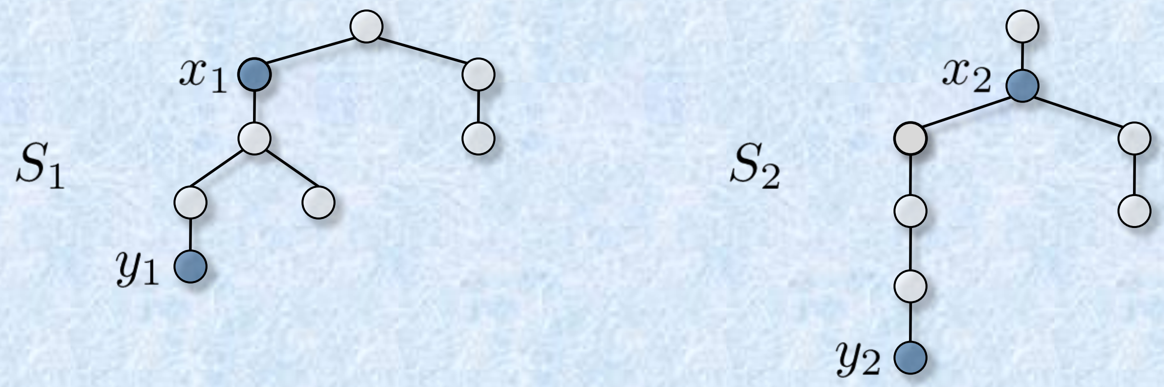
(i) Test if $S_1[x_1, y_1]$ contains x_2.

(ii) Return the topmost vertex in $S_1(x_1, y_1]$.

(iii) Test if $S_1[x_1, y_1]$ and $S_2[x_2, y_2]$ contain a common vertex.

(iv) Find the lowest ancestor of y_2 in $S_2[x_2, y_2]$ that is contained in $S_1[x_1, y_1]$.

(v) Find the highest ancestor of y_2 in $S_2[x_2, y_2]$ that is contained in $S_1[x_1, y_1]$.



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- A query uses a constant number of these operations.
- We give an $O(n)$- space data structure with $O(\log^2 n)$ time per operation.

Example : Pairs of Disjoint Paths in the New York Area

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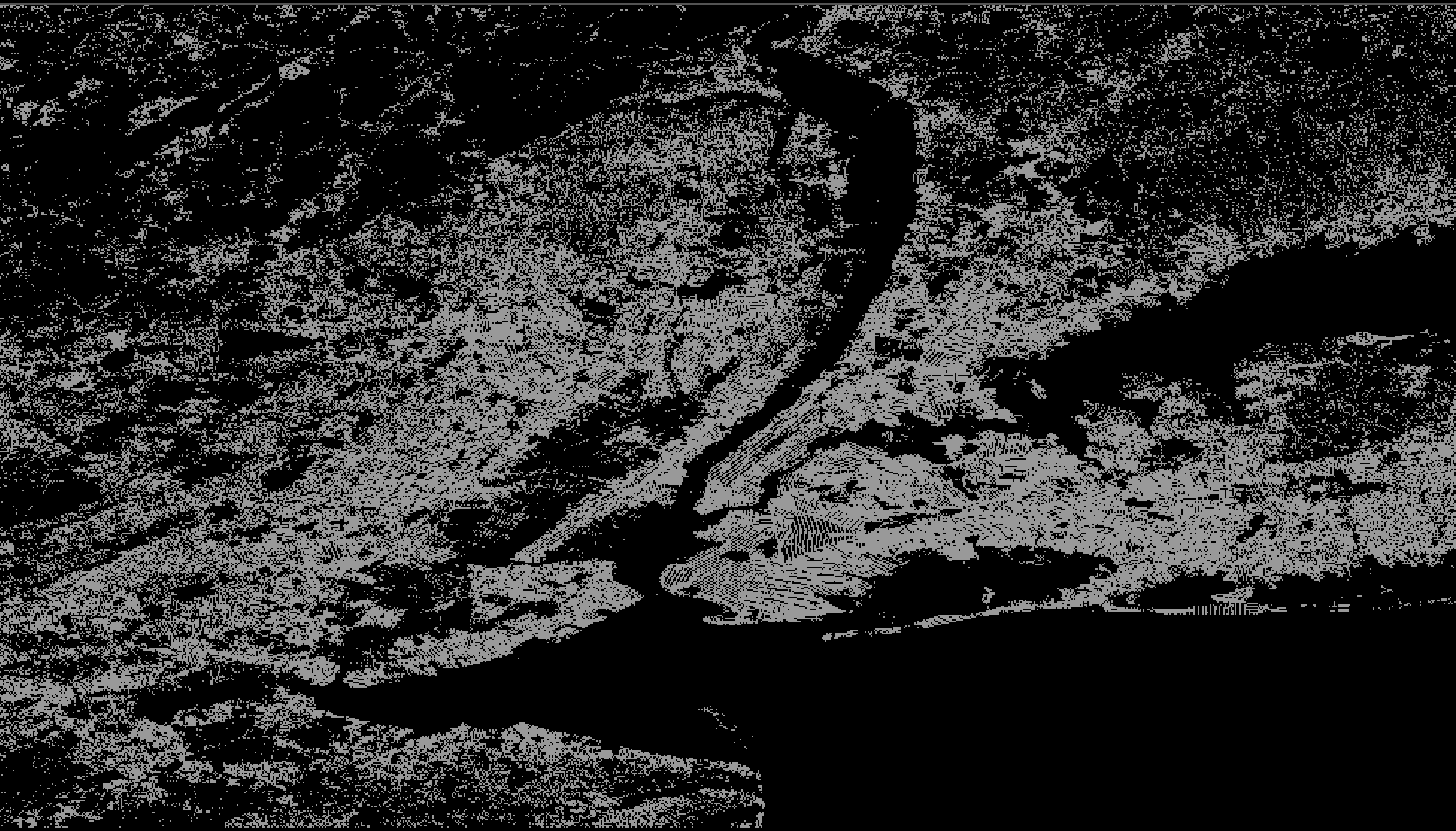

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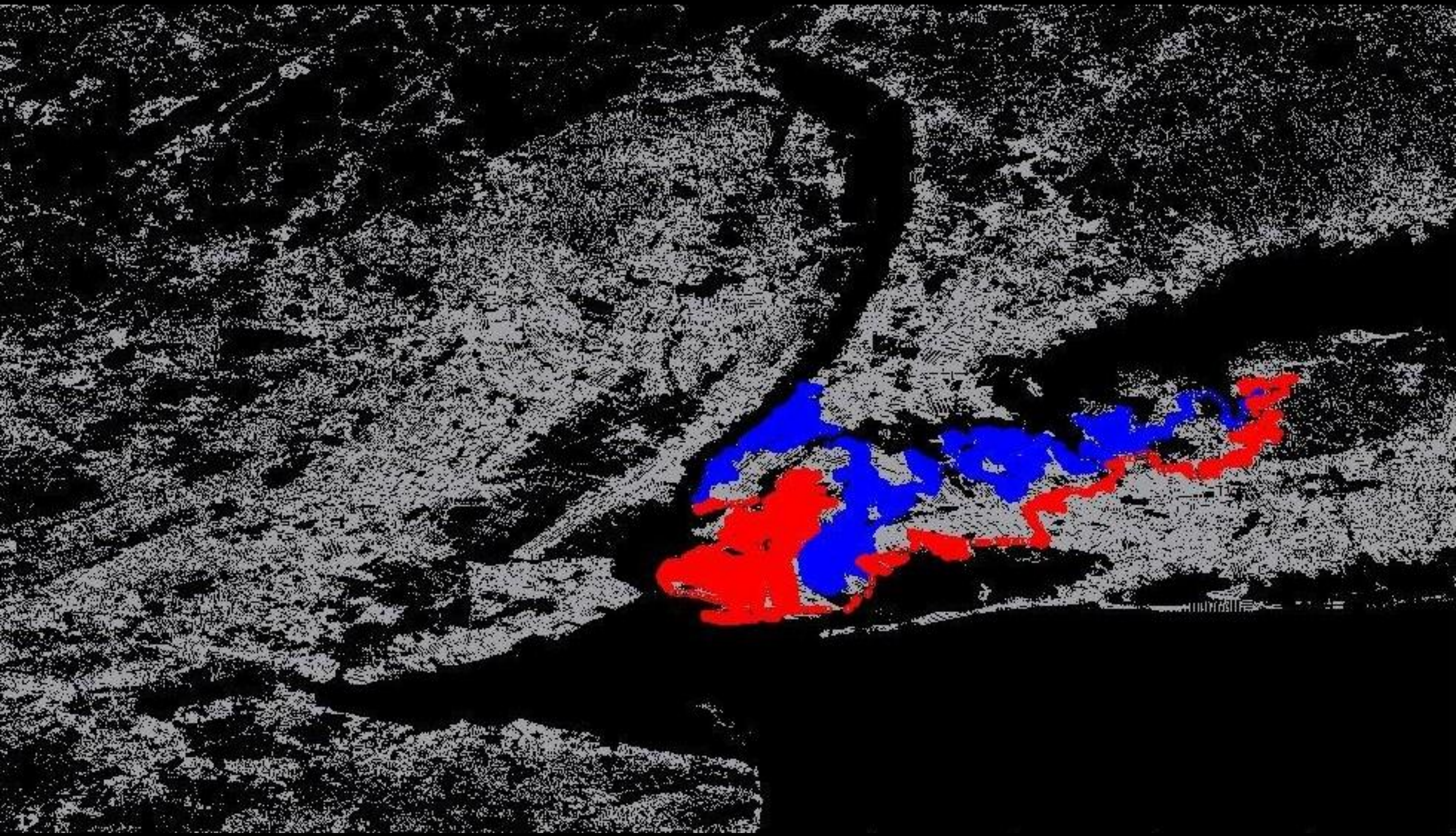

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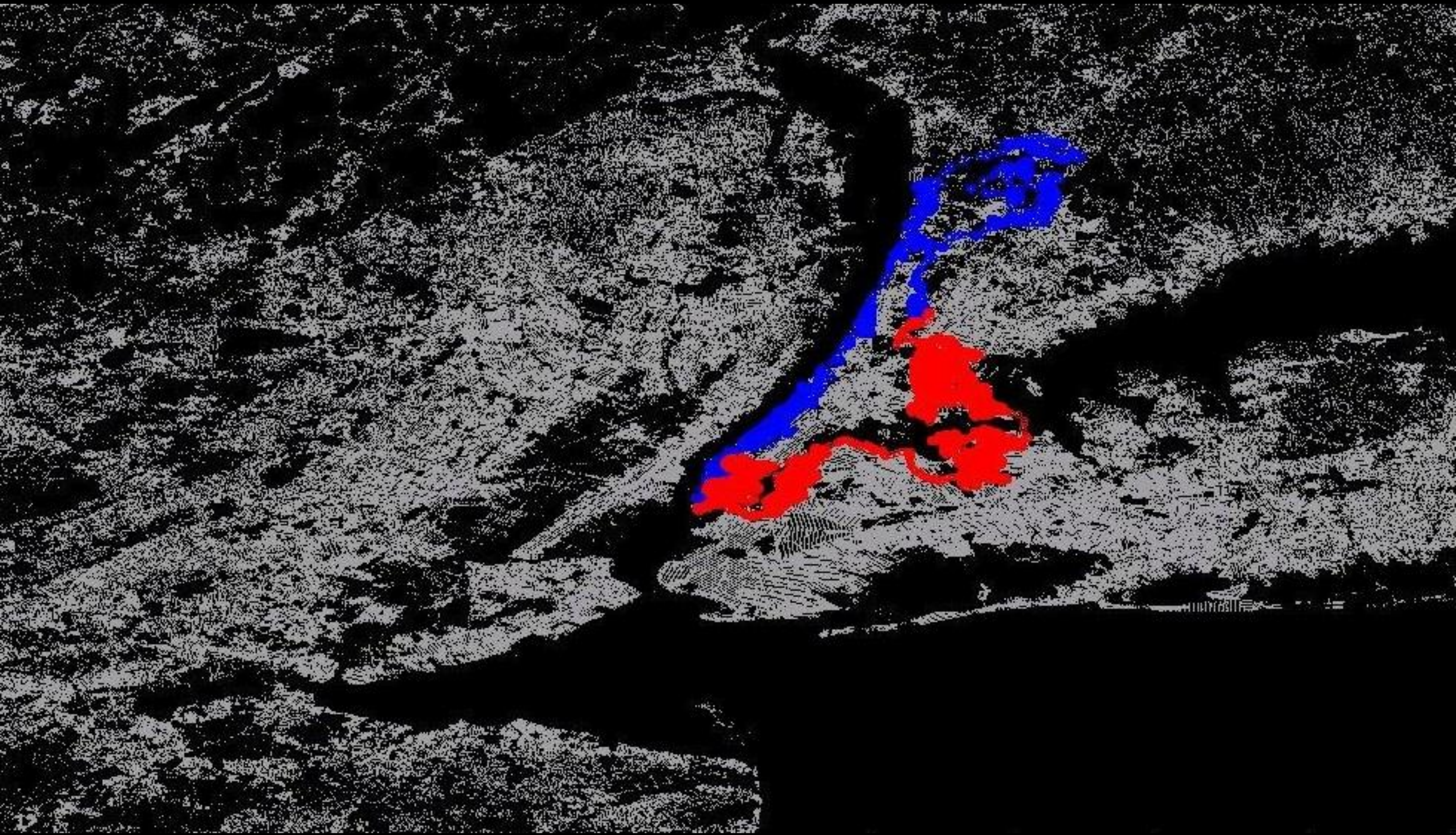

Computational Morphological Analysis

Morphological Analysis : the study of the internal structure of words

Fundamental Aim : identification of the constituents of words and the properties they express.

e.g.

play	kind	read
play-ed	kind-ness	read-ing
play-ing		read-er
play-er		read-er-s
play-er-s		read-able

Issues:

- What morphological units languages consist of?
- What features are represented in each morpheme?
- How do morphemes and features interact with one another?
- Are there any constraints in the selection of morphemes in specific environments?

Computational Morphological Analysis

Computational Approach: Morphological patterns as graph reachability and path selection problems