DWT-based Additive Image Watermarking using the Student-t Prior

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***Abstract*—In this work, a class of new blind watermark detectors is proposed for the DWT (Discrete Wavelet Transform)-based additive image watermarking problem. More specific, we model the marginal subband wavelet distributions with the Student-t probability density function (pdf) deriving a new watermark detector. The proposed detector shows high performance with regard to the watermark detection and increased robust properties against intentional or unintentional attacks. Experimental results on real images demonstrate these properties comparing the proposed detector with other state of the art methods in the transform domain.**

I. Introduction

In recent years watermarking has risen to prominence as a powerful technology for the protection of copyright information by embedding hidden information (watermark) in the digital content (e.g. image), trying to detect its presence or absence. In this framework, additive watermarking can be viewed as a binary hypothesis problem where we need to determine the existence or nonexistence of the hidden information suggesting the need for a statistical model [1], [2].

Blind watermarking requires that the detection of the watermark can be performed without access to the original unwatermarked image [1], whereas the choice of the embedding/detection domain is crucial. Usually watermarking techniques are developed in a transform domain like DWT or DCT (Discrete Cosine Transform) exploiting the imperceptibility properties along with the inherent robust properties of the transform domain [4]-[6].

Over the years a number of watermark detectors have been developed based on a transform domain [1]-[6]. These works assume that the watermark signal is the known information that we want to communicate via the noise, which in our case is the host signal's transform coefficients.

It is widely known that DWT coefficients do not obey a Gaussian distribution and the application of linear correlator detector has been proved to be suboptimal under these conditions [3]-[6],. Utilization of Gaussian pdf in wavelet domain suffers from the fact that wavelet coefficients obey in a more heavy-tailed distribution [1]-[6]. In addition, the above model is highly limited, since does not encourage sparsity in any particular useful way [13].

The modelling of wavelet coefficients by a parametric family of statistical distributions like GG (Generalized Gaussian) or SaS (Symmetric alpha Stable) pdfs has been proved an effective representation and in consequence a solid basis for the development of statistical detectors for the watermarking problem in transform domain [5], [6].

Working in a hierarchical Bayesian framework we can set up our problem in a much richer and flexible class of heavy-tailed prior distributions, meaning the scale mixture of normals [11], [13]. This class includes a wide-range of important heavy-tailed distributions and Student-t is one of them. This prior has been applied successfully in a product form in image recovery and in blind deconvolution problems [11], [12]. To the best of our knowledge Student-t pdf has been used in image watermarking problem only with different imaging model [9].

As a consequence we propose a new class of watermark detectors for the blind additive watermarking problem in DWT domain. Based on a suitable formulation for the proposed prior we can estimate our parameters of interest based on the iterative EM (Expectation-Maximization) algorithm,

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leading to an improved and robust detection structure.

The rest of this paper is organized as follows. In section 2 we define the proposed statistical model for the DWT coefficients. The proposed test statistic for the additive watermarking problem in DWT domain is presented in Section 3 and the corresponding parameter's estimation is found in Section 4. In the next Section we review the two methods of comparison, where in Section 6 we provide the experimental results. In the last Section we have the conclusions and the extensions of the proposed work.

II. Statistical Model

In order to define our model we denote the DWT coefficients of interest in vector notation with $\mathbf{x}$ where N is the total amount of coefficients of the host signal. We treat coefficients as random variables and assume that $\mathbf{x}(i)$'s for $i=1,\ldots,N$ are independent and identical distributed (i.i.d) samples from the same Student-t distribution with zero mean and scale parameter λ and ν degrees of freedom. That means:

$$p(\mathbf{x})=\prod_{i=1}^{N} St(\mathbf{x}(i)\,|\,0,\lambda,\nu) \tag{1}$$

Introducing the independent hidden variables $\tau=[\tau(1),\ldots,\tau(N)]^{\mathrm{T}}$ which obey the Gamma pdf, with parameters a,b as those defined in [8]:

$$p(\boldsymbol{\tau})=\prod_{i=1}^{N} Gamma(\boldsymbol{\tau}(i)\,|\,a,b) \tag{2}$$

the Student-t pdf could be written as the integral $p(\mathbf{x})=\int p(\mathbf{x}\,|\,\boldsymbol{\tau}^{-1})p(\boldsymbol{\tau})d\boldsymbol{\tau}$ where [8]:

$$p(\mathbf{x}\,|\,\boldsymbol{\tau})=N(\mathbf{x}\,|\,0,\boldsymbol{\tau}^{-1})\,. \tag{3}$$

Thus, we can define the Student-t pdf with zero mean, precision λ and degrees of freedom ν as:

$$St(\mathbf{x}(i)\,|\,0,\lambda,\nu)=\frac{\Gamma\big((\nu+1)/2\big)}{\Gamma\big(\nu/2\big)}\left(\frac{\lambda}{\nu\pi}\right)^{\frac{1}{2}}\left(1+\frac{\lambda}{\nu}\mathbf{x}(i)^2\right)^{-\frac{\nu+1}{2}} \tag{4}$$

$$i=1,\ldots,N$$

The proposed pdf can be considered as a generalization of the Gaussian distribution. It is noticeable that depending on parameter's selection it can have heavy tails. As the degrees of freedom increases the pdf approaches the Normal distribution, whereas ν goes towards zero, the pdf becomes uninformative [9].

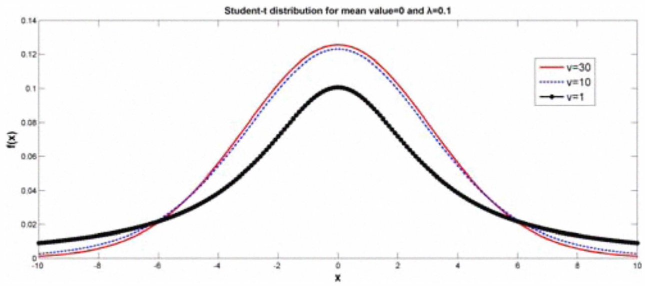

Fig. 1 Plot of Student t pdf for different values of ν (1, 10, 30) for fixed $\lambda=0.1$.

We have to notice that when $\nu=1$ the Student-t distribution is equivalent to the Cauchy distribution with tails that decay in the same manner. Based on this fact, the proposed modelling is a generalization of Cauchy's pdf. Thus, we expect the proposed watermark detector to exploit this generalization property in terms of watermark sensitivity and robustness. In addition, the validity of Student-t assumption for DWT coefficients is justified from the approach about departure from Gaussian law given in [3].

The assumption of a Student-t distribution implies a two-level data generation process [7]-[9]. Assuming that we have the i.i.d random variable $\boldsymbol{\tau}(i)$ that follows Gamma pdf: $p(\boldsymbol{\tau}(i))=Gamma\big(\nu/2,\nu/2\big)$, then the wavelet coefficient $\mathbf{x}(i)$ is drawn from a Gaussian distribution with zero mean and precision parameter $\lambda\boldsymbol{\tau}(i)$:

$$p\big(\mathbf{x}\,|\,\boldsymbol{\tau}(i)\big)=N\Big(0,(\lambda\boldsymbol{\tau}(i))^{-1}\Big) \tag{5}$$

The degree of freedom parameter ν dictates the shape of the distribution as this illustrated in Figure 1, where we make use of different values of ν for fixed λ.

In comparison with the other known distributions, the Student-t prior has the advantage to offer a supplementary hyperparameter $\boldsymbol{\tau}(i)$ which in watermarking problem suggest a framework for constructing a new perceptual mask as this depicted in Figure 2. This mask can easily be applied in wavelet coefficients providing a visual mask capable to weight the embedded watermark and as a net result to improve watermark detection in a spirit similar to [10] and [17]. Favour to limited space we won't give results here based on this assumption.

Fig.2 Plot of hyperparameter values $\boldsymbol{\tau}(i)$ that constitute a new visual mask based on the second level of DWT coefficients. In this work we don't employ this mask for watermark scaling.

III. Proposed Test Statistic for the Additive Watermarking Problem

Defining a statistical model for our data in the transform domain, we can derive a test statistic for this problem (e.g. log-likelihood ratio test) [14]. In order to define the test statistic, we treat the watermark detection problem as a binary hypothesis problem of the form:

$$\begin{aligned} H_0 &: \mathbf{y} = \mathbf{x} \\ H_1 &: \mathbf{y} = \mathbf{x} + \alpha \mathbf{w} \end{aligned} \tag{6}$$

where α represents the known embedding strength. The decision about the existence of the watermark is a blind procedure without knowledge of the un-watermarked image.

Let $\mathbf{x} = [x_1(1) \ldots x_1(N_1), \ldots, x_{\mathrm{K}}(1) \ldots x_{\mathrm{K}}(N_{\mathrm{K}})]$ be the sequence of wavelet coefficients where N_k is the total number of coefficients in the k-th band and K is the number of wavelet bands we make use. Defining the watermark signal as $\mathbf{w} = [w_1(1) \ldots w_1(N_1), \ldots, w_{\mathrm{K}}(1) \ldots w_{\mathrm{K}}(N_{\mathrm{K}})]$, $\mathbf{w'} = \alpha \mathbf{w}$ then using the additive embedding rule of Eq.(6) we receive the watermarked coefficients denoted as $\mathbf{y} = [\mathbf{y}_1(1) \ldots \mathbf{y}_1(N_1), \ldots, \mathbf{y}_{\mathrm{K}}(1) \ldots \mathbf{y}_{\mathrm{K}}(N_{\mathrm{K}})]$.

Denoting with index k the corresponding wavelet subband dependent parameters and with $\boldsymbol{\lambda} = \{\lambda^{\kappa}\}$, $\mathbf{v} = \{\nu^{\kappa}\}$ the model parameters, the conditional pdfs under the two hypotheses are [14]:

$$p(\mathbf{y} \mid H_0; \boldsymbol{\lambda}, \mathbf{v}) = \prod_{k=1}^{K} \prod_{i_k=1}^{N_k} St\left(\mathbf{y}_k(i_k); 0, \lambda^k, \nu^k\right) \tag{7}$$

$$p(\mathbf{y} \mid H_1; \boldsymbol{\lambda}, \mathbf{v}) = \prod_{k=1}^{K} \prod_{i_k=1}^{N_k} St\left(\mathbf{y}_k(i_k); w_k'(i_k), \lambda^k, \nu^k\right) \tag{8}$$

Then, assuming independence under the two hypotheses the decision rule for the above test is:

$$T_{STT}(\mathbf{y}; \boldsymbol{\lambda}, \mathbf{v}) = \sum_{k=1}^{K} (\nu^k + 1) \sum_{i=1}^{N_k} \log \left(\frac{\nu^k + \lambda^k \mathbf{y}_k^2(i_k)}{\nu^k + \lambda^k \left(\mathbf{y}_k(i_k) - \mathbf{w}_k'(i_k)\right)^2} \right) \tag{9}$$

The form of the resulting test statistic provide us with a flexible formula. Except of using the parameter's estimated values, we can also fix some of them and propose alternative forms of Eq. (9). These forms have different properties with regard to detection sensitivity, computational cost or run time requirements. In this work, we didn't use any fixed value for the employed parameters (e.g. in order to gain in computational costs without sacrifice detection performance) but we used for every image the proposed methods of estimation. In a future work we will show results based on these assumptions.

IV. Parameter's Estimation

Based on the fact that watermark is weakly embedded, it is likely that marked transformed image approximately follows a Student-t distribution with the same parameters as that of the original transform we use. Then, without having available the original image at the receiver's side, parameter estimation can be performed using the marked image [1]. In order to find the employed parameters of the proposed model, we resort to the iterative EM algorithm as this described in [7], [9] and given by the following steps:

E-step: the mean of the conditional of the hidden variable given the observations is:

$$\langle \tau(i) \rangle = \frac{\nu + 1}{\nu + \lambda\left(\mathbf{y}(i) - \mathbf{w}'(i)\right)} \tag{10}$$

M-step: in this step we derive the Maximum Likelihood (ML) estimates of parameters λ, ν:

$$\lambda = N \Big/ \sum_{i=1}^{N} \left(\mathbf{y}(i) - \mathbf{w}'(i)\right)^2 \langle \tau(i) \rangle \tag{11}$$

where we can find parameter ν by solving the equation:

$$\begin{aligned} &\frac{1}{N}\left(\sum_{i=1}^{N} \log\langle \tau(i) \rangle - \sum_{i=1}^{N} \langle \tau(i) \rangle_{\tilde{a}} \right) + \psi\left((\nu)^t \frac{1}{2} + \frac{1}{2} \right) + \\ &-\log\left((\nu)^t \frac{1}{2} + \frac{1}{2} \right) - \psi\left(\frac{\nu}{2}\right) + \log\left(\frac{\nu}{2}\right) + 1 = 0 \end{aligned} \tag{12}$$

In order to solve this method we make use of the known numerical bisection method. Parameter

$\mathbf{w}' = \alpha\mathbf{w}$ embodies the knowledge of watermark's power and $\psi(x)$ is the known digamma function, given by: $\psi(x) = d/dx(\ln\Gamma(x)) = \Gamma'(x)/\Gamma(x)$. The proposed approach converges after few iterations and depending on convergence criterion we can speed up the above procedure.

V. Methods of Comparison

Marginal distributions of the subband coefficients of natural images are highly non-Gaussian. Although the GGD is generally the best known model for the DWT detail subband coefficients [2], [4], Briassouli et. al [5] proposed the Cauchy member of SaS as an alternative distribution to model the DCT coefficients. Recently Kwitt et. al [6], applied the same pdf to detail subband coefficients of DWT. The parameterization of the wavelet-based GGD and Cauchy models, assumes that wavelet coefficients are modelled as i.i.d random variables with pdfs given by Eqs. (13) and (15).

For all the detectors of this work, we use $K = 3$ bands of the second-level of DWT transform, $N_k = (M/4)^2$ is the total number of coefficients in $k-th$ band, though $k = 1,2,3$, for image sizes of MxM where M=512. Location i_k denotes the i_k coefficient in the corresponding k-th band. For GGD's model parameters $\{c_k, b_k\}$ we used the ML estimates of each wavelet band and for the Cauchy's model parameters $\{\gamma_k\}$ we resort to the estimation which is given by [6].

A. GGD-based detector

The parameterization of the wavelet-based GGD model is given by: $p(\mathbf{x}(i)) = A\exp(-|b\mathbf{x}(i)|^c)$ (13)
where c is the known distribution's shape parameter $b = (1/\sigma)n(c)$, $A = bc/2\Gamma(1/c)$, $\Gamma(t) = \int_0^\infty u^{t-1}e^{-u}du$ is the Gamma function and $n(c) = \sqrt{(\Gamma(3/c)/\Gamma(1/c))}$. Then the test statistic is given by:

$$T_{GGD}(\mathbf{y};\mathbf{b},\mathbf{c}) = \sum_{k=1}^{K}\sum_{i_k=1}^{N_K} b_k^{c_k}\left(|\mathbf{y}_k(i_k)|^{c_k} - |\mathbf{y}_k(i_k) - \mathbf{w}'_k(i_k)|^{c_k}\right) \quad (14)$$

where $\mathbf{b} = [b_1, \ldots b_K]$, $\mathbf{c} = [c_1, \ldots c_K]$ denotes the GGD model parameters.

B. Cauchy-based detector

For the wavelet-based Cauchy model we used the parameterization [6] :

$$p(\mathbf{x};\boldsymbol{\gamma},\delta) = \frac{1}{\pi}\frac{\gamma}{\gamma^2 + (\mathbf{y}-\delta)^2} \quad (15)$$

where $-\infty < \delta < +\infty$ is the location parameter and $\gamma > 0$ is the shape parameter with $-\infty < \mathbf{x} < +\infty$. In our experiments $\delta = 0$ and the corresponding detector is:

$$T_{CAUCHY}(\mathbf{y};\boldsymbol{\gamma}) = \sum_{k=1}^{K}\sum_{i_k=1}^{N_k}\log\left(\frac{(\gamma_k)^2 + \mathbf{y}_k(i_k)^2}{(\gamma_k)^2 + (\mathbf{y}_k(i_k) - \mathbf{w}'_k(i_k))^2}\right) \quad (16)$$

where $\boldsymbol{\gamma} = \{\gamma_k\}$ is the unknown model's parameter. It is important to note that the Cauchy's test statistic is similar to the proposed t-based test statistic in Eq. (9) except from the employed parameters that depends on the underlying model.

VI. Numerical Results

In this section we evaluate the detection performance and the robust properties of the proposed method. We conducted two kind of experiments, where in the first one we have a fixed image and a set of 100 random 1-bit spread spectrum watermarks and in the second case, for the statistical significance verification of our proposal, we used 200 representative images of the Microsoft Image Recognition data base interpolated at the size of 512x512 and a fixed watermark [12].

In all cases, for the fairness of the comparison, we used the same embedding method where we add the watermark in the detail sub-bands of the second level in the DWT domain. Then, in order to apply the watermark detectors, we first transform back the images in the spatial domain and then we apply the direct transform. The wavelet filter of our choice is the Daubechies-8 2-D separable filters.

For the quantification of watermark's power in our experiments we used the known WDR (Watermark to Document ratio) definition [9]

$$WDR = 20\log_{10}\left(\frac{\|\mathbf{w}'\|}{\|\mathbf{x}\|}\right)dB.$$

The PSNR (Peak Signal to Noise Ratio) values for the referenced WDR values are higher than 42dB for all of our experiments.

We have to notice that every test statistic is the addition of the responses in every sub-band we make use in the wavelet domain, considering sub-band adaptive models. The detector's performance is measured by the empirical ROCs (Receiver Operating Characteristics), which are obtained by varying the detection threshold. More specific, when we demonstrate the results in the table form, we utilize the area under the ROC (AUROC1) curve in the range of [0-0.1] describing the detector's performance at low false-alarm rates and we also make use of the total area under the ROC (AUROC2) describing the overall performance of the detector.

A. *Experiment I: Fixed Image and "Random Watermark"*

For the evaluation of detector performance we use the known test image "Lena". In Table 1, we can see the AUROC1, AUROC2 measures for very low WDRs and for comparison reasons we summarise also the results of adaptive wavelet domain GGD and Cauchy-based detectors. From these results it is obvious that t-based detector outperforms the other two detectors with respect to detection sensitivity.

In order to test the robustness of our detectors, we apply the JPEG compression attack with quality factor equal to 15 and the Wiener filtering plus AWGN (Additive White Gaussian Noise) attack with filter size equal to 5x5 and added white Gaussian noise of 5dB. In Table 2 we can observe the enhanced robust properties of the proposed t-based detector, whereas Table 3 shows that our proposal has almost the same behaviour after this particular attack.

B. *Experiment II: Fixed Watermark and "Random Image"*

In order to verify the statistical significance of our results we apply our detectors to a dataset of many images keeping the same watermark signal. In this kind of experiments the evaluation of test statistics is take place for 200 images of the Microsoft Image Recognition data base [16], evaluating every test statistic with and without the watermark. In Table 4 we summarize the results of detection performance of the detectors for all these images. It is obvious that the t-based detector has superior performance compared with the other wavelet based detectors.

The robustness property of the three detectors of comparison is verified through the ROC curves of Figure 3 and 4, where we compare the aforementioned detectors after JPEG compression and after the Wiener filtering plus AWGN attack. In the first case the quality factor is 50 with WDR=-45dB and in the second case we keep the same setting as in the previous kind of experiments with WDR=-40dB. The reason for showing results with these WDR values is that we want to keep in a high level the visual quality of the images.

VII. Conclusions and Further Work

In this work we proposed a new statistical detector for the additive image watermarking problem in DWT domain. More specifically, we modelled wavelet coefficients by a Student's-t pdf and we also proposed a method of parameter's estimation based on the iterative EM algorithm. The detection performance based on ROC curves demonstrates the enhanced detection sensitivity along with the robust properties of the proposed test statistic for the additive watermarking problem in DWT domain.

In a future work, we will provide the analytical performance analysis of the proposed detector, examining the computational costs of our proposal and the lightweight versions in a manner similar to [6]. We will also provide results that demonstrate the improvements in watermarking problem of perceptual shaping by exploiting the hyperparamers in order to construct a new perceptual model. In addition, we will investigate alternative strategies based on MCMC that may improve watermark detection performance [15].

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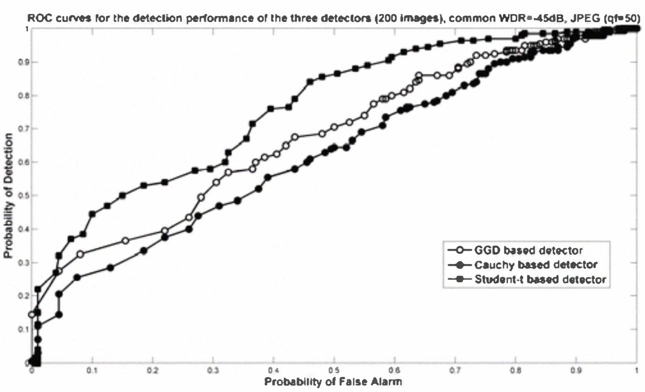

|Fig. 3 ROC curves for the GGD, Cauchy and Student-t wavelet based detectors using 200 images after JPEG attack with quality factor of 50 (common WDR=-45dB).

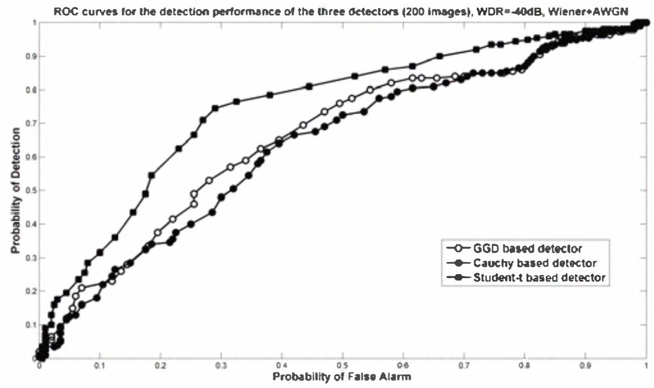

Fig. 4 ROC curves for the GGD, Cauchy and Student-t wavelet based detectors using 200 images after Wiener filtering plus AWGN attack (common WDR=-40dB)

TABLE I
AUROC RESULTS (AUROC2, AUROC1) FOR DETECTION PERFORMANCE COMPARISON OF DWT-BASED DETECTORS - "LENA"

WDR	*GGD*	*Cauchy*	*Student t*
-62	1.00,0.1000	1.00,0.1000	1.00,0.1000
-63	0.99,0.0965	0.99,0.0876	1.00,0.1000
-64	0.81,0.0279	0.83,0.0239	0.95,0.0413
-65	0.58,0.0092	0.60,0.0081	0.62,0.0050
-66	0.50,0.0056	0.50,0.0015	0.51,0.0029

TABLE II
AUROC (AUROC2, AUROC1) RESULTS FOR ROBUST COMPARISON OF DWT – BASED DETECTORS AFTER JPEG ATTACK WITH QUALITY FACTOR OF 15 - "LENA"

WDR	*GGD*	*Cauchy*	*Student t*
-50	0.994,0.0951	0.985,0.0881	1.000,0.1000
-51	0.983,0.0799	0.982,0.0877	0.999,0.0998
-52	0.939,0.0828	0.968,0.0790	0.998,0.0998
-53	0.897,0.0763	0.958,0.0805	0.905,0.0633
-54	0.874,0.0616	0.909,0.0605	0.904,0.0634
-55	0.847,0.0602	0.899,0.0632	0.904,0.0622

TABLE III
AUROC (AUROC2, AUROC1) RESULTS FOR ROBUST COMPARISON OF DWT – BASED DETECTORS AFTER WIENER FILTERING PLUS AWGN - "LENA".

WDR	*GGD*	*Cauchy*	*Student t*
-50	0.995,0.0916	0.993,0.0940	0.999,0.0940
-51	0.989,0.0876	0.985,0.0888	0.985,0.0799
-52	0.980,0.0834	0.971,0.0841	0.979,0.0834
-53	0.965,0.0713	0.953,0.0597	0.959,0.0738
-54	0.946,0.0633	0.932,0.0542	0.935,0.0687

TABLE IV
AUROC (AUROC2, AUROC1) RESULTS USING 200 IMAGES FOR DETECTION PERFORMANCE COMPARISON OF DWT- BASED DETECTORS.

WDR	*GGD*	*Cauchy*	*Student t*
-61	0.992,0.0540	0.566,0.0069	0.999,0.0735
-62	0.704,0.0166	0.538,0.0060	0.818,0.0138
-63	0.603,0.0041	0.517,0.0052	0.641,0.0500
-64	0.530,0.0052	0.506,0.0051	0.537,0.0065
-65	0.505,0.0014	0.500,0.0012	0.509,0.0019