Scalable Video Transmission Over Wireless DS-CDMA Channels Using Minimum TSC Spreading Codes

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***Abstract*—In this letter, we report results on the relative performance of scalable video transmission via a single-rate or a multirate direct sequence code-division multiple-access channel using minimum total squared correlation spreading codes. Our findings demonstrate the superiority of the multirate system on a wide range of chip rates of practical interest.**

***Index Terms*—Direct-sequence code-division-multiple-access (DS-CDMA), scalable video, spreading code design, wireless video transmission.**

I. Introduction

RESEARCH on real-time multimedia transmission over wireless channels has received a great deal of attention recently. In particular, there is significant interest in the design of multirate wireless systems that would allow efficient seamless delivery of variable data-rate video services.

In [1], video transmission from one transmitter to one receiver using binary phase-shift-keying (BPSK) modulation was analyzed. The channel behavior was modeled as nonfrequency-selective Rayleigh fading. In [2], video transmission over a direct-sequence code-division-multiple-access (DS-CDMA) link was considered. A frequency-selective (multipath) Rayleigh fading channel model was used. At the receiver, an adaptive antenna array auxiliary-vector (AV) linear filter that provides space-time RAKE-type processing (thus, taking advantage of the multipath characteristics of the channel) and multiple-access interference suppression was employed [3].

The tradeoffs of source coding, channel coding, and spreading for image transmission in CDMA systems were considered in [4]. In [5], video transmission via a single-rate CDMA channel was compared against transmission via a combination of multicode multirate CDMA and variable sequence length multirate CDMA under frequency selective Rayleigh fading. In this letter, we define and solve an operational rate-distortion problem in order to optimally select the: 1) source coding rates; 2) channel coding rates; and 3) spreading code lengths (processing gains) used for the transmission. Our spreading code domain is the newly developed class of minimum total squared correlation (TSC) optimal binary antipodal signature sets by Karystinos–Pados that are available for almost all signature lengths and number of signatures [6], [7].

The rest of the letter is organized as follows. In Section II, we describe the elements of the video transmission system under consideration, i.e., scalable video coding (Section II-A), channel encoding (Section II-B), and received signal (Section II-C). In Section III, the joint source coding optimization algorithm is described. The minimum TSC optimal binary antipodal signature sets are discussed in Section IV. Experimental results using the Karystinos–Pados minimum TSC signature sets are presented in Section V, and final conclusions are drawn in Section VI.

II. Video Transmission System

A. Scalable Video Coding

A scalable video codec produces a bit stream which can be divided into embedded subsets that can be decoded to provide video sequences of increasing quality. Here, we consider signal-to-noise ratio (SNR) scalability where the scalable layers lead to an SNR increase in the reconstructed video. Scalability is an attribute of great practical importance for video transmission from a server to multiple users over heterogeneous networks, such as the Internet.

B. Channel Coding

We use rate-compatible punctured convolutional (RCPC) codes for channel coding. The rate of a convolutional code is defined as k/n where k is the number of input bits, and n is the number of output bits. For variable rate coding, a higher rate code can be obtained by puncturing the output of a "mother" code of rate $1/n$ [8]. For rate compatibility, higher rate codes are chosen to be a subset of a lower rate code. Decoding of the convolutional codes is carried out by the soft-decision Viterbi algorithm.

C. Received Signal

The mobile radio communications link is modeled as a multipath fading channel. We consider antenna array reception. The baseband received signal at each antenna element m, $m = 1, \dots, M$, is the aggregate of the multipath received direct-sequence (DS) spread spectrum (SS) signal of interest with signature code $\mathbf{s}_0$ of length L, $K-1$ multipath received DS-SS interferers with unknown signatures $\mathbf{s}_k$, $k = 1, \dots, K-1$, and white Gaussian noise. Without loss of generality, a chip-synchronous signal setup is used. We assume that the multipath spread is of the order of a few chip intervals, P. After conventional chip-matched filtering and sampling at the chip rate over

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a multipath extended symbol interval of $L+P$ chips, the $L+P$ data samples from the mth antenna element, $m=1,\ldots,M$, are organized in the form of a vector $\mathbf{r}_m$ given by

$$\mathbf{r}_m = \sum_{k=0}^{K-1}\sum_{p=0}^{P} c_{k,p}\sqrt{E_k}\left(b_k\mathbf{s}_{k,p} + b_k^-\mathbf{s}_{k,p}^- + b_k^+\mathbf{s}_{k,p}^+\right) \times \mathbf{a}_{k,p}[m] + \mathbf{n}, m=1,\ldots,M \quad (1)$$

where, with respect to the kth SS signal, E_k is the transmitted energy per bit, b_k, b_k^-, and b_k^+ are the present, the previous, and the following transmitted bit, respectively, and $\{c_{k,p}\}$ are the coefficients of the frequency-selective slowly fading channel modeled as independent zero-mean complex Gaussian random variables that are assumed to remain constant over several symbol intervals. $\mathbf{s}_{k,p}$ represents the 0-padded by P, p-cyclic-shifted version of the signature of the kth SS signal $\mathbf{s}_k$, $\mathbf{s}_{k,p}^-$ is the 0-filled $(L-p)$-left-shifted version of $\mathbf{s}_{k,0}$, and $\mathbf{s}_{k,p}^+$ is the 0-filled $(L-p)$-right-shifted version of $\mathbf{s}_{k,0}$. Finally, $\mathbf{a}_{k,p}[\mathrm{m}]$ is the mth coordinate of the kth SS signal, pth path, antenna array response vector and $\mathbf{n}$ represents additive complex Gaussian noise.

An auxiliary-vector space-time adaptive filter [3], [5] is used for despreading and interference mitigation, since the AV filter has been shown to be a more effective solution for adaptive filtering under limited data record support in a rapidly changing wireless communications environment.

In this work, we consider a potential combination of *multicode multirate CDMA* and *variable sequence length multirate CDMA*. In multicode CDMA, more than one CDMA channel (code) can be allocated to a user. In variable sequence length CDMA, the spreading sequences of different CDMA channels can have different lengths. Consider as an example two CDMA channels: Channel 1 with a spreading code of length $L=16$ and Channel 2 with a spreading code of length $L=32$. Assuming equal energy per chip for the two channels, Channel 2 transmits twice as many chips per bit (and energy per bit) than Channel 1. However, Channel 1 transmits at twice the bit rate of Channel 2. Thus, Channel 2 exhibits a lower bit error rate than Channel 1, but Channel 1 allows for transmission at a higher data rate.

III. Optimal Resource Allocation

The optimization constraint for the single channel and multiple channel cases is the available chip rate, $R_{\mathrm{budget}}^{\mathrm{chip}}$. Fixed energy per chip is assumed.

A. Single CDMA Channel Case

If a single CDMA channel (code) is used for the transmission of all scalable layers of a video user, the layers are time-multiplexed. The available transmission bit rate is

$$R_{\mathrm{budget}} = \frac{R_{\mathrm{budget}}^{\mathrm{chip}}}{L} \quad (2)$$

where L is the spreading code length (system processing gain), and R_{budget} is the available bit rate.

The formal statement of the problem at hand is as follows, Given an overall bit rate R_{budget}, allocate bits optimally between source and channel coding such that the overall mean-square distortion D_{s+c} is minimized, i.e.,

$$\min D_{s+c} \text{ subject to } R_{s+c} \le R_{\mathrm{budget}} \quad (3)$$

where R_{s+c} is the total bit rate used for source and channel coding for all layers, and D_{s+c} is the resulting expected squared-error distortion which is due to both source coding errors and channel errors.

Via Lagrangian optimization, the constrained problem in (3) is transformed to the unconstrained problem of minimizing

$$\mathcal{L}(\lambda) = D_{s+c} + \lambda R_{s+c} \quad (4)$$

where λ is the Lagrange multiplier.

For T scalable layers, R_{s+c} is equal to

$$R_{s+c} = \sum_{l=1}^{T} R_{s+c,l} \quad (5)$$

where $R_{s+c,l}$ is the bit rate used for source and channel coding for the scalable layer l. $R_{s+c,l}$ is equal to

$$R_{s+c,l} = \frac{R_{s,l}}{R_{c,l}} \quad (6)$$

where $R_{s,l}$ and $R_{c,l}$ are the source and channel rates, respectively, for the scalable layer l. It should be emphasized that $R_{s,l}$ is in bits per second and $R_{c,l}$ is a dimensionless number.

The task described in (3) is a discrete optimization problem: $R_{s,l}$ and $R_{c,l}$ can only take values from discrete sets that are defined as part of the problem. Optimization can be carried out by a Lagrange procedure. The overall distortion D_{s+c} can be written as the sum of distortions per scalable layer

$$D_{s+c} = \sum_{l=1}^{T} D_{s+c,l}. \quad (7)$$

In this letter, we define the distortion per layer as the *differential improvement* due to the inclusion of this layer in the reconstruction. Therefore, in the absence of channel errors, only the distortion for layer 1 would be positive and the distortions for all other layers would be negative, since inclusion of these layers reduces the mean squared error (mse). Of course, in the presence of channel errors, it is possible for the distortion of any layer to be positive since inclusion of a badly damaged enhancement layer can increase the mse [1]. The differential improvement in mse due to a given layer depends on the rates of the previous layers. Therefore, the distortion per layer is better written as

$$D_{s+c} = \sum_{l=1}^{T} D_{s+c,l}(R_{s+c,1},\ldots,R_{s+c,l}). \quad (8)$$

The optimization problem in (3) reduces to the problem of finding the *operational rate-distortion functions* (ORDF) $D_{s+c,l}(\cdot,\ldots,\cdot)$ for each scalable layer. If the ORDFs were directly obtained, we would need to conduct simulations for all possible combinations of source and channel coding as well as all channel conditions under consideration. This would have high computational complexity. Furthermore, the ORDFs would need to be recalculated if different channel coding or channel model were used. Due to the high computational complexity of experimentally obtaining the ORDFs, we use here the *universal rate-distortion characteristics* (URDC) [1], [9] at the expense of a small performance penalty.

B. Multiple CDMA Channel Case

In this case, each scalable video layer is transmitted over a separate DS-CDMA channel. The available bit rate for layer i is

$$R_{s+c,i} = \frac{R_{\text{budget}}^{\text{chip}}}{L_i} \tag{9}$$

where L_i is the spreading length for layer i. Thus, if two layers are assumed the ratio $R_{s+c,1}/R_{s+c,2}$ is fixed and equal to L_2/L_1.

For the case of T layers, the optimization problem is now as follows:

$$\min D_{s+c} \text{ subject to } L_i R_{s+c,i} \leq R_{\text{budget}}^{\text{chip}}, \qquad i = 1, \ldots, T. \tag{10}$$

For each layer, the source coding rate $R_{s,i}$, the channel coding rate $R_{c,i}$, and the spreading length L_i are to be determined. As mentioned previously, the total bit rate allocated to a scalable layer depends only on L_i and not on any decisions made for another layer. Since $D_{s+c} = \sum_{i=1}^{T} D_{s+c,i}(R_{s+c,1}, \ldots, R_{s+c,i})$, the problem can be broken into separate problems for each layer, thus simplifying the optimization when compared to the single CDMA channel case. For the case of two layers, the two problems to be solved can be written as follows:

$$\begin{aligned} \{R_{s,1}^*, R_{c,1}^*, L_1^*\} &= \arg\min D_{s+c,1}(R_{s+c,1}) \\ &\qquad \text{subject to} L_1 R_{s+c,1} \leq R_{\text{budget}}^{\text{chip}} \qquad (11), \\ \{R_{s,2}^*, R_{c,2}^*, L_2^*\} &= \arg\min D_{s+c,2}\left(R_{s+c,1}^*, R_{s+c,2}\right), \\ &\qquad \text{subject to } L_2 R_{s+c,2} \leq R_{\text{budget}}^{\text{chip}}. \qquad (12) \end{aligned}$$

Thus, optimization for the two-channel video transmission case is algebraically simpler than optimization in the single-channel case. Since there is no dependency between layers, the above two problems can be solved independently using Lagrangian optimization. As in the single-channel case, we need to obtain the rate-distortion functions for each layer $D_{s+c,i}(.)$,$i = 1, 2$.

The total rate R_{s+c} for each combination of source and channel codes is obtained from the channel characteristic plots and the URDCs [1], [9] thereby generating the rate-distortion operating points for the given channel conditions.

IV. Minimum TSC Binary Spreading Code Sets

For the optimization problems in Section III, we choose spreading codes from the class of minimum total-squared-correlation optimal binary spreading codes [6], [7] that are available for all spreading code lengths L and number of signatures K except $K = L = 4n+1$, $n = 1, 2, \ldots$.

The TSC is a fundamental measure of the cross-correlation properties of a signature set. If $\mathcal{S} = \{\mathbf{s}_0, \mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_{K-1}\}$, $\mathbf{s}_i \in \mathcal{C}^L$, $\|\mathbf{s}_i\| = 1$, $i = 0, 1, 2, \ldots, K-1$ is a set of complex valued user signatures of length L, then the TSC of set $\mathcal{S}$ is

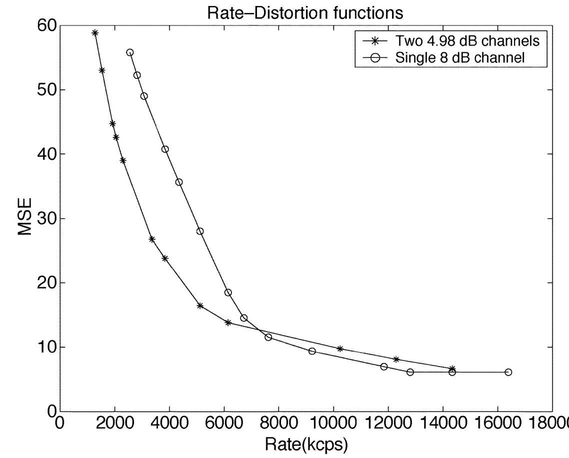

Fig. 1. Rate-distortion performance of scalable video over DS-CDMA channels using Karystinos–Pados codes.

defined [10] as the sum of the squared magnitudes of all inner products between signatures

$$\text{TSC}(\mathcal{S}) \triangleq \sum_{i=0}^{K-1} \sum_{j=0}^{K-1} \left|\mathbf{s}_i^H \mathbf{s}_j\right|^2 \tag{13}$$

where H is the Hermitian transpose operator. Welch showed [11] that for $K \geq L$, $\text{TSC}(\mathcal{S}) \geqslant (K^2/L)$ (for $K < L$, $\text{TSC}(\mathcal{S}) \geq K$ trivially). While for real/complex-valued signature sets, the Welch bound (K^2/L for $K \geq L$ and K for $K < L$) is always achievable, this is not the case for binary antipodal signature sets. In [6], new bounds were derived on the TSC of binary antipodal signature sets for all possible combinations of the values of K (number of users) and L (processing gain). The new bounds were shown to be tight in all cases except when $K = L \equiv 1 (\text{mod } 4)$, which remains an open problem [6], [7]. For all other K, L, optimum binary signature sets that achieve the new bounds were designed via simple Hadamard matrix transformations [6].

A. Karystinos–Pados Bounds for Non-Overloaded Systems

Let $\mathcal{S} = \{\mathbf{s}_i\}_{i=0}^{K-1}$ be a binary antipodal signature set where $\mathbf{s}_i \in \{\pm 1^L\}$, $i = 0, 1, 2, \ldots, K-1$, $K \leqslant L$. The TSC of this signature set $\mathcal{S}$ is given by $\text{TSC}(\mathcal{S}) = \sum_{i=0}^{K-1} \sum_{j=0}^{K-1} (\mathbf{s}_i^T \mathbf{s}_j)^2 = KL^2 + \sum_{i=0}^{K-1} \sum_{j=0, j\neq i}^{K-1} (\mathbf{s}_i^T \mathbf{s}_j)^2$, where T is the transpose operator. The second, double-summation term is the TSC between different users in $\mathcal{S}$.

Equations defining new Karystinos–Pados bounds on the TSC of binary antipodal signature sets for nonoverloaded ($K \leqslant L$) systems can be written as follows:

$$\text{TSC}(\mathcal{S}) \geqslant \begin{cases} KL^2, & L \equiv 0 \,(\text{mod } 4) \\ KL^2 + 2K(K-2), & L \equiv 2 \,(\text{mod } 4) \\ & K \equiv 0 \,(\text{mod } 2) \\ KL^2 + 2(K-1)^2, & L \equiv 2 \,(\text{mod } 4) \\ & K \equiv 1 \,(\text{mod } 2) \\ KL^2 + K(K-1), & L \equiv 1 \,(\text{mod } 2) \end{cases}. \tag{14}$$

A flowchart showing simple algorithms based on Hadamard matrix transformations for the design of optimum binary signature sets that achieve the TSC bound (for both underloaded and overloaded systems) can be found in [6].

TABLE I
OPTIMAL RATE ALLOCATION FOR TWO-LAYER SNR SCALABLE VIDEO OVER A SINGLE 8-dB DS-CDMA CHANNEL WITH KARYSTINOS–PADOS SPREADING CODES

Total Rate	Total Distortion	Base Layer			Enhancement Layer			Base Distortion
R_{budget}^{chip*}	D_{s+c}^{*}	$R_{s,1}^{*}$	$R_{c,1}^{*}$	L_1^{*}	$R_{s,2}^{*}$	$R_{c,2}^{*}$	L_2^{*}	$D_{s+c,1}^{*}$
2560	55.83	64	4/5	16	64	2/3	16	60.89
2816	52.24	64	2/3	16	64	4/5	16	55.30
3072	49.03	64	2/3	16	64	2/3	16	55.30
3584	43.54	96	2/3	16	64	4/5	16	53.10
3840	40.78	96	2/3	16	64	2/3	16	53.10
4352	35.64	96	1/2	16	64	4/5	16	47.70
4608	33.34	128	2/3	16	64	2/3	16	46.80
4800	31.41	96	4/5	24	64	4/5	24	43.11
5120	28.02	128	2/3	16	64	1/2	16	46.80
6144	18.48	128	4/5	24	64	2/3	24	43.21
6720	14.54	128	4/5	28	64	4/5	28	39.93
7616	11.53	128	2/3	28	64	4/5	28	39.91
8960	9.73	128	2/3	28	64	1/2	28	39.91
9216	9.36	128	1/2	24	64	1/2	24	39.55
11200	7.64	256	4/5	28	64	4/5	28	9.33
11840	6.96	256	1/2	20	64	4/5	20	7.24
13312	6.12	256	4/5	32	64	2/3	32	6.86
14336	6.11	256	4/5	32	64	1/2	32	6.56

TABLE II
OPTIMAL RATE ALLOCATION FOR TWO-LAYER SNR SCALABLE VIDEO OVER TWO SEPARATE 4.98-dB DS-CDMA CHANNELS USING KARYSTINOS–PADOS CODES

Total Rate	Total Distortion	Base Layer			Enhancement Layer			Base Distortion
R_{budget}^{chip*}	D_{s+c}^{*}	$R_{s,1}^{*}$	$R_{c,1}^{*}$	L_1^{*}	$R_{s,2}^{*}$	$R_{c,2}^{*}$	L_2^{*}	$D_{s+c,1}^{*}$
1280	58.84	64	4/5	16	64	4/5	16	66.86
1536	53.02	64	2/3	16	64	2/3	16	60.73
1920	44.73	64	2/3	20	64	4/5	24	52.70
2048	42.60	64	1/2	16	64	1/2	16	47.72
2304	39.02	96	2/3	16	64	2/3	24	40.25
3360	26.75	96	4/5	28	96	4/5	28	32.22
3840	23.79	96	4/5	32	96	4/5	32	27.37
5120	16.45	128	4/5	32	128	4/5	32	17.51
6144	13.81	128	2/3	32	96	1/2	32	17.37
10240	9.78	256	4/5	32	256	4/5	32	12.82
12288	8.11	256	1/2	24	256	1/2	24	10.65
14336	6.66	256	1/2	28	256	1/2	28	7.27

V. EXPERIMENTAL RESULTS

In this section, we present experimental results that compare scalable video transmission over one and two DS-CDMA channels for the signal model in (1) and an MPEG-4 compatible video source codec. SNR scalability was implemented using the "Spatial Scalability" mode of MPEG-4 (MoMuSys implementation). The "Foreman" test sequence was used. All SNR values identified in the experimental study refer to the total SNR per chip, defined as $(\sum_{p=1}^{P} E\{|c_{k,p}|^2\}E_k/\sigma^2)$, $k=0,1,\ldots,K-1$.

For both the single and dual CDMA channel case, the admissible source coding rates are set at 64 000, 96 000, 128 000, and 256 000 bits per second for both the base and enhancement layer. RCPC codes of rates 1/2, 2/3, and 4/5 from [8] were used for channel coding. Minimum TSC binary signature sequences of length 16, 20, 24, 28, and 32 that correspond to the case of $L \equiv 0 (\mathrm{mod}\ 4)$ were generated according to [6]. Fig. 1 shows a comparison of the rate-distortion performance characteristics of scalable video transmission over DS-CDMA channels with minimum TSC codes when the video user occupies a single channel or two CDMA channels. The mse distortion is plotted against the total chip rate. Tables I and II show the exact distortion values and the optimal allocation of source coding rate, channel coding rate and spreading sequence lengths for the base and enhancement layers under single and two CDMA channel transmission, respectively. For the SNR values under consideration, the 4.98-dB two-channel case exhibits lower mse than the single 8-dB channel case at chip rates lower than 7.5 Mcps. Two CDMA channel transmission allows for higher bit rates at the expense of higher bit error rates. These higher bit rates allow for higher source coding rates and lower channel coding rates (to combat the increased bit error rate) to be used. As can be seen in Tables I and II, this is important for relatively low chip rates (e.g., see the case 6144 kcps). At high chip rates, two-channel transmission can still support higher source coding rates than single-channel transmission but increasing the source coding rate beyond a point does not significantly improve video quality. Thus, single CDMA channel transmission (with lower bit error rates) outperforms two CDMA channel transmission at high chip rates. Other SNR values were tested and confirm that the trends are similar. Third-generation mobile communication systems based on W-CDMA standards typically employ a chip rate of 3.84 Mcps. As seen in Fig. 1, the two-channel multirate video access scheme under investigation performs exceptionally well at this chip rate.

VI. CONCLUSION

The performance of scalable video transmission via a single-rate CDMA channel was compared against transmission via multirate CDMA channels. A rate-distortion optimization procedure was carried out for the choice of source coding rate, channel rate, and minimum TSC spreading sequence length for each of the scalable layers in the single-rate and multirate scenarios. The results show that with the availability of a greater number of spreading codes the two-channel scalable video system outperforms the single CDMA channel system over a large range of chip rates.

REFERENCES

[1] L. P. Kondi, F. Ishtiaq, and A. K. Katsaggelos, "Joint source-channel coding for motion-compensated DCT-based SNR scalable video," *IEEE Trans. Image Processing*, vol. 11, pp. 1043–1054, Sept. 2002.

[2] L. P. Kondi, S. N. Batalama, D. A. Pados, and A. K. Katsaggelos, "Joint source-channel coding for scalable video over DS-CDMA multipath fading channels," in *Proc. IEEE Int. Conf. Image Processing*, vol. 1, Thessaloniki, Greece, Oct. 2001, pp. 994–997.

[3] D. A. Pados and G. N. Karystinos, "An iterative algorithm for the computation of the MVDR filter," *IEEE Trans. Signal Processing*, vol. 49, pp. 290–300, Feb. 2001.

[4] Q. Zhao, P. Cosman, and L. B. Milstein, "Tradeoffs of source coding, channel coding and spreading in CDMA systems," in *Proc. MILCOM*, vol. 2, Los Angeles, CA, Oct. 2000, pp. 846–850.

[5] L. P. Kondi, D. Srinivasan, D. A. Pados, and S. N. Batalama, "Layered video transmission over multirate DS-CDMA wireless systems," *IEEE Trans. Circuits Syst. Video Technol.*, submitted for publication.

[6] G. N. Karystinos and D. A. Pados, "New bounds on the total squared correlation and optimum design of DS-CDMA binary signature sets," *IEEE Trans. Commun.*, vol. 51, pp. 48–51, Jan. 2003.

[7] C. S. Ding, M. Golin, and T. Klove, "Meeting the Welch and Karystinos-Pados bounds on DS-CDMA binary signature sets," *Designs, Codes, Cryptography*, vol. 30, no. 1, pp. 73–84, Aug. 2003.

[8] J. Hagenauer, "Rate-compatible punctured convolutional codes (RCPC codes) and their applications," *IEEE Trans. Commun.*, vol. 36, pp. 389–400, Apr. 1988.

[9] M. Bystrom and T. J. W. Modestino, "Combined source-channel coding for video transmission over an additive white Gaussian noice channel," *IEEE J. Select Areas Commun.*, vol. 18, pp. 880–890, June 2000.

[10] J. L. Massey and T. Mittelholzer, *Sequences II, Methods in Communication, Security, and Computer Sciences*. New York: Springer-Verlag, 1993, ch. Welch's bound and sequence sets for code-division multiple-access systems.

[11] L.-R. Welch, "Lower bounds on the maximum cross correlation of signals," *IEEE Trans. Inform. Theory*, vol. IT-20, pp. 397–399, May 1974.