Resolution Enhancement of Video Sequences with Simultaneous Estimation of the Regularization Parameter

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ABSTRACT

In this paper, we extend our previous image resolution enhancement results in [1] by proposing a technique for the estimation of the regularization parameter based on the assumption it should satisfy the following properties: It should be a function of the regularized noise power of the data and its choice should yield a convex functional whose minimization would give the desired high-resolution image. Experimental results are presented and conclusions are drawn.

Keywords: Resolution enhancement, superresolution, MAP estimation, image restoration, regularization

1. INTRODUCTION

In many imaging systems, the resolution of the detector array of the camera is not sufficiently high for a particular application. Furthermore, the capturing process introduces additive noise and the point spread function of the lens and the effects of the finite size of the photo-detectors further degrade the acquired video frames. The goal of resolution enhancement is to estimate a high-resolution image from a sequence of low-resolution images while also compensating for the above-mentioned degradations.

Resolution enhancement using multiple frames is possible when there exists subpixel motion between the captured frames. Thus, each of the frames provides a unique look into the scene. An example scenario is the case of a camera that is mounted on an aircraft and is imaging objects in the far field. The vibrations of the aircraft will generally provide the necessary motion between the focal plane array and the scene, thus yielding frames with subpixel motion between them and minimal occlusion effects.

In this paper, we extend our previous results [1] by utilizing a technique for the estimation of the regularization parameter. The rest of the paper is organized as follows. In section 2, a MAP-based resolution enhancement technique from [1] is introduced, which leads to solving a regularized cost function. In section 3, we discuss our technique for the estimation of the regularization parameter. In section 4, experimental results are presented. Finally, in section 5, conclusions are drawn.

2. MAP-BASED RESOLUTION ENHANCEMENT

The problem of video resolution enhancement using multiple frames has been studied for a long time. Tsai and Huang first addressed frequency domain approach in [3]. The unaliased image can be solved for from a set of equations if enough frames with known shifts are available, however, it is impractical in many applications. Another method is Projection Onto Convex Sets (POCS). This method has been extended to treat motion blur and noise and can be applied to the space-varying restoration problem using a number of space-domain constraints [4].

This problem has been approached from the class of Bayesian methods. Maximum *a posteriori* (MAP) estimation with an edge preserving Huber-Markov random field image prior is studied in [5], [6], [7]. MAP based resolution enhancement with simultaneously estimation of registration parameters (motion between frames) has been proposed [1], [8]. In the following, we use the same model and notation as in [1].

We order all vectors lexicographically. We assume that p low-resolution frames are observed, each of size $N_1 \times N_2$. The desired high-resolution image $\mathbf{z}$ is of size $N = L_1 N_1 L_2 N_2$ and L_1 and L_2 represent the down-sampling factors in the horizontal and vertical directions respectively. Thus, the observed low resolution images are related to the high resolution image through blurring, motion shift and subsampling. Let the kth low-resolution frame be denoted as $\mathbf{y}_k = [y_{k,1}, y_{k,2}, \cdots y_{k,M}]^T$ for $k = 1,2,\cdots p$ and where $M = N_1 N_2$. The full set of p observed low-resolution images can be denoted as

$$\mathbf{y} = [\mathbf{y}_1^T, \mathbf{y}_2^T, \cdots, \mathbf{y}_p^T]^T = [y_1, y_2, \cdots, y_{pM}]^T \tag{1}$$

The observed low resolution frames are related to the high-resolution image through the following model:

$$y_{k,m} = \sum_{r=1}^{N} w_{k,m,r}(\mathbf{s}_k) z_r + \eta_{k,m} \tag{2}$$

for $m = 1,2,\cdots M$ and $k = 1,2,\cdots p$. The weight $w_{k,m,r}(s_k)$ represents the "contribution" of the rth high-resolution pixel to the mth low resolution observed pixel of the kth frame. The vector $s_k = [s_{k,1}, s_{k,2}, \cdots s_{k,K}]^T$, is the K registration parameters for frame k, representing global translational shift, rotation, affine transformation parameters, or other motion parameters. This motion is measured in reference to a fixed high resolution grid. The term $\eta_{k,m}$ represents additive noise samples that will be assumed to be independent and identically distributed (i.i.d.) Gaussian noise samples with variance σ_η^2.

It will be convenient to represent the observation model in matrix notation.

$$\mathbf{y} = \mathbf{W_s z} + \mathbf{n} \tag{3}$$

where matrix $\mathbf{W_s}$ contains the values $w_{k,m,r}$ and $\mathbf{n} = [\eta_1, \eta_2, \cdots \eta_{pM}]^T$. Note that since the elements of $\mathbf{n}$ are i.i.d. Gaussian samples, the multivariate pdf of $\mathbf{n}$ is given by

$$P_r(\mathbf{n}) = \frac{1}{(2\pi)^{\frac{pM}{2}} \sigma_\eta^{pM}} \exp\left\{-\frac{1}{2\sigma_\eta^2} \mathbf{n}^T \mathbf{n}\right\} = \frac{1}{(2\pi)^{\frac{pM}{2}} \sigma_\eta^{pM}} \exp\left\{-\frac{1}{2\sigma_\eta^2} \sum_{m=1}^{pM} \eta_m^2\right\} \tag{4a}$$

and

$$P_r(\mathbf{y}|\mathbf{z},\mathbf{s}) = \frac{1}{(2\pi)^{\frac{N}{2}} \sigma_\eta^{N}} \exp\left\{-\frac{1}{2\sigma_\eta^2} (\mathbf{y} - \mathbf{W_s z})^T (\mathbf{y} - \mathbf{W_s z})\right\} \tag{4b}$$

We can form a MAP estimate of the high-resolution image $\mathbf{z}$ and the registration parameters $\mathbf{s}$ simultaneously, given the bserved $\mathbf{y}$. The estimates can be computed as

$$\hat{\mathbf{z}}, \hat{\mathbf{s}} = \arg\max_{\mathbf{z},\mathbf{s}} P_r(\mathbf{z},\mathbf{s}|\mathbf{y}) \tag{5}$$

Using Bayes rule, this can alternatively be expressed as

$$\hat{\mathbf{z}}, \hat{\mathbf{s}} = \arg\max_{\mathbf{z},\mathbf{s}} \frac{P_r(\mathbf{y}|\mathbf{z},\mathbf{s}) P_r(\mathbf{z},\mathbf{s})}{\Pr(\mathbf{y})} \quad (6)$$

Since the denominator is not a function of $\mathbf{z}$ or $\mathbf{s}$, and that $\mathbf{z}$ and $\mathbf{s}$ are statistically independent, the estimates can be written as

$$\hat{\mathbf{z}}, \hat{\mathbf{s}} = \arg\max_{\mathbf{z},\mathbf{s}} P_r(\mathbf{y}|\mathbf{z},\mathbf{s}) P_r(\mathbf{z}) P_r(\mathbf{s}) \quad (7)$$

It is more convenient, and equivalent, to minimize the negative logarithm of the functional in (7). This yields

$$\hat{\mathbf{z}}, \hat{\mathbf{s}} = \arg\min_{\mathbf{z},\mathbf{s}} L(\mathbf{z},\mathbf{s}) = \arg\min_{\mathbf{z},\mathbf{s}} \left\{ -\log\left[P_r(\mathbf{y}|\mathbf{z},\mathbf{s})\right] - \log\left[P_r(\mathbf{z})\right] - \log\left[P_r(\mathbf{s})\right] \right\} \quad (8)$$

If all possible vectors $\boldsymbol{s}$ are equally probable, we can drop the term for $Pr(\boldsymbol{s})$

$$\hat{\mathbf{z}}, \hat{\mathbf{s}} = \arg\min_{\mathbf{z},\mathbf{s}} L(\mathbf{z},\mathbf{s}) = \arg\min_{\mathbf{z},\mathbf{s}} \left\{ -\log\left[P_r(\mathbf{y}|\mathbf{z},\mathbf{s})\right] - \log\left[P_r(\mathbf{z})\right] \right\} \quad (9)$$

The prior image can be chosen to be a Gauss-Markov random field with density of the form:

$$\Pr(\mathbf{z}) = \frac{1}{(2\pi)^{\frac{N}{2}} |C|^{\frac{1}{2}}} \exp\left\{ -\frac{1}{2} \mathbf{z}^T C^{-1} \mathbf{z} \right\} \quad (10)$$

where C is the $N \times N$ covariance matrix of z. For a specific choice of the covariance matrix C, the above equation can be written as

$$\Pr(\mathbf{z}) = \frac{1}{(2\pi)^{\frac{N}{2}} |C|^{\frac{1}{2}}} \exp\left\{ -\frac{1}{2\lambda} \sum_{i=1}^{N} \left(\sum_{j=1}^{N} d_{i,j} z_j \right)^2 \right\} \quad (11)$$

where $d_i = \left[d_{i,1}, d_{i,2}, \cdots d_{i,N}\right]^T$ is the coefficient vector and λ is called temperature or tuning parameter. Thus

$$C_{i,j}^{-1} = \frac{1}{\lambda} \sum_{i=1}^{N} d_{r,i} d_{r,j} \quad (12)$$

where

$$d_{i,j} = \begin{cases} 1 & for\ i = j \\ -1/4 & for\ j : z_j\ is\ a\ cardinal\ neighbor\ of\ z_j \end{cases} \quad (13)$$

Thus, the cost function we need to minimize becomes

$$L(\mathbf{z},\mathbf{s}) = \frac{1}{2\sigma^2} (\mathbf{y} - \mathbf{W_s z})^T (\mathbf{y} - \mathbf{W_s z}) + \frac{1}{2} \mathbf{z}^T C^{-1} \mathbf{z} \quad (14)$$

If we express the above equation using the individual elements of the matrices and vectors, we get

$$L(\mathbf{z},\mathbf{s}) = \frac{1}{2\sigma_\eta^{\ 2}}\sum_{m=1}^{pM}\left(y_m - \sum_{r=1}^{N} w_{m,r}(s)z_r\right)^2 + \frac{1}{2\lambda}\sum_{i=1}^{N}\left(\sum_{j=1}^{N} d_{i,j}z_j\right)^2 \tag{15}$$

The above cost function can be minimized using the coordinate-descent method. This iterative method starts with an initial estimate of **z** obtained using interpolation from a low resolution frame. Then, for a fixed **z**, the cost function is minimized with respect to **s**. Thus, the motion of each frame is estimated. Then, for fixed **s**, a new estimate for **z** is obtained. This procedure continues until convergence is reached, i.e., **z** and **s** are updated in a cyclic fashion. **z** can be updated recursively as

$$\hat{z}_k^{n+1} = \hat{z}_k^{n} - \varepsilon^n g_k(\hat{\mathbf{z}}^n, \hat{\mathbf{s}}^n) \tag{16}$$

until convergence is reached. n is the iteration number starting from 0. In order to update the estimate **z**, we first estimate

$$\hat{s}_k^n = \arg\min_{s_k}\left\{\sum_{m=1}^{M}\left(y_m - \sum_{r=1}^{N} w_{m,r}(s)z_r\right)^2\right\} \tag{17}$$

Also, the gradient can be obtained from

$$g_k(\mathbf{z},\mathbf{s}) = \frac{\partial L(\mathbf{z},\mathbf{s})}{\partial z_k} = \frac{1}{\sigma_\eta^{\ 2}}\sum_{m=1}^{pM} w_{m,k}(\mathbf{s})(\sum_{r=1}^{N} w_{m,r}(\mathbf{s})z_r - y_m) + \frac{1}{\lambda}\sum_{i=1}^{N} d_{i,k}\left(\sum_{j=1}^{N} d_{i,j}z_j\right) \tag{18}$$

To find the optimal step size, we take the partial derivative

$$\begin{aligned}
&\frac{\partial L(\hat{\mathbf{z}}^{n+1},\hat{\mathbf{s}}^n)}{\partial \varepsilon^n} \\
&= \frac{1}{\sigma_\eta^{\ 2}}\sum_{m=1}^{pM}\left\{(\sum_{r=1}^{N} w_{m,r}(\hat{\mathbf{s}}^n)g_r)(y_m - \sum_{r=1}^{N} w_{m,r}(\hat{\mathbf{s}}^n)(\hat{z}_r^n - \varepsilon^n g_r))\right\} + \frac{1}{\lambda}\sum_{i=1}^{N}\left(\sum_{j=1}^{N} d_{i,j}(\hat{z}_j^n - \varepsilon^n g_j)\right)\left(-\sum_{j=1}^{N} d_{i,j}g_j\right) \\
&= (-1)\times\left\{\frac{1}{\sigma_\eta^{\ 2}}\sum_{m=1}^{pM}\left\{(\sum_{r=1}^{N} w_{m,r}(\hat{\mathbf{s}}^n)g_r)(\sum_{r=1}^{N} w_{m,r}(\hat{\mathbf{s}}^n)\hat{z}_r^n - y_m)\right\}\right\} \\
&\quad + \varepsilon^n \times\left\{\frac{1}{\sigma_\eta^{\ 2}}\sum_{m=1}^{pM}\left\{\left(\sum_{r=1}^{N} w_{m,r}(\hat{\mathbf{s}}^n)g_r\right)\left(\sum_{r=1}^{N} w_{m,r}(\hat{\mathbf{s}}^n)g_r\right)\right\}\right\} \\
&\quad + (-1)\times\left\{\frac{1}{\lambda}\sum_{i=1}^{N}\left(\sum_{j=1}^{N} d_{i,j}\hat{z}_j^n\right)\left(\sum_{j=1}^{N} d_{i,j}g_j\right)\right\} \\
&\quad + \varepsilon^n \times\left\{\frac{1}{\lambda}\sum_{i=1}^{N}\left(\sum_{j=1}^{N} d_{i,j}g_j\right)\left(\sum_{j=1}^{N} d_{i,j}g_j\right)\right\}
\end{aligned} \tag{19}$$

By defining

$$\gamma_m = \sum_{r=1}^{N} w_{m,r}(\hat{\mathbf{s}}^n) g_r \tag{20}$$

and

$$\bar{g}_i = \sum_{j=1}^{N} d_{i,j} g_j \tag{21}$$

we can rewrite (19) as

$$\begin{aligned} &\frac{\partial L(\hat{\mathbf{z}}^{n+1}, \hat{\mathbf{s}}^n)}{\partial \varepsilon^n} \\ &= (-1) \times \left\{ \frac{1}{\sigma_\eta^{\ 2}} \sum_{m=1}^{pM} \left\{ \gamma_m \left(\sum_{r=1}^{N} w_{m,r}(\hat{\mathbf{s}}^n) \hat{z}_r^n - y_m \right) \right\} \right\} + \varepsilon^n \times \left\{ \frac{1}{\sigma_\eta^{\ 2}} \sum_{m=1}^{pM} \gamma_m^2 \right\} \\ &\quad + (-1) \times \left\{ \frac{1}{\lambda} \sum_{i=1}^{N} \bar{g}_i \left(\sum_{j=1}^{N} d_{i,j} \hat{z}_j^n \right) \right\} + \varepsilon^n \times \left\{ \frac{1}{\lambda} \sum_{i=1}^{N} \bar{g}_i^2 \right\} \end{aligned} \tag{22}$$

We make the derivative equal to zero and solve for ε^n

$$\varepsilon^n = \frac{\dfrac{1}{\sigma_\eta^{\ 2}} \displaystyle\sum_{m=1}^{pM} \left\{ \gamma_m \left(\sum_{r=1}^{N} w_{m,r}(\hat{\mathbf{s}}^n) \hat{z}_r^n - y_m \right) \right\} + \dfrac{1}{\lambda} \displaystyle\sum_{i=1}^{N} \bar{g}_i \left(\sum_{j=1}^{N} d_{i,j} \hat{z}_j^n \right)}{\dfrac{1}{\sigma_\eta^{\ 2}} \displaystyle\sum_{m=1}^{pM} \gamma_m^2 + \dfrac{1}{\lambda} \displaystyle\sum_{i=1}^{N} \bar{g}_i^2} \tag{23}$$

3. ESTIMATION OF REGULARIZATION PARAMETERS

Image restoration is a typical ill-posed problem. A problem is ill-posed if the solution is not unique or it is not a continuous function of the data, i.e., if an arbitrary small perturbation of the data can cause an arbitrary large perturbation of the solution. Discrete ill-posed problems can be solved by numerical regularization methods in which the solution is stabilized by including appropriate additional information.

The noise in the measurements **y**, in combination with the ill conditioning of matrix $\mathbf{W_s}$, means that the exact solution of the standard LS problem (or Pseudo Inverse Filtering) usually deviates strongly from the noise free solution and is therefore useless. This follows because the pseudo inverse of the blur transfer function usually has a very large magnitude at high frequencies where the blur transfer function has zeros. This results in excessive amplification of the noise at those frequencies. To alleviate this problem, we can apply regularization. After regularization, the ill-posed problem becomes a better-posed problem.

The functional in (15) has two terms: a term representing the fidelity of the solution to the received data and a term representing a priori information about the high-resolution image. The latter involves a high pass filter and thus dictates that the solution be smooth by penalizing discontinuities. The relative weighting of the two terms is determined by a regularization parameter. We show that the above formulation is equivalent to a Maximum A Posteriori (MAP) formulation assuming a Gaussian prior model. The covariance matrix of the Gaussian random field corresponds to the high pass filter in the regularization formulation. It is possible to define a covariance matrix that depends on a constant, which is equivalent to the regularization parameter in the regularization formulation. Therefore, the regularization approach to the resolution enhancement problem is equivalent to the MAP approach for specific choices of the prior model.

The cost function in (15) can be rewritten as

$$\begin{aligned} L(\mathbf{z},\mathbf{s}) &= \frac{1}{2\sigma_\eta^{\ 2}}\sum_{m=1}^{pM}\left(y_m - \sum_{r=1}^{N} w_{m,r}(\mathbf{s})z_r\right)^2 + \frac{1}{2\lambda}\sum_{i=1}^{N}\left(\sum_{j=1}^{N} d_{i,j}z_j\right)^2 \\ &= \frac{1}{2\sigma_\eta^{\ 2}}\left\|\mathbf{y}-\mathbf{W_s z}\right\|^2 + \frac{1}{2\lambda}\left\|D\mathbf{z}\right\|^2 \end{aligned} \tag{24}$$

or equivalently

$$L(\mathbf{z},\mathbf{s}) = \left\|\mathbf{y}-\mathbf{W_s z}\right\|^2 + \alpha(\mathbf{z})\left\|D\mathbf{z}\right\|^2 \tag{25}$$

where D is the $N^2 \times N^2$ matrix representing a high-pass filter. We define the regularization parameter

$$\alpha(\mathbf{z}) \equiv \frac{\sigma_\eta^{\ 2}}{\lambda} \tag{26}$$

which is, in general, a function of the original high resolution image **z**. Furthermore, we can rewrite the cost function as the sum of individual smoothing functionals for each of the p low-resolution images as:

$$L(\mathbf{z},\mathbf{s}) = \sum_{k=1}^{p}\left\{\left\|y_k - W_{s,k}\mathbf{z}\right\|^2 + \alpha_i(\mathbf{z})\left\|C_i\mathbf{z}\right\|^2\right\} \tag{27}$$

In our case, $C_i = D$., i.e., the same high pass filter is assumed for all functionals. We define the regularization parameter for each observed frame as

$$\alpha_k(\mathbf{z}) \equiv \frac{\sigma_\eta^{\ 2}}{\lambda_k} \tag{28}$$

In order for Eqs. (25) and (27) to be equivalent, we should have

$$\sum_{k=1}^{p}\frac{\sigma_\eta^{\ 2}}{\lambda_k} = \sum_{k=1}^{p}\alpha_k(\mathbf{z}) = \alpha(\mathbf{z}) = \frac{\sigma_\eta^{\ 2}}{\lambda} \tag{29a}$$

or

$$\sum_{k=1}^{p}\frac{1}{\lambda_k} = \frac{1}{\lambda} \tag{29b}$$

Now each term in the cost function is the same as the smoothing functional in reference [2]. We choose a regularization parameter that satisfies the following properties: it should be a function of the regularized noise power of the data and its choice should yield a convex functional whose minimization would give the high-resolution image. That will give a global minimum to the cost function regardless to choice of initial value of the high-resolution image. The choice of regularization parameter for the multichannel regularization functional is given by

$$\alpha_k(\mathbf{z}) = \frac{\left\| y_k - W_{s,k}\mathbf{z} \right\|^2}{\frac{1}{\gamma} - \left\| C_k \mathbf{z} \right\|^2} \tag{30}$$

for a linear function $f(\cdot)$ between $\alpha_k(\mathbf{z})$ and each term of the cost function.

$$\alpha_k(\mathbf{z}) = f\left(\left\| y_k - W_{s,k}\mathbf{z} \right\|^2 + \alpha_i(z) \left\| C_i \mathbf{z} \right\|^2 \right) = \gamma \left\{ \left\| y_k - W_{s,k}\mathbf{z} \right\|^2 + \alpha_i(\mathbf{z}) \left\| C_i \mathbf{z} \right\|^2 \right\} \tag{31}$$

where γ is determined by the sufficient conditions for convergence

$$\frac{1}{\gamma} > \left\| y_k - W_{s,k}\mathbf{z} \right\|^2 + \left\| C_i \mathbf{z} \right\|^2 \tag{32}$$

Now, $\left\| y_k \right\|^2 \geq \left\| y_k - W_{s,k}\mathbf{z} \right\|^2$, since all elements of $W_{s,k}\mathbf{z}$ are positive, and $\left\| C_k \mathbf{z} \right\|^2 \leq \left\| \mathbf{y}_k \right\|^2$ since $\mathbf{z}$ is assumed to have more energy at low than at high frequencies. Therefore the choice $\frac{1}{\gamma} = 2\left\| \mathbf{y}_\mathbf{k} \right\|^2$ satisfies the condition for convergence and also provides a positive $\alpha_k(\mathbf{z})$

$$\alpha_k(\mathbf{z}) = \frac{\left\| y_k - W_{s,k}\mathbf{z} \right\|^2}{2\left\| y_k \right\|^2 - \left\| C_k \mathbf{z} \right\|^2} \tag{33}$$

By substituting $\alpha_k(z)$ in (25), we get

$$\lambda_k \equiv \frac{\sigma_\eta^{\ 2}}{\alpha_k(\mathbf{z})} \tag{34}$$

Then from (29b), (33), we obtain the final form for tuning parameter λ

$$\lambda = \frac{1}{\sum_{k=1}^{p} \frac{1}{\lambda_k}} = \frac{1}{\sum_{k=1}^{p} \frac{\alpha_k(\mathbf{z})}{\sigma_\eta^{\ 2}}} = \frac{\sigma_\eta^{\ 2}}{\sum_{k=1}^{p} \alpha_k(\mathbf{z})} = \frac{\sigma_\eta^{\ 2}}{\sum_{k=1}^{p} \left\{ \frac{\left\| y_k - W_{s,k}\mathbf{z} \right\|^2}{2\left\| y_k \right\|^2 - \left\| C_k \mathbf{z} \right\|^2} \right\}} \tag{35}$$

It is clear to see that λ is a function of z. That implies we can adaptively "update" tuning parameter λ according to the current estimate of high-resolution image z in the cyclic fashion.

4. EXPERIMENTAL RESULTS

We used real data of an infrared video sequence provided to us by the Naval Research Laboratory, Washington, DC, to test the proposed algorithm. 20 frames of low-resolution image with size 128x128 pixels are used. Up-sample ratio is $L_1 = L_2$.=4. We assumed a Gaussian point spread function with variance 1.6. The variance of noise can be estimated as a byproduct of the modified cost function $\left\| \mathbf{y} - D\widehat{\mathbf{z}} \right\| / N^2$.

Bilinear interpolation of the first frame is chosen as the first estimate of high-resolution image **z**. We assumed that convergence is reached when $\left\|\hat{\mathbf{z}}^{n+1} - \hat{\mathbf{z}}^{n}\right\| / \left\|\hat{\mathbf{z}}^{n}\right\| < 1e-6$.

The first frame of the low video sequence is shown in Fig.1. In Fig. 2, bilinear interpolation of the first frame is shown as a comparison to the reconstructed high-resolution image using Joint MAP with estimation of regularization parameter λ shown in Fig 3.

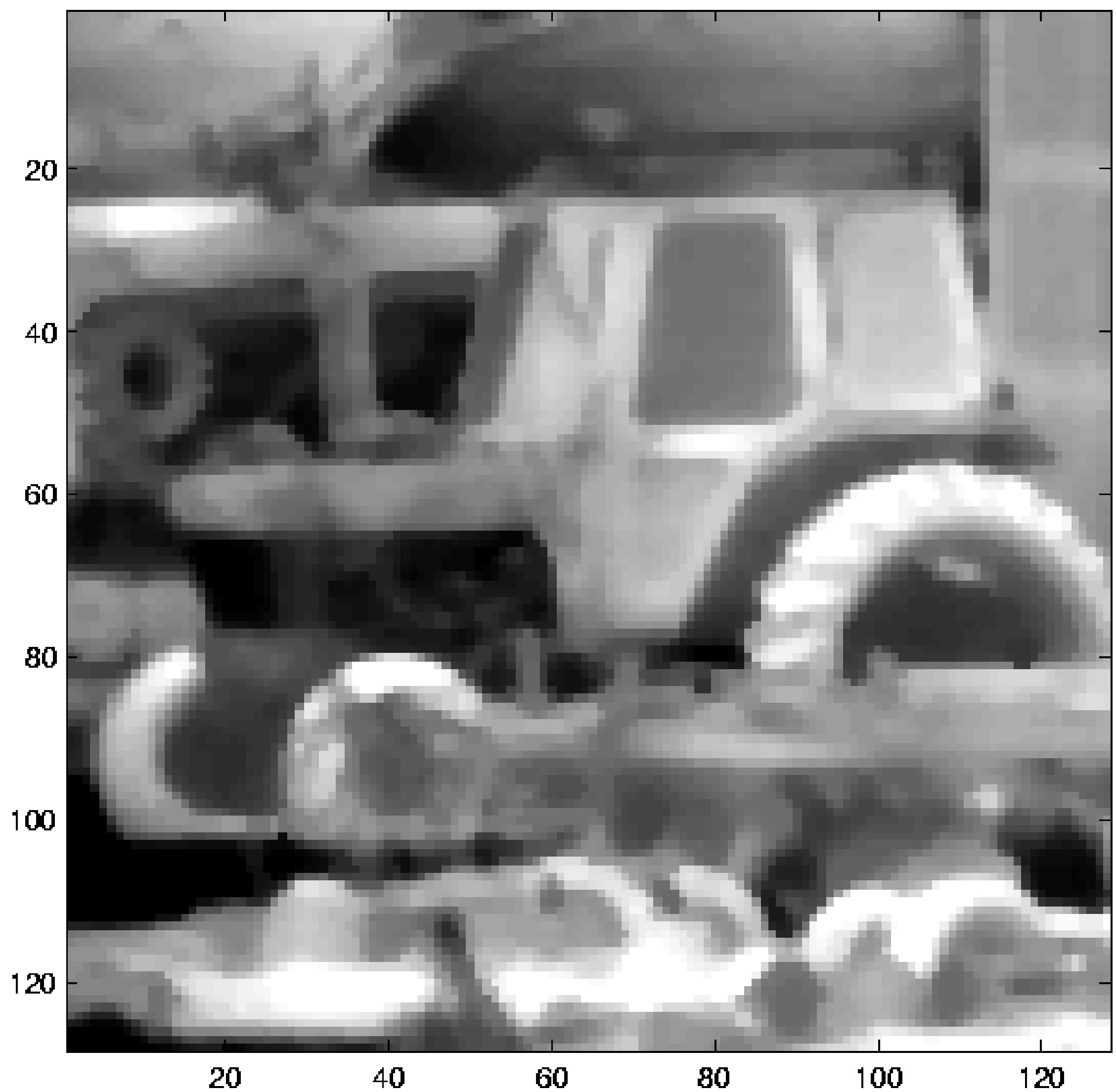

Fig. 1 Original image (first frame of truck sequence)

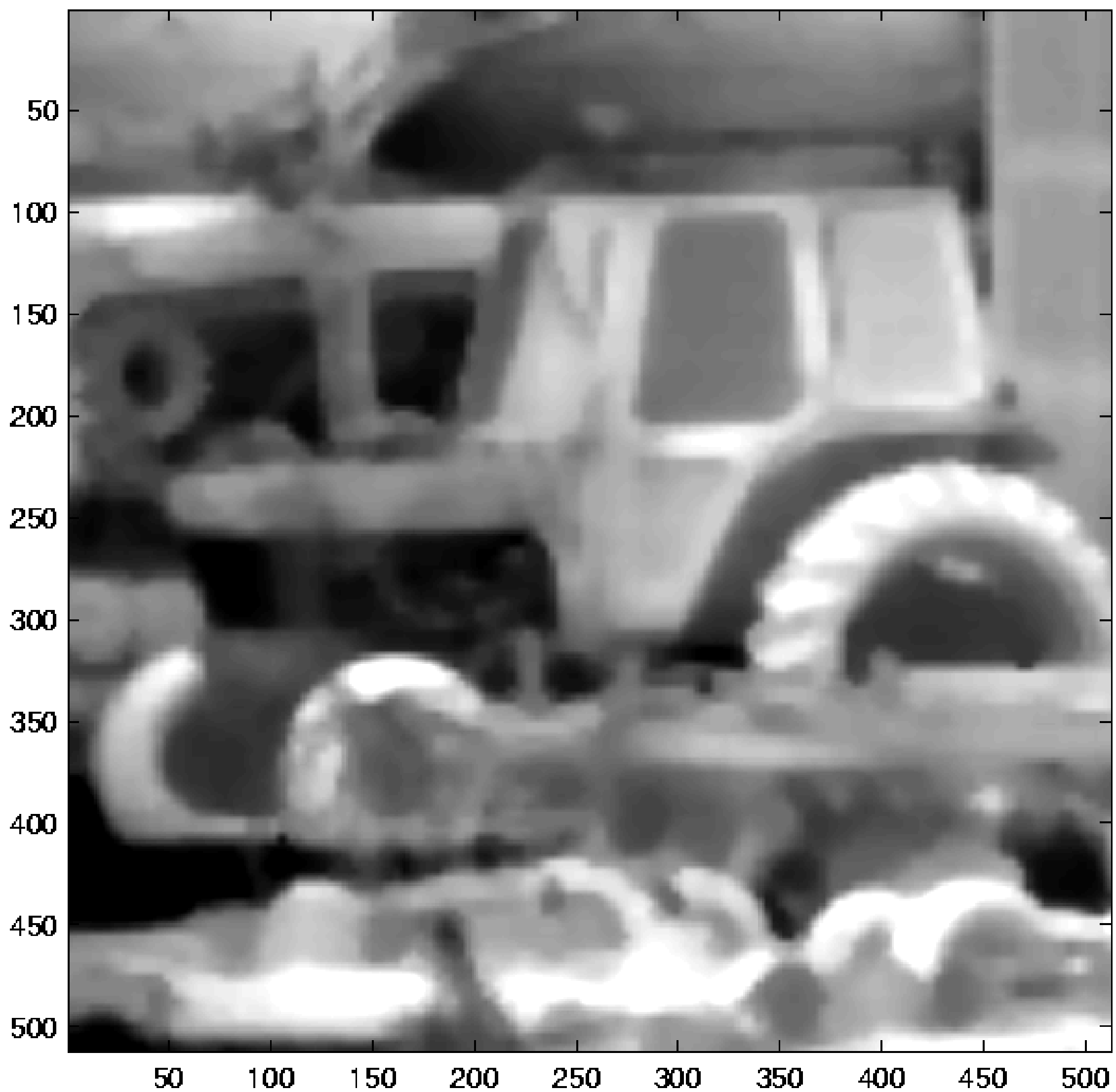

Fig. 2 Bilinear interpolation of first frame

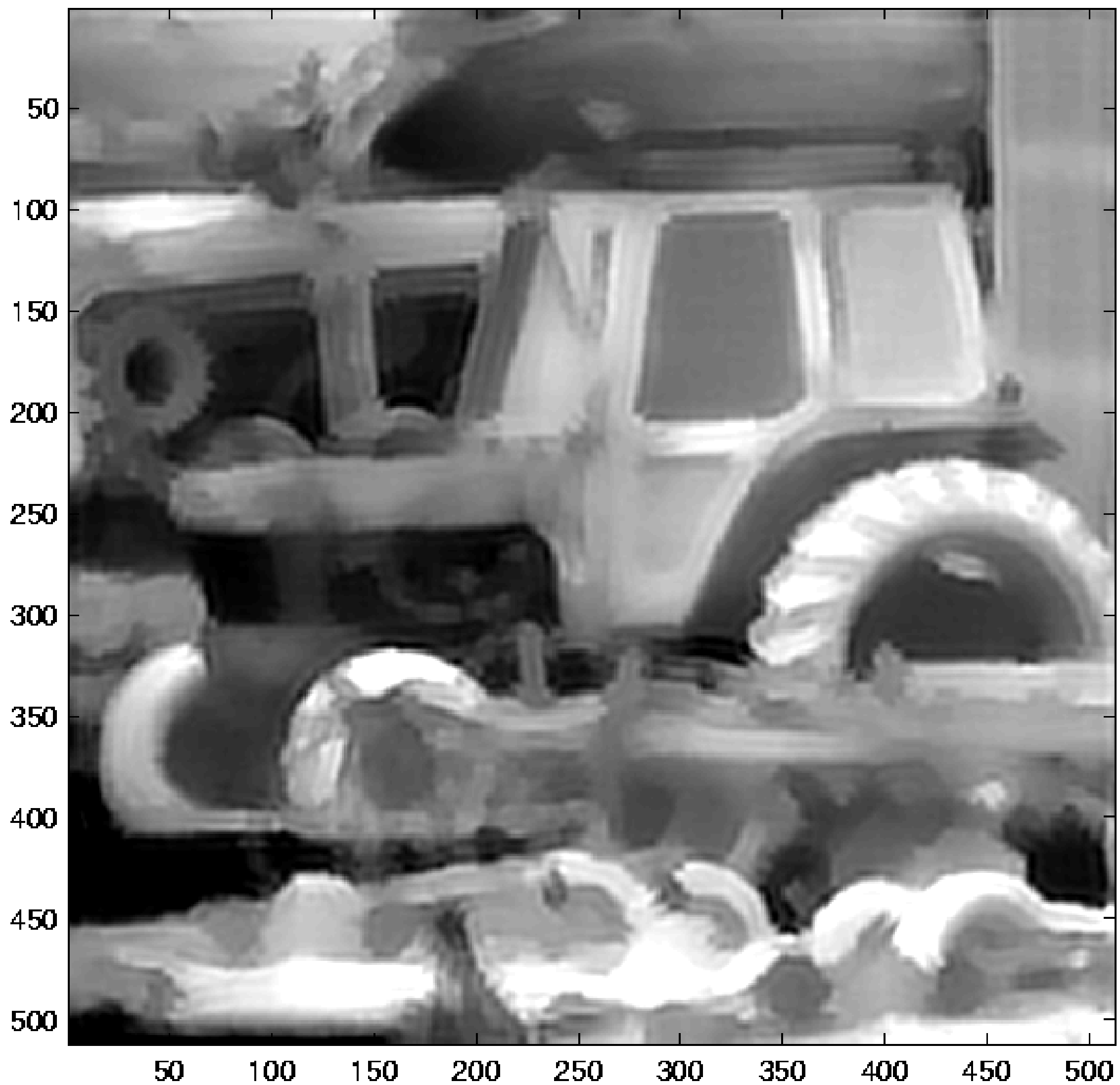

Fig. 3 Reconstructed High-resolution Image using Joint MAP with Estimation of Regularization Parameter λ

5. CONCLUSIONS

We have proposed a technique for the estimation of the regularization parameter for digital image resolution enhancement. Our experimental results demonstrate the performance of the proposed algorithm. The high-resolution image looks much clear than both the original low-resolution image and the one from bilinear interpolation. The regularization parameter acts adaptively as a weight to the fidelity between data and prior model from the multi-channels.

6. REFERENCES

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