Joint Optimal Pilot Placement and Space Frequency (SF) Code Design for MIMO-OFDM Systems

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***Abstract*—In this paper, we propose an optimal power allocation scheme for joint pilot placement and Space Frequency (SF) code design for Multiple Input Multiple Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) wireless channels. OFDM is combined with multiple antennas to increase data diversity gain on a time varying multipath fading channel, resulting in a MIMO-OFDM system. A set of fast time-varying and frequency-selective fading channels is considered. The proposed pilot assisted transmission scheme multiplexes known symbols with information bearing data to estimate the channel in the presence of Inter Symbol Interference (ISI). The channel is assumed to be unchanged for the duration of one OFDM block and change independently from one OFDM block to the other. The number of pilot symbols for each OFDM block is held constant and the symbols are dispersed throughout the block for efficient channel estimation. However, equally distributing the total power between the data and pilots doesn't optimize the transmission over a frequency-selective fading channel. We propose an optimal power allocation scheme for the joint pilot placement and Space Frequency code design scheme based on the channel response and the Bit Error Rate (BER) values. This leads to substantial improvement in performance for a 2×2 MIMO-OFDM system. The results can be extended to MIMO-OFDM systems with any number of transmit and receive antennas.**

***Index Terms*—Multiple Input Multiple Output (MIMO), Orthogonal Frequency Division Multiplexing (OFDM), Pilot Assisted Transmission (PAT), Space Frequency (SF) Codes.**

I. Introduction

During the past few years, there has been an increased interest in multimedia communication over different types of channels. In recent days, a significant amount of research has been focused on multimedia transmission over wireless channels. Multicarrier communication becomes a natural choice to transmit multimedia content at high data rates. Among the multicarrier communication techniques, Orthogonal Frequency Division Multiplexing (OFDM) has become a popular technique for multimedia transmission over wireless channels. It converts a frequency selective fading channel into a parallel collection of frequency-flat subchannels. In [1], we optimized a wireless system for video transmission over an OFDM system. In this work, Unequal Error Protection (UEP) is applied to the layers of the scalable bitstream.

On the other hand, in a Multiple Input Multiple Output (MIMO) communication system, multiple antennas can be used at the transmitter and the receiver. A MIMO system can be implemented in a number of ways to achieve diversity gain [2], [3] to account for signal fading or to obtain capacity gain [4], [5], [6]. Hence, the combination of OFDM technology with MIMO systems becomes a natural choice for high data rate wireless communication [7]. However, the performance of such systems depends upon the knowledge of the channel state information at the receiver.

The channel state information can be obtained by three different methods. One is called blind channel estimation [7], [8] which uses the statistical property of the channel and properties of the transmitted signals. The second technique is a pilot data based technique where a set of training symbols, known beforehand at the receiver is transmitted with data. Channel estimation is done based purely on these pilot symbols, known as non blind channel estimation. The third technique involves channel estimation utilizing information from both the pilot and data symbols. Such a technique is known as semi blind estimation. In [9], [10], [11], estimation for MIMO-OFDM channels has been studied. In [11], a pilot tone design technique was proposed. The pilot tone design was connected to Space Frequency (SF) codes utilizing the ideas of simplified channel estimation algorithm in [9], [10]. In this paper, we propose an alternative approach by extending the pilot placement design in [11].

The rest of the paper is organized as follows. In section II, the basic model for a MIMO-OFDM system along with the signal model are presented. In section III, the optimal pilot placement problem for channel estimation is formulated. The optimal power allocation between the data and pilots is formulated in Section IV. In Section V, experimental results are presented. Finally, in section VI, conclusions are drawn.

II. System Model

In this section, we provide the signal and channel model for the system. The block diagram of the MIMO-OFDM transceiver is shown in Fig. 1. As shown in Fig. 1, the source data bit stream is channel coded and modulated before being fed to a MIMO Space-Frequency Encoder. The individual bit streams from each of the antennas are then subjected to an adaptive pilot sequence insertion along with optimal power allocation between data and pilots. The bit streams are then fed to OFDM modulators at each of the Tx antennas which perform a inverse FFT (IFFT) operation followed by

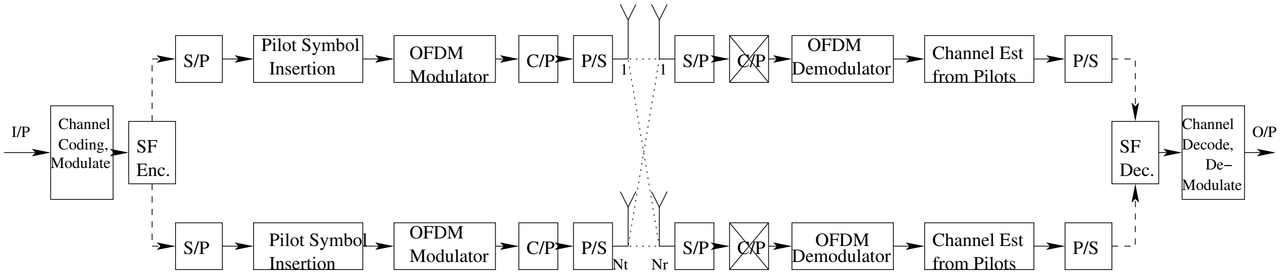

Fig. 1: MIMO-OFDM System model with Pilot Assisted Transmission.

Cyclic Prefix (CP) insertion in order to mitigate the effect of Inter Symbol Interference (ISI). The resultant OFDM blocks from the N_t transmit antennas are sent over a time-varying frequency-selective fading channel. At the N_r receive antennas, the inverse operations of those at the transmitters are performed in addition to the channel estimation, which is done using pilot symbols that are extracted from the OFDM blocks. Optimal Maximum Likelihood (ML) detection is then performed in the MIMO SF decoder and the resultant bitstream is demodulated / channel decoded to obtain the final estimate of the source bitstream.

A. Channel Model

In our system, the assumption made is that the channel is block fading, i.e. the channel coefficients remain constant over one OFDM block but change from one OFDM block to another. Let us denote $h_l(i,j)$ to be the lth coefficient of the channel impulse response to the jth transmit antenna from the ith receive antenna. From the Discrete Fourier Transform (DFT) relation we have,

$$H_k(i,j) = \sum_{l=0}^{L-1} h_l(i,j) W_N^{kl} \qquad l = 0,1,\ldots,(L-1)$$
$$k = 0,1,\ldots,(N-1) \qquad (1)$$

where $H_k(i,j)$ is the kth tone of channel frequency response at the jth transmit antenna from ith receive antenna. Here, N is the length of the OFDM block, also representing the number of FFT/IFFT points, L represents the channel length and $W_N = e^{-j2\pi/N}$. For a $N_r \times N_t$ MIMO-OFDM system, $h = [h_0^T, \ldots, h_{L-1}^T]^T$, $H = [H_0^T, \ldots, H_{N-1}^T]^T$, $\underline{\Im}_N = \Im_N \bigotimes I_{N_r}$ where $\Im_N$ is a $N \times N$ DFT matrix and $\bigotimes$ denotes the Kronecker product. Also, $(.)^T$ denotes the transpose operation while I_{N_r} is a $N_r \times N_r$ identity matrix. Then, we have $H = \underline{\Im}_N\,(:,1:N_r \times L)\,\underline{h}$ where $\underline{\Im}_N(:,1:N_r \times L)$ denotes the first $N_r \times L$ columns of $\underline{\Im}_N$. The above equation defines the mathematical relation between the Channel Impulse Response (CIR) and Channel Frequency Response (CFR) in matrix form for the complete MIMO-OFDM system.

B. Signal Model

Let us denote the transmitted OFDM block from the ith transmit antenna by $\mathbf{S(i)} = [S_0(i), S_1(i), \ldots, S_{N-1}(i)]$. Also, let us denote the received OFDM block at the jth receive antenna to be $\underline{Y}(j) = [Y_0(j), Y_1(j), \ldots, Y_{N-1}(j)]$. Also, if we denote the CFR matrix at the jth receive antenna by

$$\underline{H}(j) = \begin{pmatrix} H_0(j,1) & \cdots & H_0(j,N_t) \\ \vdots & \ddots & \vdots \\ H_{N-1}(j,1) & \cdots & H_{N-1}(j,N_t) \end{pmatrix}, \qquad (2)$$

then the received and transmitted OFDM blocks are given by the relation $\underline{Y}(j) = S\underline{H}(j) + \underline{Z}(j)$ where, $S = [diag\{\mathbf{S}(1)\}, \ldots, diag\{\mathbf{S}(\mathbf{N_t})\}]$ and $\underline{Z}(j)$ is the zero mean additive white Gaussian noise at the jth receive antenna.

III. Optimal pilot-placement scheme and channel estimation

The proposed pilot tone design is dependent on the number of transmit antennas N_t in the system [9]. In general and for all practical purposes, it can be assumed that $N = mL$ and $m > 1$ since the size of a OFDM block N is always much greater than the channel length L, in a MIMO-OFDM system. If we define p as any integer such that $0 \leq p \leq m-1$, then a pth down sampled version of $\underline{H}$ is given by, $\underline{H}^{(p)} = [H_p^T, H_{m+p}^T, \ldots, H_{(L-1)m+p}^T]$ Also, we define $\underline{Y} = [\underline{Y}_0^T, \ldots, \underline{Y}_{N-1}^T]^T$, $\underline{Z} = [\underline{Z}_0^T, \ldots, \underline{Z}_{N-1}^T]^T$, $\underline{S}(i) = \{diag[S_0(i), S_1(i), \ldots, S_{N-1}(i)]\} \bigotimes I_{N_r}$. Then, the pth downsampled version of the above is denoted by $\underline{Y}^{(p)} = [\underline{Y}_p^T, \underline{Y}_{m+p}^T, \ldots, \underline{Y}_{(L-1)m+p}^T]^T$, $\underline{Z}^{(p)} = [\underline{Z}_p^T, \underline{Z}_{m+p}^T, \ldots, \underline{Z}_{(L-1)m+p}^T]^T$, $\underline{S}^{(p)}(i) = \{diag[S_p(i), S_{m+p}(i), \ldots, S_{(L-1)m+p}(i)]\} \bigotimes I_{N_r}$. In the above equation, $\{S_p(i), S_{m+p}(i), \ldots, S_{(L-1)m+p}(i)\}$ represents pilot tones transmitted at a known index p along L sub-carriers with the index known at the receiver a priori from the ith transmit antenna. Having defined the above equations, the pth down sampled version of the OFDM block at the receiver of the system is related to the channel coefficients as follows:

$$\underline{Y}^{(p)} = \underline{S}^{(p)}(1)\underline{\Im}_L W_N^{(p)} \underline{h}(:,1) + \ldots + \underline{Z}^{(p)} \qquad (3)$$

where $\underline{\Im}_L$ is a $LN_r \times LN_r$ DFT matrix and $W_N^{(p)} = diag\{1, W_N^{(p\times 1)}, \ldots, W_N^{(p\times(L-1))}\} \bigotimes I_{N_r}$. From Eq. (3), it can be clearly seen that at least N_t disjoint sets of pilot tones are required for a proper estimation of the CIR. Accordingly, if we choose N_t different indexes $p_1, p_2, \ldots, p_{N_t}$ such that

$0 \leq p_1, p_2, \ldots, p_{N_t} \leq (m-1)$ and $p_1 \neq p_2 \neq \ldots \neq p_{N_t}$, then the coefficients as derived from the solutions to the Least Squares (LS) channel estimation is given by

$$\underline{h}_{LS} = (S^H_{p-design1}.S_{p-design1})^{-1} S^H_{p-design1} \overline{Y} \quad (4)$$

where $S_{p-design1}$ is the pilot tone design matrix and $\overline{Y}$ is the demodulated OFDM block which are defined as follows: $\overline{Y} = S_{p-design1}\overline{h} + \overline{Z}$, $\overline{h} = [\underline{h}^T(:,1), \ldots, \underline{h}^T(:,N_t)]^T$, $\overline{Z} = [\underline{Z}^{T(P_1)}, \ldots, \underline{Z}^{T(P_{N_t})}]^T$, $\overline{Y} = [\underline{Y}^{T(P_1)}, \ldots, \underline{Y}^{T(P_{N_t})}]^T$.

The principle of the above tone design is extended by increasing the number of pilot tone indexes from $p_1, p_2, \ldots, p_{N_t}$ to $p_1, p_2, \ldots, p_{N_t}, \ldots, p_{N_t+2}, \ldots, p_x$ where $x \leq m$ at the same time not necessitating an increase in number of transmit antennas N_t in the system and also maintaining the unitary property of the pilot-tone matrix. The pilot tone matrix for the system is given by

$$S_{p-design2} = [diag\{\mathbf{S}(1)\}\Im_N, \ldots, diag\{\mathbf{S}(N_t)\}\Im_N] \quad (5)$$

The solution to the LS channel estimation is then given by

$$\underline{h}_{LS}(j) = (S^H_{p-design2}.S_{p-design2})^{-1} S^H_{p-design2} \underline{Y}(j) \quad (6)$$

We outline the operation of this design by means of an example of a 2×2 MIMO-OFDM system with channel length $L = 8$ and of size of OFDM block $N = 64$. The pilot sequence placement is as shown in Fig. 2.

The reduction in the decoding complexity is achieved by exploiting the orthogonality structure of the Alamouti design.

IV. Optimal power-allocation between data and pilots

The power between data symbols and pilot symbols along with the pilot placement are the two most critical factors that govern the performance of a pilot assisted MIMO-OFDM system.

Let us denote S_k to be a symbol (data/pilot) on the kth tone of a OFDM block. This symbol can be represented mathematically as $S_k = \sqrt{P_k^{(p)}} S_k^{(p)} + \sqrt{P_k^{(d)}} S_k^{(d)}$ where $S_k^{(p)}$ and $S_k^{(d)}$ correspond to the pilot and data parts of the symbol and $P_k^{(p)}$ and $P_k^{(d)}$ are the respective powers. However, since in our system each symbol in an OFDM block is either completely a data or a pilot symbol, we can set the $\sqrt{P_k^{(p)}} = 0$ or $\sqrt{P_k^{(d)}} = 0$ appropriately for a data or pilot symbol respectively. This being the case, the average power constraint per OFDM block can be stated as $\frac{1}{N}\sum_{k=1}^{N} E\{|S_k|^2\} = P$ or, $\frac{1}{N}\sum_{k=1}^{N}(P_k^{(p)} + P_k^{(d)}) = P$, where N is the length of the OFDM block and P is the roof power per block. The power allocation problem is the optimal distribution of P between the data and pilot symbols for superior system performance.

The optimal power distribution between the data and pilot symbols can be formulated as an Information Theoretic Formulation [12]. Let θ represent the fraction of the total power P allocated to pilots. The optimal power fraction θ is obtained by finding the fraction or an average value taken over a number of Monte-Carlo runs such that the value minimizes the probability of error or the BER for given SNR/channel conditions. For this we define, $\tilde{\theta} = \arg\min_{\theta \in [0.1, \ldots, 0.9]}\{P_e(\rho)\}$ where, $P_e(\rho)$ denotes the BER for a particular SNR value and channel conditions. In a practical system, θ can be adaptively determined by means of a training algorithm until its value converges to a particular factor which minimizes the BER for a given set of channel conditions.

Consequently, the distribution of power between data and pilots according to this converged factor θ would yield a superior BER performance compared to an arbitrary power allocation system.

V. Simulation Results

We next present experimental results for the joint pilot placement and Space Frequency code design scheme. The admissible channel coding rates are $1/4$, $1/3$, $2/3$ and $1/2$ using Rate Compatible Punctured Convolutional (RCPC) codes from [13]. The proposed MIMO-OFDM transmission system has two transmitter (N_t) and two receiver (N_r) antennas. The number of sub-carriers, i.e the length of a OFDM block, is chosen to be $N = 64$. The Cyclic Prefix (CP) length, i.e. the guard length, is chosen to be $G = 11$. This CP is prefixed to each OFDM block before being transmitted over the channel. The channel being simulated is a Jakes' model. The number of channel taps or the channel length $L = 8$. Thus, for the results shown $L < G$ and ISI does not come into play. However, it can be shown that the trend of the results remain the same even during the presence of ISI. The four channels, namely $h_{11}, h_{12}, h_{21}, h_{22}$ are assumed to be independent from each other and vary from one OFDM block to the next. The OFDM symbol duration is given by $T_s = N \times T_a$, where T_a is the sampling interval defined as $T_a = 1/B$ where B is the bandwidth. The sampling duration chosen is $t_s = 50ns$, typical for Hyper-LAN applications. The modulation scheme used in the simulation is 4-QAM which reads and modulates two bits at a time. The noise variance is chosen to be 1. Data and pilot symbols are Space Frequency coded using Alamouti's design.

In Experiment 1, a comparison is made between the $N_t = 2$ design and the $N_t = 4$ design for the equal power distribution and the optimal power distribution cases for a joint SF coded and Pilot Assisted Transmission over a MIMO-OFDM system. Comparisons of the performance of the equal power allocation and the optimal power distribution cases for the $N_t = 2$ design and $N_t = 4$ design are shown in Figs. 3 and 4 respectively. For the optimal power allocation case, we find the optimal power fraction θ or an average value taken over a number of Monte-Carlo runs such that the value minimizes the probability of error or the BER for given SNR/channel conditions.

The pilot-tone design 1 ($N_t = 2$ design) requires a lesser number of pilot-tones to be placed along sub-carriers of a OFDM block compared to pilot-tone design 2 ($N_t = 4$ design). This implies that a higher number of sub-carrier slots can be used to transmit data in design 1 than in design 2. For example, for the 2×2 system with $L = 8$ and $N = 64$, while design 1 allows 48 sub-carriers for data symbol Tx, design 2 provides

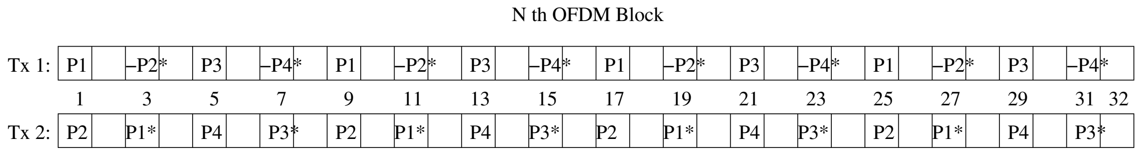

Fig. 2: Pilot placement scheme with $N_t = N_r = 2$, $L = 8$, $p_1 = 0$, $p_2 = 2$, $p_3 = 4$, $p_4 = 6$ and $N = 32$.

just 32 sub-carriers resulting in a 50% decrease in data rate. However, intuitively and also as shown by our experimental results from the Figs. 3 and 4, design 2 provides better BER performance compared to design 1.

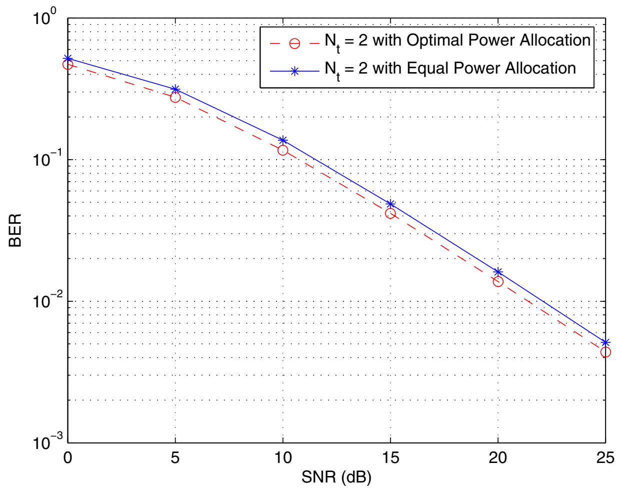

Fig. 3: Comparison of the performance of the equal power allocation and the optimal power distribution cases for $N_t = 2$ design.

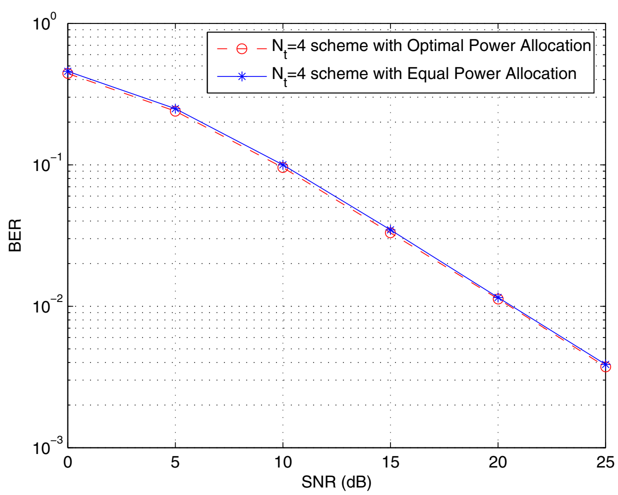

Fig. 4: Comparison of the performance of the equal power allocation and the optimal power allocation cases for the $N_t = 4$ design.

In Experiment 2, we draw a comparison between the equal power allocation and the optimal power allocation case for the $N_t = 2$ and $N_t = 4$ design. The results for specific channel coding rates are shown in Figs. 5 and 6.

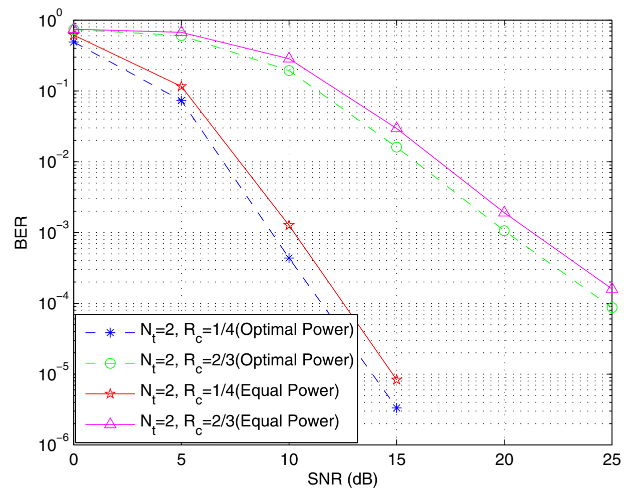

Fig. 5: Comparison of the performance of the equal power allocation and the optimal power allocation cases for the $N_t = 2$ for the channel coding rates of $R_c = 1/4$ and $2/3$.

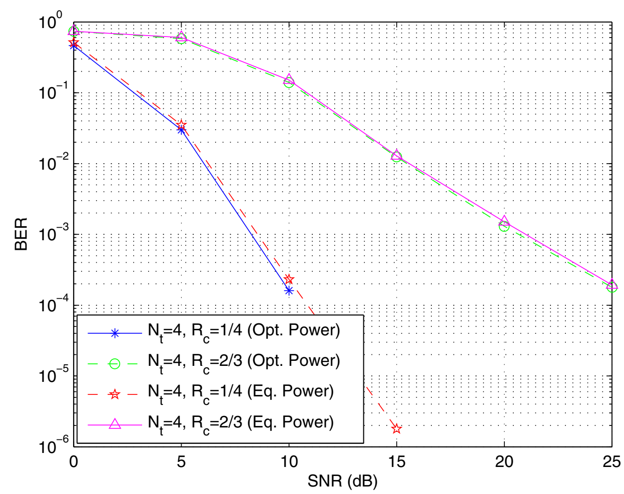

Fig. 6: Comparison of the performance of the equal power allocation and the optimal power allocation cases for the $N_t = 4$ for the channel coding rates of $R_c = 1/4$ and $2/3$.

In Experiment 3, we draw a comparison between the $N_t = 2$ and $N_t = 4$ designs. Fig. 7 shows the comparison of performance of the $N_t = 2$ design and $N_t = 4$ design for the equal power distribution case. The comparison of the performance of the $N_t = 2$ design and $N_t = 4$ design for the optimal power allocation case is shown in Fig. 8.

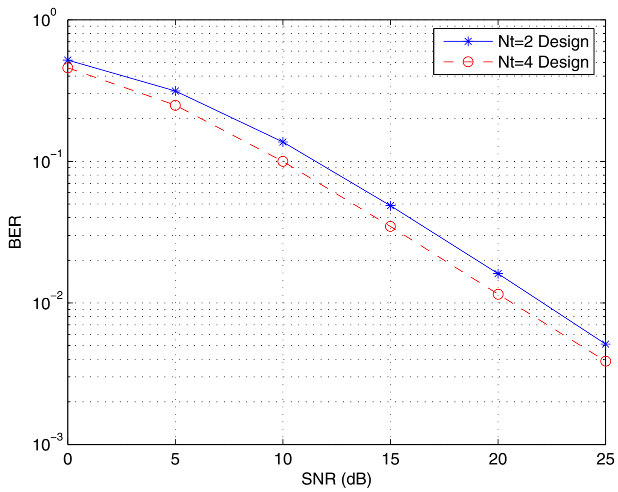

Fig. 7: Comparison of performance of the $N_t = 2$ design and $N_t = 4$ design for the equal power distribution case.

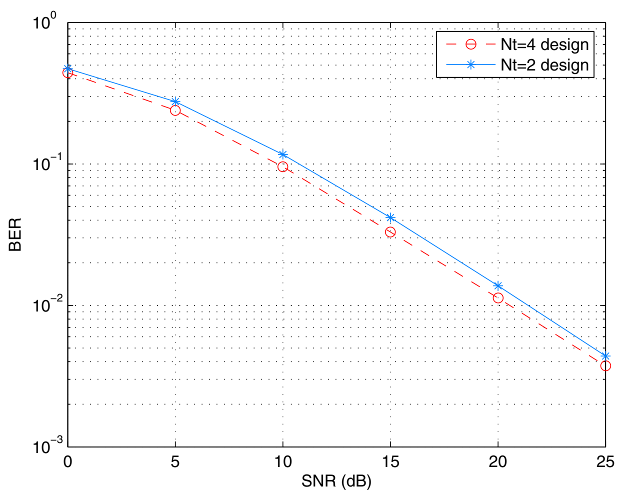

Fig. 8: Comparison of the performance of the $N_t = 2$ design and $N_t = 4$ design for the optimal power allocation case.

VI. Conclusion

In this paper, we have proposed a jointly adaptive pilot-tone placement design and Space Frequency code design for a MIMO-OFDM system. The design has N_t sets of L pilot-tones that are space-frequency coded and transmitted over an OFDM block from each transmit antenna. An extension to this design where $N_t + x$ sets of L pilot tones where x can be any multiple of N_t was proposed which resulted in a superior BER performance, albeit at the cost of decreased data rate. A comparison is drawn between the $N_t = 2$ design and the $N_t = 4$ design for the equal power allocation and the optimal power allocation cases.

The pilot assisted transmission multiplexes known symbols with information bearing data. The pilot symbols are known at the receiver. Pilot assisted transmission is used for channel estimation, receiver adaptation and optimal decoding. The pilot tones are grouped into equally spaced clusters due to their superior channel estimation capability for time varying channels [14]. A channel estimator method based on the Least Square estimator is used with different training sequences for each transmit antenna. OFDM transforms the frequency selective MIMO channel to a set of parallel frequency flat MIMO channels and therefore reduces receiver complexity.

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