# A theoretical model for the x-ray luminescence of granular phosphor screens

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A theoretical model for the x-ray luminescence of a phosphor screen is presented. The model takes into account the granular structure of the screen and the random deposition of the phosphor grains on the screen substrate. Results based on this model are in very good agreement with the experimental data already known for the x-ray luminescence of the various phosphor screens.

## I. INTRODUCTION

All theoretical calculations and predictions concerning the efficiency and resolution of fluorescent screens have been based until now on the Hamaker–Ludwig model,[1,2] according to which a phosphor screen is assumed to be a homogeneous and uniform layer of phosphor material. As a result the model is suitable for the nongranular screens like the ones prepared by evaporation, but it is generally unsuitable for the granular screens like the ones prepared by sedimentation. In a previous paper[3] we have studied the case of a low-thickness granular phosphor screen excited by an electron beam; in this paper we complete that study considering the case of granular screens excited by x rays.

## II. DESCRIPTION OF THE MODEL

The vast majority of phosphor screens used in the x-ray applications are prepared by sedimentation. As a result the structure of the screen is as shown in Fig. 1(a); the transparent screen substrate is covered by randomly deposited phosphor grains of irregular shape, but of approximately the same size. It is generally expected that some regions of the screen will not be covered by the phosphor grains, some regions will be singly covered, some others doubly covered, etc. One expects, as the number of the phosphor grains is increased (which is equivalent to an increase of the screen's thickness) the size of the low-coverage regions to be reduced.

In the model proposed[3] the phosphor grains are simulated by particles rectangular in shape and equal to each other, which can overlap and are randomly distributed over the substrate, as shown in Fig. 1(b). It has already been proved[3] that, if we designate the mean area of the covered regions of the substrate by $D_k^{(N)}$, where $N$ is the number of particles on the screen and $k=0,1,2,\ldots,N$ denotes the order of coverage of the substrate region under consideration, then

$$D_k^{(N)}/A=(N_k)\alpha^k(1-\alpha)^{N-k}, \tag{1}$$

with $\alpha=S/A$, where $S$ is the cross section of a particle and $A$ is the area of the screen's substrate.

In the following we derive a formula concerning the most important quantity of an x-ray phosphor screen, its efficiency $n$ and the dependence of it on the various screens' and phosphor's parameters (e.g., x-ray and light absorption coefficients, thickness, etc.).

Let us consider that such a screen of $N$ particles is excited to luminescence by a monochromatic x-ray beam. A monochromatic x-ray beam passing through a phosphor layer of thickness $d$ suffers an attenuation given by the relation

$$I=I_0\exp(-\mu d), \tag{2}$$

where $\mu$ is the x-ray-absorption coefficient of the phosphor material. The above relation can be simplified to

$$I=I_0(1-\mu d), \tag{3}$$

if $\mu d\ll 1$; the latter is always true when considering the thickness of only one particle (phosphor grain), since the mean grain size of the phosphor materials used in x-ray screens in 4–15 $\mu$m.

If the fluorescent screen consists of $N$ grain layers the attenuation is obviously given by

$$I=I_0(1-\mu d)^N. \tag{4}$$

According to this relation the x-ray beam power $I_{ab}(i)$ absorbed by the $i$th particle layer in a region having an order of coverage $k$ will be given by

$$\begin{aligned} I_{ab}(i)&=I_{in}(i)-I_{out}(i)\\ &=I_0(1-\mu d)^{k-i}-I_0(1-\mu d)^{k-i+1},\\ I_{ab}(i)&=I_0\mu d(1-\mu d)^{k-i}. \end{aligned} \tag{5}$$

As it is well known[4] the light flux created in each phosphor layer is proportional to the x-ray beam power absorbed in it; so we can write for the light flux $\phi_i$ created in the $i$th layer the expression

$$\phi_i=tI_{ab}(i), \tag{6}$$

where $t$ is the intrinsic efficiency of the phosphor material under consideration (a proportionality factor for the conversion of the x-ray photon power to light power).

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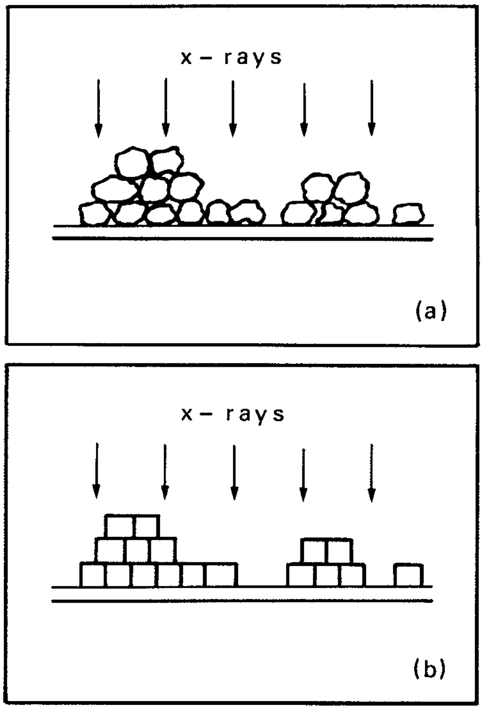


FIG. 1. Cross sections of a phosphor screen: (a) real structure and (b) modeled structure.

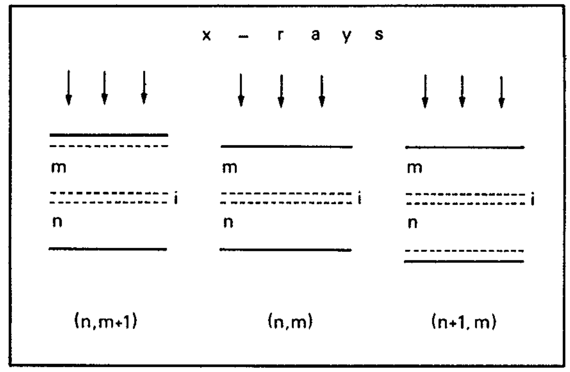


FIG. 2. Cross section of a part of a phosphor screen (for the development of the recursion relation for the light fluxes emitted from it).

If we consider for the propagation of the light in the screen's mass a one-dimensional model (as it is also the Hamaker–Ludwig one) the light flux $\phi_i$ is divided into two equal parts propagating towards opposite directions and giving rise to the two basic modes of screen observation; the transmission mode and the reflection mode.[5]

In order to estimate the total light flux emitted from the screen towards each of the two directions we have to calculate the light flux emitted from every region of it having a different order of coverage and sum all these flux amounts. For the calculation of the total light flux emitted from a region of order of coverage $k$ we calculate at first the flux coming out the screen from the $i$th phosphor layer (where $i$ runs from 0 to $k$) and then we sum all the corresponding results.

The light created in every phosphor layer in its way out of the screen suffers attenuation due to absorption and scattering. Referring to Fig. 2 we build a recursion relation for the calculation of the light flux coming out from the screen in transmission (and also in reflection) mode observation due to the light created in the $i$th layer. Let $n$ be the number of phosphor grain layers between the $i$th layer and the transmission mode observation side of the screen and $m$ the number of phosphor grain layers between the $i$th layer and the reflection mode observation side of it; let also $\phi_{it}(n,m)$ and $\phi_{ir}(n,m)$ be the light fluxes coming out from the screen in transmission and in reflection mode observation, respectively, due to the light created in the $i$th layer. Then for the corresponding $\phi_{it}(n+1,m)$, $\phi_{it}(n,m+1)$, $\phi_{ir}(n+1,m)$, and $\phi_{ir}(n,m+1)$ fluxes the following relations hold:

$$\phi_{it}(n+1,m)=\phi_{it}(n,m)(1-s)(1-ad), \tag{7}$$

$$\phi_{ir}(n+1,m)=\phi_{ir}(n,m)+\phi_{it}(n,m) \times s(1-ad)^{m+n+1}, \tag{8}$$

$$\phi_{it}(n,m+1)=\phi_{it}(n,m)+\phi_{ir}(n,m) \, s(1-ad)^{m+n+1}, \tag{9}$$

$$\phi_{ir}(n,m+1)=\phi_{ir}(n,m)(1-s)(1-ad), \tag{10}$$

where $a$ and $s$ are the light-absorption and scattering coefficients of the phosphor material, respectively. It is obvious that $\phi_{it}(0,0) = \phi_{ir}(0,0) = \phi_i/2$. Relations (7)–(10) are, of course, accurate only when $ad \ll 1$, a condition which is always true as a consequence of the very low thickness of the phosphor grains.

It is interesting to notice here that in the case of absence of scattering phenomena ($s=0$), which is the case of a homogeneous and uniform screen, the above relations are reduced to

$$\phi_{it}(n+1,m)=\phi_{it}(n,m)(1-ad), \tag{11}$$

$$\phi_{ir}(n+1,m)=\phi_{ir}(n,m), \tag{12}$$

$$\phi_{it}(n,m+1)=\phi_{it}(n,m), \tag{13}$$

$$\phi_{ir}(n,m+1)=\phi_{ir}(n,m)(1-ad), \tag{14}$$

and lead to the relations

$$\phi_{it}(n,m)=\phi_i/2(1-ad)^n \approx \phi_i/2\exp(-and), \tag{15}$$

$$\phi_{ir}(n,m)=\phi_i/2(1-ad)^m \approx \phi_i/2\exp(-amd), \tag{16}$$

which are the well-known relations for attenuation of light passing through a uniform absorbing layer of thickness $nd$ or $md$, respectively.

According to relation (1) and relations (7)–(10) the total light flux coming out from a region of the screen having a degree of coverage $k$ in transmission and in reflection mode observation is given by

$$\phi_t(k)=\left(\sum_{i=0}^{k}\phi_{it}(i,k-i)\right)D_k^{(N)}/A, \tag{17}$$

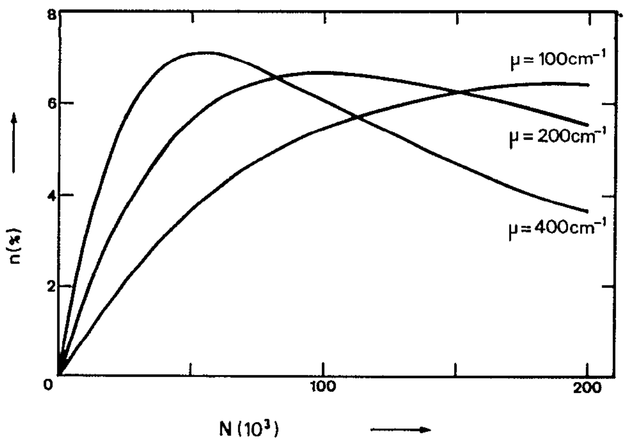


FIG. 3. Calculated x-ray luminescence efficiency vs screen's thickness (number of phosphor grains on the screen's substrate) for three different x-ray-absorption coefficients.

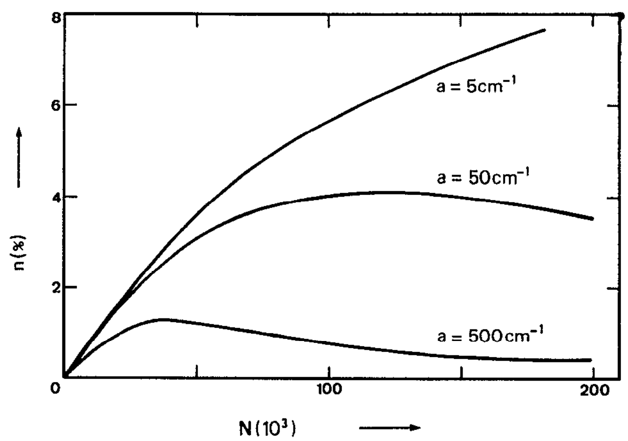


FIG. 4. Calculated x-ray luminescence efficiency vs screen's thickness (number of phosphor grains on the screen's substrate) for three different light-absorption coefficients.

$$\phi_r(k)=\left(\sum_{i=0}^{k}\phi_{ir}(i,k-i)\right)D_k^{(N)}/A. \tag{18}$$

As a result the total light flux emitted from the whole screen will be given by the relations

$$\phi_{\text{tot},t}=\sum_{k=1}^{V}\phi_t(k), \tag{19}$$

$$\phi_{\text{tot},r}=\sum_{k=1}^{V}\phi_r(k), \tag{20}$$

respectively, where $v$ is the highest order of coverage on the screen. In practice we have not taken into account layers having an area less than 1% of the total screen area.

As already mentioned the interesting parameter for the screen is not the light emitted from it, but its efficiency $n$, which is defined as the ratio of the light flux emitted to the x-ray beam power exciting the screen. Hence we have

$$n_t=\phi_{\text{tot},t}/I_0 \tag{21}$$

and

$$n_r=\phi_{\text{tot},r}/I_0. \tag{22}$$

## III. RESULTS AND DISCUSSION

Using relations (21) and (22) we have calculated the efficiency of a phosphor screen as a function of its thickness (number of phosphor grains $N$) for a number of cases. In Figs. 3–5 we present the results of our calculations and we demonstrate the effect of the various parameters (x-ray-absorption coefficient, light-absorption coefficient, light-scattering coefficient, etc.) on the shape of the $N=f(N)$ curves. All the results presented in these figures concern the efficiency of the screens in the transmission mode observation, because the effect of the various parameters is more obvious in this case than in the case of the reflection mode observation. We observe in all these three groups of curves the same typical shape, already known from the experiment;[6,7] an initial almost linear increase followed by a more or less well-expressed peak and a slower decrease thereafter.

In Fig. 3 we show the effect of the x-ray-absorption coefficient on the shape of the curves. It is clear that as this coefficient increases the efficiency peak moves towards lower thicknesses. This is well expected from a physical point of view, since a high x-ray-absorption coefficient means increased stopping power for the low-thickness screens, which suffer a smaller efficiency attenuation due to light absorption and scattering in their mass; that explains also the increased efficiency values as the x-ray-absorption coefficient increases.

In Fig. 4 we illustrate the effect of the light-absorption coefficient on the shape of the curves. It is obvious that as this coefficient increases the peak of the curves moves towards smaller grain numbers (lower thicknesses) and the overall efficiency decreases. This behavior is quite reasonable. Increasing the light-absorption coefficient means an increased attenuation for the light created in the screen before coming out from it, which causes the overall reduction of the efficiency; on the other hand the peak in the efficiency curves being the result of a balance between the increase of the light emitted due to x-rays absorption (that increases with the screen's thickness) and its reduction due to the light absorption within the phosphor mass (that also increases with the screen's thickness) moves towards smaller grain numbers as the light-absorption coefficient increases.

In Fig. 5 we show finally the effect of the light-scattering coefficient on the shape of the curves. It is interesting to notice that the scattering coefficient generally has a small effect on the shape (the peak position moves slowly towards lower thicknesses as this coefficient increases) and on the corresponding values (variations of the order of 10%) of the curves. Figure 6 shows that the dependence of the efficiency on the scattering coefficient is not monotonic, but shows initially a slow decrease (in the range $s=0.0$–$0.3$) and an almost linear increase thereafter. This increase

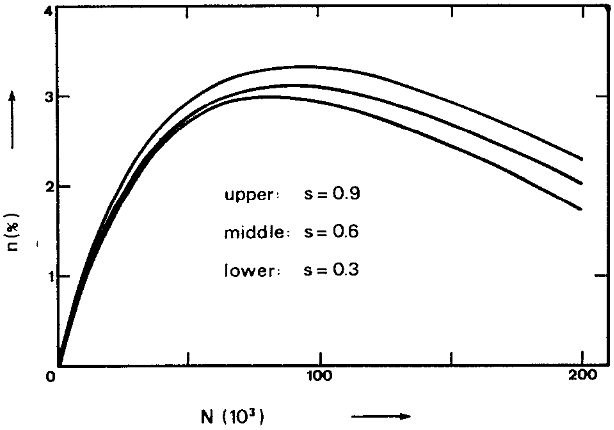


FIG. 5. Calculated x-ray luminescence efficiency vs screen's thickness (number of phosphor grains on the screen's substrate) for three different light-scattering coefficients.

of efficiency as the scattering phenomena are increased (in the range $s = 0.3$–$1.0$) is due to the increase of the light "reflected" from the higher-order grain layers towards the transmission mode observation side.

To investigate the reliability of the model we compared for a number of cases the $n=f(w)$ model curves to the experimental ones. In Fig. 7 we present the results of these comparisons. The phosphor material is ZnCdS:Ag. The experimental data for the transmission mode curve were available from a previous work,[6] while the data for the reflection mode curve come from measurements carried out specifically for the needs of this comparison. Concerning the various parameters in Eqs. (21) and (22) we have used $p = 4.4$ mg/cm$^2$ for the specific density,[8] $t = 0.22$ for the intrinsic efficiency[9] and $\mu = 30$ cm$^{-1}$ for the x-ray-absorption coefficient.[10,11] The light and scattering coefficients have been obtained by a $\chi^2$ fit and the resulting values $a = 60$ cm$^{-1}$ and $s = 0.3$ are in accord with estimates suggested by an earlier work.[12] As can be seen from

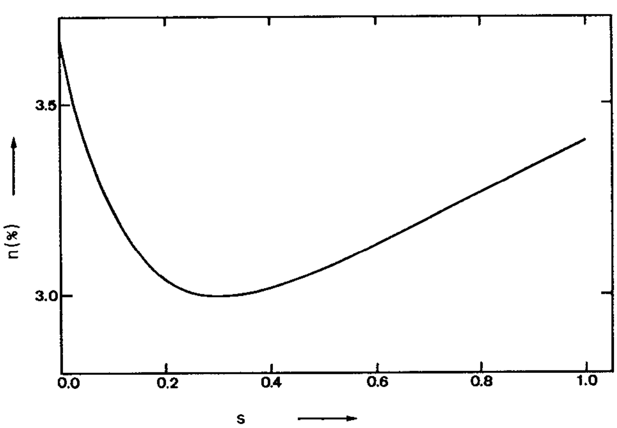


FIG. 6. Calculated x-ray luminescence efficiency (peak values) of a phosphor screen vs the light-scattering coefficient.

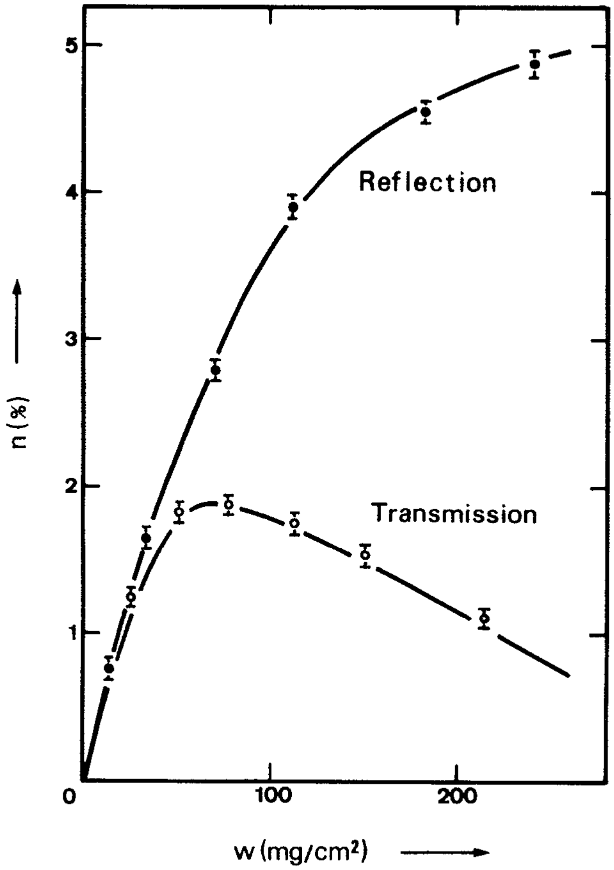


FIG. 7. Calculated and experimental x-ray luminescence efficiency vs screen's thickness for ZnCdS:Ag screens in transmission and in reflection mode observation. Dots: experimental data; lines: theoretical results.

Fig. 7 the agreement between the theoretical results and the experimental data is almost excellent.

We would like to make one more comment related to the shape of the efficiency versus thickness curve in reflection mode observation. This curve—and all others concerning this mode of observation—does not show any peak, but a continuous increase with thickness towards a saturation value. This is well known from previous experimental studies[5] and well expected from a physical point of view, because, as already mentioned, in reflection mode observation the light emitted from the screen is observed from the very same side the screen is excited by the x-rays; as a result the addition of more phosphor grains (increase of the screen's thickness) simply results to an increase of the light reflected towards the observation side, which increase is smaller and smaller, but of course can never reduce the overall efficiency.


[1] H. C. Hamaker, Philips Res. Rep. **2**, 55 (1947).
[2] G. Ludwig, J. Electrochem. Soc. **118**, 1152 (1971).
[3] G. E. Giakoumakis and I. E. Lagaris, J. Appl. Phys. **64**, 5106 (1988).
[4] A. Blazhevich, Sov. Phys. Semicond. **1**, 1997 (1968).
[5] G. E. Giakoumakis, C. D. Nomicos, E. N. Yiakoumakis, and E. K. Evangelou, Phys. Med. Biol. **35**, 1017 (1990).
[6] C. D. Nomicos, G. E. Giakoumakis, P. E. Euthymiou, and P. X. Sandilos, J. Appl. Phys. **49**, 3636 (1978).

[7] G. E. Giakoumakis, C. D. Nomicos, and P. X. Sandilos, Phys. Med. Biol. **34**, 673 (1989).
[8] *CRC Handbook of Chemistry and Physics*, 51st ed. (CRC, Cleveland, OH, 1971), B-76 and B-105.
[9] G. Ludwig and J. Kingsley, J. Electrochem. Soc. **117**, 348 (1970).
[10] E. Storm and H. Israel, Report No. LA-3753, Los Alamos Scientific Laboratory of the University of California, 1967.
[11] E. B. Saloman, J. H. Hubbell, and J. H. Scofield, At. Data Nucl. Data Tables, **38**, 1 (1988).
[12] G. E. Giakoumakis and C. D. Nomicos, J. Appl. Phys. **58**, 2742 (1985).