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GENERALIZED RESONANCE MODES IN FERROMAGNETIC SPHERES

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Abstract—In the present work we deal with the resonance modes in small ferromagnetic spheres. The analysis is based on the theory of micromagnetism, proposed by W. F. Brown and an optimization technique. Numerical results are presented for the resonance field, as well as for the resonance modes and the magnetization configuration in the material. The resonance modes described in this work could be named *generalized modes*.

1. INTRODUCTION

Magnetic resonance phenomena in magnetitite (Fe_3O_4) have been investigated by Bickford [1] and others in the late 1950s and early 1960s. Rajagopalan and Furdyna [2] studied magnetic resonances in Fe_3O_4 spheres by microwave magnetotransmission experiments and they observed, in addition to ferromagnetic resonances, new size-dependent ones. In Ref. [2] these new resonances have been studied as a function of sphere diameter and microwave frequency. It is noted that the diameter of spheres considered in Ref. [2], was of the order of mm and the behaviour of the new resonances could not be described in terms of ordinary Walker modes [3]. Rajagopalan and Furdyna named these new resonances *dimensional* ones because of their dependence on the sphere diameter. Such spheres are obviously too large for the exchange forces to play any significant role in these resonances. Nowadays there are techniques for making very small ferromagnetic spheres [4, 5] which should eventually lead to resonance measurements in a size range where the exchange contributions are important. Aharoni [6] studied the possible effect of the exchange forces on the resonance modes in sufficiently small ferromagnetic particles. He offered a mechanism for some sort of size -dependent resonances in small spheres, by neglecting the magnetostatic energy and named these resonance modes *exchange resonance modes* as an analogue to the magnetostatic ones.

In the present work we deal with the solution of the mathematical problem describing the general theory of resonance in ferromagnetic spherical particle under the assumption of cylindrical symmetry, introduced by Aharoni [6]. The proposed analysis, in comparison to that used by Aharoni [7], has the advantage of computing the eigenvectors of the problem and it is therefore possible to plot the shape of the magnetization configuration in the material. We reduced our problem to minimizing a suitable objective function that guarantees the satisfaction of the boundary conditions. We used the optimization package MERLIN-2.1 [8, 9]. Finally numerical results are presented for the resonance field and the associated shape of the magnetization configuration and the role of the particle size and frequency on the resonance field is discussed. The shape of the magnetization configuration, as it appears, could be named *generalized curling mode*.

2. PROBLEM FORMULATION

The general theory of resonance in a ferromagnetic particle, in the absence of losses, is described by the following equation [10, 11]

$$\frac{d\mathbf{v}}{dt} = \gamma_0 \mathbf{v} \times \mathbf{H}_{eff} \tag{2.1}$$

where

$$\mathbf{H}_{eff} = \frac{C}{M_s} \nabla^2 \mathbf{v} - \frac{1}{M_s} \frac{\partial \omega_\alpha}{\partial \mathbf{v}} + \mathbf{H} \tag{2.2}$$

is the effective field $(\partial/\partial \mathbf{v})_i = \partial/\partial v_i$ $(i = x, y, z)$, $\mathbf{v}$ is a unit vector parallel to the magnetization, $C = 2A$ is the exchange constant, ω_α is the anisotropy energy density, M_s is the saturation magnetization, t is the time, γ_0 is the gyromagnetic ratio and $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_m$ is the magnetic field which is composed of the applied field $\mathbf{H}_0$, and that, $\mathbf{H}_m$, created by the volume and surface charge of the magnetiziation distribution. The boundary conditions for the set of equations (2.1) are

$$\frac{\partial \mathbf{v}}{\partial n} = 0 \tag{2.3}$$

where $\partial/n = \mathbf{v} \cdot \nabla$ and $\mathbf{n}$ denotes the unit outward normal to the particle surface. To the equations (2.1–3) we have to add the equations for the potential problem which in the present case are [12]

$$\nabla^2 V_{in} = 4\pi M_s \nabla \cdot \mathbf{v} \tag{2.4}$$

inside the particle and

$$\nabla^2 V_{out} = 0 \tag{2.5}$$

outside it. The boundary conditions on particle surface are

$$V_{in} = V_{out} \tag{2.6}$$

$$-\frac{\partial V_{in}}{\partial n} + 4\pi M_s v_n = -\frac{\partial V_{out}}{\partial n} \tag{2.7}$$

where $v_n = \mathbf{n} \cdot \mathbf{v}$. In the experimental studies of resonances, a large dc field $\mathbf{H}_0$ is applied; its direction is identified here with the z-axis. The field $\mathbf{H}_0$ keeps the magnetization almost parallel to the z-axis so that v_x and v_y are small. To a first order in these small quantities, the differential equations (2.1) for a steady-state solution, $(\ )e^{i\omega t}$, become

$$\left(\frac{C}{M_s}\nabla^2 - H_z\right)v_x - \left(\frac{i\omega}{\gamma_0}\right)v_y = \frac{\partial V_{in}}{\partial x} \tag{2.8}$$

$$\left(\frac{C}{M_s}\nabla^2 - H_z\right)v_y + \left(\frac{i\omega}{\gamma_0}\right)v_x = \frac{\partial V_{in}}{\partial y} \tag{2.9}$$

where V_{in} is the potential due to the transverse magnetization $\mathbf{m} = M_s(v_x\mathbf{i} + v_y\mathbf{j})$ and ω is the resonance frequency. In order a confusion to be avoided, we note here that we keep the same symbols for the time independent components of $\mathbf{v}$ and for the potentials. The potential due to z-component is included in H_z. In the case under discussion (spherical particle) H_z includes the dc field H_0, the demagnetizing field

$$H_D = -\frac{4\pi}{3} M_s$$

and the anisotropy field H_K, that is

$$H_z = H_0 - \frac{4\pi}{3} M_s + \frac{2K_1}{M_s} \tag{2.10}$$

where K_1 is the anisotropy constant. It is noted that for the above linearized equations cubic or uniaxial anisotropies lead to the same expression provided that z is an easy axis.

3. PROBLEMS SOLUTION

Following Aharoni [6] we suppose that $\mathbf{v}$ does not depend on the coordinate ϕ (cylindrical symmetry) and use the components of $\mathbf{v}$ in a cylindrical coordinate system (ϱ, ϕ, z) but express the spatial dependence in spherical coordinates (r, θ, ϕ). Under these considerations the equations (2.8) and (2.9) are transformed as

$$\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{2}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\cos\theta}{r^2 \sin\theta}\frac{\partial}{\partial\theta} - \frac{1}{r^2 \sin^2\theta} - \frac{M_s H_z}{C}\right) v_\phi + \frac{iM_s\omega}{\gamma_0 C} v_\rho = 0 \tag{3.1}$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{2}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\cos\theta}{r^2 \sin\theta}\frac{\partial}{\partial\theta} - \frac{1}{r^2 \sin^2\theta} - \frac{M_s H_z}{C}\right) v_\varrho - \frac{iM_s\omega}{\gamma_0 C} v_\phi = \frac{M_s}{C}\left(\sin\theta\frac{\partial}{\partial r} + \frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\right) V_{in}, \tag{3.2}$$

respectively. The equations (2.4) and (2.5) of the potential problem are written as

$$\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{2}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\cos\theta}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\right) V_{in} = 4\pi M_s \left(\frac{1}{r\sin\theta} + \frac{\cos\theta}{r}\frac{\partial}{\partial\theta} + \sin\theta\frac{\partial}{\partial r}\right) v_\varrho \tag{3.3}$$

for $r \le R$, where R is the radius of the spherical particle, and

$$\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{2}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\cos\theta}{r^2 \cos\theta}\frac{\partial}{\partial\theta}\right) V_{out} = 0 \tag{3.4}$$

for $r \ge R$. The boundary conditions (2.3), (2.6) and (2.7) on $r = R$ become:

$$\frac{\partial v_\varrho}{\partial r} = \frac{\partial v_\phi}{\partial r} = 0 \tag{3.5}$$

$$V_{in} = V_{out} \tag{3.6}$$

and

$$\frac{\partial V_{in}}{\partial r} - \frac{\partial V_{out}}{\partial r} = 4\pi M_s v_\varrho \sin\theta, \tag{3.7}$$

respectively. Expanding the solution of the problem in a series of Legendre functions

$$v_\varrho = \sum_{n=1}^{\infty} A_n(r) P_n^1(\cos\theta) \tag{3.8}$$

$$v_\phi = i \sum_{n=1}^{\infty} B_n(r) P_n^1(\cos\theta) \tag{3.9}$$

$$V_{in} = \sum_{n=0}^{\infty} V_n(r) P_n(\cos\theta) \tag{3.10}$$

$$V_{out} = \sum_{n=0}^{\infty} V_n(R) \left(\frac{R}{r}\right)^{n+1} P_n(\cos\theta) \tag{3.11}$$

the set of partial differential equations (3.1–7) is transformed to an infinite set of ordinary differential equations. We note that (3.11) is the solution of (3.4) that is regular at infinity and satisfies the boundary condition (3.6).

Substituting the solution (3.8–11) into the equations (3.1–7) and using the appropriate relations between the Legendre functions and their recurrence relations [13] the variable θ can be eliminated.

Using the dimensionless quantities,

$$\tau = r/R, \qquad h = H_z/2\pi M_s, \qquad h_r = \omega/2\pi M_s\gamma_0, \qquad u = V/2\pi M_s R_0$$
$$R_0 = \sqrt{A}/M_s, \qquad A = C/2, \qquad S = R/R_0$$

and the notations

$$L_n = \frac{\mathrm{d}}{\mathrm{d}\tau} + \frac{n+1}{\tau}, \qquad M_n = \frac{\mathrm{d}}{\mathrm{d}\tau} - \frac{n}{\tau}$$

the infinite set of ordinary differential equations, in which the set of equations (3.1–3) is reduced, is given by:

$$(L_{n+1}M_n - \pi S^2 h)B_n(\tau) + \pi S^2 h_r A_n(\tau) = 0, \qquad n \geq 1 \tag{3.12}$$

$$(L_{n+1}M_n - \pi S^2 h)A_n(\tau) + \pi S^2 h_r B_n(\tau) = \pi S\left(M_{n-1}\frac{u_{n-1}(\tau)}{2n-1} - L_{n+1}\frac{u_{n+1}(\tau)}{2n+1}\right), \qquad n \geq 1 \tag{3.13}$$

$$L_{n+1}M_n u_n(\tau) = 2S\left[\frac{(n+1)(n+2)}{(2n+3)}L_{n+1}A_{n+1}(\tau) - \frac{n(n-1)}{(2n-1)}M_{n-1}A_{n-1}(\tau)\right], \qquad n \geq 0 \tag{3.14}$$

with the following boundary conditions on $\tau = 1$,

$$\frac{\mathrm{d}A_n}{\mathrm{d}\tau} = \frac{\mathrm{d}B_n}{\mathrm{d}\tau} = 0, \qquad n \geq 1 \tag{3.15}$$

$$L_n u_n(\tau) = 2S\left[\frac{(n+1)(n+2)}{(2n+3)}A_{n+1}(\tau) - \frac{n(n-1)}{(2n-1)}A_{n-1}(\tau)\right], \qquad n \geq 0. \tag{3.16}$$

We seek a solution of the set of equations (3.12–14) in the following form:

$$A_n(\tau) = \sum_{k=1}^{\infty} a_{n,k} j_n(\mu_k \tau), \qquad n \geq 1 \tag{3.17}$$

$$B_n(\tau) = \sum_{k=1}^{\infty} b_{n,k} j_n(\mu_k \tau), \qquad n \geq 1 \tag{3.18}$$

$$u_n(\tau) = \sum_{k=1}^{\infty} c_{n,k} j_n(\mu_k \tau), \qquad n \geq 0 \tag{3.19}$$

where $j_n(x)$ are the n-th spherical Bessel functions and $a_{n,k}$, $b_{n,k}$, $c_{n,k}$ and μ_k are unknown coefficients.

By substitution of the above trial solution into equations (3.12–14) we obtain

$$\sum_{k=1}^{\infty} [\pi S^2 h_r a_{n,k} - (\mu_k^2 + \pi S^2 h)b_{n,k}] j_n(\mu_k \tau) = 0, \qquad n \geq 1 \tag{3.20}$$

$$\sum_{k=1}^{\infty}\left[\pi S^2 h_r b_{n,k} - (\mu_k^2 + \pi S^2 h)a_{n,k} + \pi S\mu_k\left(\frac{c_{n-1,k}}{2n-1} + \frac{c_{n+1,k}}{2n+3}\right)\right] j_n(\mu_k \tau) = 0, \qquad n \geq 1 \tag{3.21}$$

$$\sum_{k=1}^{\infty}\left[2S\mu_k\left(\frac{(n+1)(n+2)}{(2n+3)}a_{n+1,k} + \frac{n(n-1)}{(2n-1)}a_{n-1,k}\right) + \mu_k^2 c_{n,k}\right] j_n(\mu_k \tau) = 0, \qquad n \geq 0. \tag{3.22}$$

In order (3.20-22) to be valid $\forall \tau$, the following relations have to be satisfied,

$$\pi S^2 h_r a_{n,k} - (\mu_k^2 + \pi S^2 h) b_{n,k} = 0, \qquad n \geq 1, \qquad k \geq 1 \tag{3.23}$$

$$\pi S^2 h_r b_{n,k} - (\mu_k^2 + \pi S^2 h) a_{n,k} + \pi S \mu_k \left(\frac{c_{n-1,k}}{2n-1} + \frac{c_{n+1,k}}{2n+3} \right) = 0, \qquad n \geq 1, \qquad k \geq 1 \tag{3.24}$$

$$2S \left[\frac{(n+1)(n+2)}{(2n+3)} a_{n+1,k} + \frac{n(n-1)}{(2n-1)} a_{n-1,k} \right] + \mu_k c_{n,k} = 0, \qquad n \geq 0, \qquad k \geq 1. \tag{3.25}$$

From (3.23) and (3.24) we obtain

$$b_{n,k} = \frac{\pi S^2 h_r}{\mu_k^2 + \pi S^2 h} a_{n,k}, \qquad n \geq 1, \qquad k \geq 1, \qquad \mu_k^2 + \pi S^2 h \neq 0 \tag{3.26}$$

and

$$c_{n,k} = -\frac{2S}{\mu_k} \left[\frac{(n+1)(n+2)}{(2n+3)} a_{n+1,k} + \frac{n(n-1)}{(2n-1)} a_{n-1,k} \right], \qquad n \geq 1, \qquad k \geq 1, \tag{3.27}$$

respectively, and (3.25) leads to the following recurrence relation

$$a_{n+2,k} = \beta_{nk} a_{n,k} + \xi_n a_{n-2,k}, \qquad n \geq 1, \qquad k \geq 1 \tag{3.28}$$

where $a_{-m,k} = 0$ for $m = 0, 1, 2, \ldots$ and

$$\beta_{nk} = \frac{(2n+3)(2n+5)}{2\pi S^2 (n+2)(n+3)} \left[\frac{(\pi S^2 h_r)^2}{\mu_k^2 + \pi S^2 h} - (\mu_k^2 + \pi S^2 h) - 4\pi S^2 \frac{n(n+1)}{(2n-1)(2n+3)} \right]$$

$$\xi_n = -\frac{(n-1)(n-2)(2n+3)(2n+5)}{(2n-1)(2n-3)(n+2)(n+3)}.$$

In view of the solution (3.17–19), the boundary conditions (3.15) and (3.16) become

$$\sum_{k=1}^{\infty} \begin{Bmatrix} a_{n,k} \\ b_{n,k} \end{Bmatrix} \mu_k j_n(\mu_k) = 0, \qquad n \geq 1 \tag{3.29}$$

$$\sum_{k=1}^{\infty} \left[2S \left(\frac{(n+1)(n+2)}{2n+3} j_{n+1}(\mu_k) a_{n+1,k} - \frac{n(n-1)}{2n-1} j_{n-1}(\mu_k) a_{n-1,k} \right) - \mu_k j_{n-1}(\mu_k) c_{n,k} \right] = 0, \qquad n \geq 0. \tag{3.30}$$

Taking into account (3.26), (3.27) and the recurrence relation,

$$j_n'(x) = j_{n-1}(x) - \frac{n+1}{x} j_n(x)$$

where $(\ )' = \mathrm{d}/\mathrm{d}x$, the conditions (3.29) and (3.30) are written as:

$$\sum_{k=1}^{\infty} a_{n,k} [\mu_k j_{n-1}(\mu_k) - (n+1) j_n(\mu_k)] = 0, \qquad n \geq 1 \tag{3.31}$$

$$h_r \sum_{k=1}^{\infty} a_{n,k} \frac{\mu_k j_{n-1}(\mu_k) - (n+1) j_n(\mu_k)}{\mu_k^2 + \pi S^2 h} = 0, \qquad n \geq 1, \tag{3.32}$$

and

$$2S \sum_{k=1}^{\infty} \frac{(n+1)(n+2)(2n+1)}{2n+3} \frac{j_n(\mu_k)}{\mu_k} a_{n+1,k} = 0, \qquad n \geq 0, \tag{3.33}$$

respectively. Equation (3.33) can equivalently be written as

$$\sum_{k=1}^{\infty} \frac{j_{n-1}(\mu_k)}{\mu_k} a_{n,k} = 0, \qquad n \geq 1. \tag{3.34}$$

The satisfaction of the boundary conditions (3.31), (3.32) and (3.34) is guaranteed when the following quadratic form:

$$\sum_{n=1}^{\infty} [(\text{Eq. (3.31)})^2 + (\text{Eq. (3.32)})^2 + (\text{Eq. (3.34)})^2] \tag{3.35}$$

vanishes. This condition can be written as

$$f(\mu_k, a_{1,k}, a_{2,k}, h, h_r, S) = 0 \tag{3.36}$$

since from the recurrence relation (3.28), every $a_{n,k}$ can be expressed in terms of $a_{1,k}$ and $a_{2,k}$. The equation (3.36) is solved numerically, by using an optimization technique proposed by Papageorgiou *et al.* [8, 9].

4. NUMERICAL RESULTS AND DISCUSSION

A numerical solution of equation (3.36) was carried out by the aid of the optimization package MERLIN-2.1 [8, 9]. For given S and h_r, equation (3.36) determines the eigenvalue h for finite n and k. The number of terms in the sums was increased until the difference between consecutive eigenvalues of (h) was negligible. For a matter of convenience we chose μ_k to be the positive zeros of $j_1'(x) = 0$. The numerical calculation of μ_k was carried out by the use of function FindRoot, of the package Mathematica [14].

The results of the computation are shown in Figs 1–4. The variation of the resonance field (h) with the resonance frequency (h_r) is plotted in Fig. 1 and with the size parameter (S) in Fig. 2, for various values of h and h_r, respectively. It is noted that h is always larger than the nucleation field of the static problem and h_r is real, because only then the precession about the reference state is stable. Values of h smaller than the static nucleation field have no physical significance because the corresponding oscillations occur around an unstable reference state [12]. It is also seen that the zero frequency limit ($h_r = 0$) of the resonance field coincides with the nucleation field of the static problem, which, in the case under discussion, corresponds to the *curling mode* [15–17]. The resonance field is a decreasing function of S, a behaviour at least qualitatively the same with the experimental results [2], though far from the corresponding size range ($R \ll 1$ mm) [6].

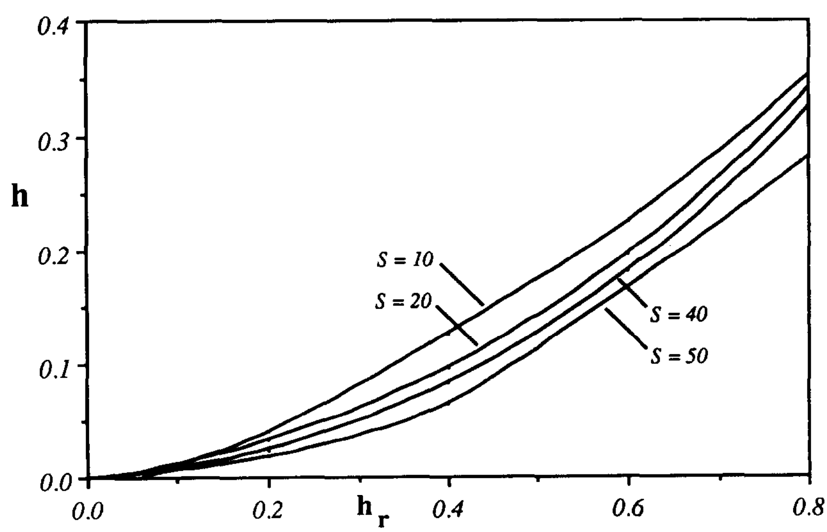

Fig. 1. Variation of h with h_r.

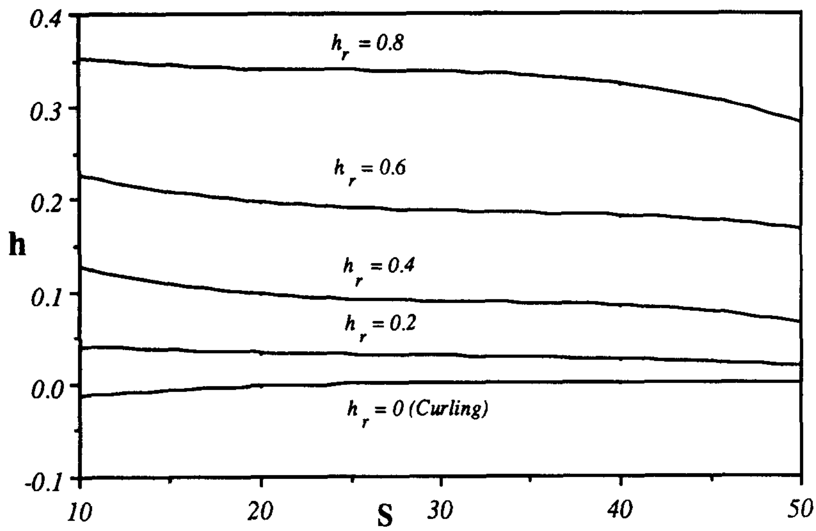

Fig. 2. Variation of h with S.

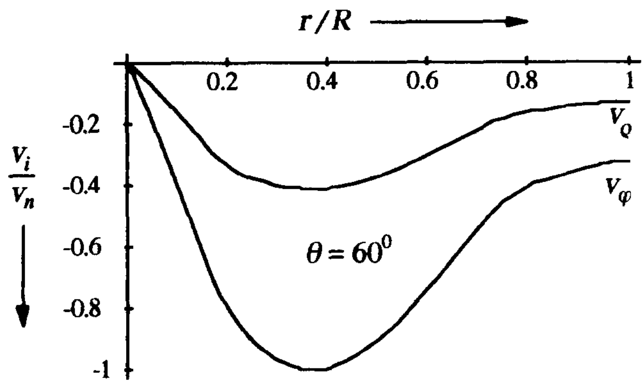

Fig. 3. Reduced components v_ϱ, v_φ as a function of the reduced radial distance r/R for $h = 0.324$, $h_r = 0.8$, $S = 40$ for $i = \varrho$, φ and $v_n = \max(|v_\varrho|, |v_\varphi|)$.

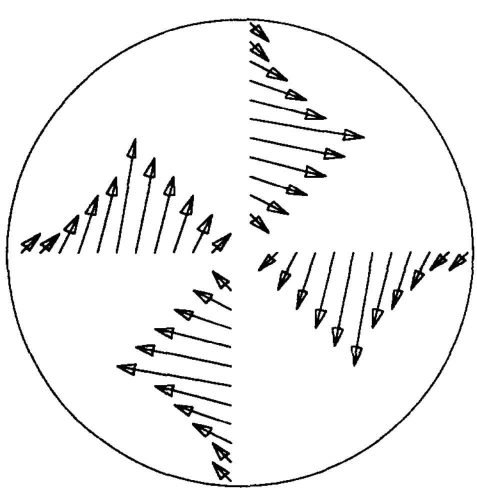

Fig. 4. Configuration of the transverse magnetization for $\theta = 60°$, $h = 0.324$, $h_r = 0.8$, $S = 40$.

For the case of magnetite (Fe_3O_4) with $A \simeq 10^{-7}$ erg/cm, $M_s \simeq 4.5 \times 10^5$ A/m ($R_0 = 1.98$ nm), $\gamma_0 = 1.105 \times 10^5 g \times$ m/Asec and $g \simeq 2.001$ we obtain for $10 \le S \le 50 \Leftrightarrow 19.8\,\text{nm} \le R \le 99\,\text{nm}$ that the resonance field H_z varies as $0.019 \le h \le 0.352 \Leftrightarrow 676\,\text{Oe} \le H_z \le 12.523\,\text{kOe}$ for frequencies $f(\omega = 2\pi f \Rightarrow h_r = f/M_s\gamma_0)$, $0.2 \le h_r \le 0.8 \Leftrightarrow 19.9\,\text{GHz} \le f \le 79.6\,\text{GHz}$, or wavelengths λ, in the following range: $3.8\,\text{mm} \le \lambda \le 1.5\,\text{cm}$.

The proposed analysis, in comparison to that presented by Aharoni [7], has the advantage of computing the eigenvectors of the problem and it is therefore possible to plot the shape of the magnetization configuration in the material. The results for $\theta = 60°$, $h_r = 0.8$, $S = 40$ and $h = 0.324$ are plotted in Fig. 3. The vertical axis corresponds to the normalized components v_ϱ and v_Φ and the horizontal to the distance from the center of the sphere. It is also seen that the dominant contribution is of the circumferential component of the transverse magnetization vector. A more realistic picture of the vector field of the transverse magnetization is shown in Fig. 4. It is obvious that it looks like some kind of curling. That is why it could be named *generalized curling*.

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