# OPTIMAL POWER ALLOCATION AND JOINT SOURCE-CHANNEL CODING FOR WIRELESS DS-CDMA VISUAL SENSOR NETWORKS USING THE NASH BARGAINING SOLUTION

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# ABSTRACT

We consider the problem of resource allocation for a Direct Sequence Code Division Multiple Access (DS-CDMA) wireless visual sensor network (VSN). We use the Nash Bargaining Solution (NBS) from game theory in order to determine the transmission power and source and channel coding rate for each node. The NBS assumes that the nodes negotiate (using the help of a centralized control unit) in order to jointly determine their transmission parameters. The transmission powers are allowed to take continuous values, whereas the source and channel coding rate combination can only assume discrete values. Thus, the resulting optimization problem is a mixedinteger optimization task and is solved using Particle Swarm Optimization (PSO). Experimental results are provided and conclusions are drawn.

*Index Terms*— Visual sensor network, cross–layer optimization, Nash Bargaining Solution, Particle Swarm Optimization, game theory.

# 1. INTRODUCTION

Wireless visual sensor networks (VSN) consist of low-weight nodes that are equipped with video cameras and are able to compress and transmit video to a centralized control unit. In this work, we assume that Direct Sequence Code Division Multiple Access (DS-CDMA) is used for wireless transmission. The use of DS-CDMA is very appropriate for this application because it allows nodes that image a low-motion scene to use a lower transmission power. This is because low-motion video requires a lower source coding bit rate, thus a larger bit rate may be used for channel coding, which will allow a lower transmission power to be used. Transmission with lower power has the dual benefit of battery conservation and reduced interference to the other nodes.

In this paper, we propose a scheme for selecting the transmission power, source coding rate and channel coding rate for each node. Increasing the transmission power of a node will increase the received quality of the video it transmits but it will also degrade the received qualities of the other nodes' videos, due to increased interference. Thus, it is necessary for the transmission powers of all nodes to be jointly selected. An appropriate optimization criterion that considers the qualities of the videos of all nodes should be used. In previous work [1, 2], we proposed cross–layer optimization schemes that minimize either the average video distortion of all nodes, or the maximum video distortion among the nodes.

Here, we propose a resource allocation scheme that is based on the Nash Bargaining Solution (NBS) from Game Theory [3]. The NBS allocates resources as a result of a negotiation between the nodes with the help of the centralized control unit. Each node is guaranteed a minimum video quality if negotiations fail (disagreement point). The NBS has been used before in video streaming for the allocation of the total bit rate among several users [4]. However, in [4], no specific network setup is assumed. In [5], we used the NBS for the cross–layer optimization of a DS-CDMA VSN. However, in that work, the transmission powers of the nodes were restricted to take values from a discrete set. Thus, a discrete optimization problem was formulated and solved. In the present paper, we assume that the transmission powers can take values from a continuous set. Since the source coding–channel coding rate combination can only take discrete values, the resulting optimization problem is a mixed– integer problem and is solved using Particle Swarm Optimization [6]. Also, in the present paper, the utility function and the disagreement point are defined differently than in [5].

The rest of the paper is organized as follows. In section 2, the proposed VSN is described. In section 3, the proposed optimal resource allocation based on the NBS is presented. In section 4, experimental results are provided and in section 5, conclusions are drawn.

# 2. VISUAL SENSOR NETWORK

In this section, we describe the basic architecture of the considered wireless visual sensor network that utilizes DS-CDMA.

## 2.1. DS-CDMA

In DS-CDMA, all nodes transmit on the same frequency. To transmit a single bit, a node actually transmits L "chips". Each node k is assigned a unique spreading code  $s_k$ , which is a vector of length L. In order to transmit the *i*th bit of a bit stream, node k actually transmits  $b_k(i)s_k$ , which is a vector of L chips and  $b_k(i)$  is either 1 or -1, depending on the value of the bit that is being transmitted. The node of interest receives interference from the other nodes. It is reasonable to assume that the interference can be approximated by Additive White Gaussian Noise (AWGN) [7]. Since node k has an associated power level,  $S_k = E_k R_k$  and assuming that the thermal noise is negligible compared to the interference, the energy-per-bit to Multiple Access Interference (MAI) ratio becomes:

$$\frac{E_k}{N_0} = \frac{\frac{S_k}{R_k}}{\sum_{j \neq k}^K \frac{S_j}{W_t}}; k = 1, 2, 3, ..., K,$$
(1)

where  $E_k$  is the energy-per-bit,  $N_0/2$  is the two-sided noise power spectral density due to MAI in Watts/Hertz,  $S_k$  is the power of the node of interest in Watts,  $R_k$  is the transmitted bit rate (total bit rate used for source and channel coding) in bits per second,  $S_j$  is the power of interfering node j in Watts, and  $W_t$  is the total bandwidth in Hertz [7].

#### 2.2. Source and Channel Coding

In our setup, we assume that the video captured by the nodes is compressed using the H.264/AVC video coding standard. This coding standard has been used for video transmission over various networking environments and it has two major layers, the Video Coding Layer (VCL) and the Network Abstraction Layer (NAL). VCL contains specifications of the video–encoding engine including motion compensation, transform coding of coefficients, and entropy coding. NAL is responsible for the encapsulation of the coded slices into transport entities of the network, the NAL units.

Regarding channel coding, we use Rate Compatible Punctured Convolutional (RCPC) codes [8] that allow us to utilize Viterbi's upper bounds on the bit error probability,  $P_b$ .

### 3. OPTIMAL RESOURCE ALLOCATION

A centralized control unit at the network layer of a DS-CDMA VSN determines how network resources should be allocated amongst the nodes. It can request changes in transmission parameters, such as the source coding rates, channel coding rates, and power levels. The constraint for the NBS is that the chip rate be the same for all nodes. As will be explained later in this section, assuming that the spreading code length is the same for all nodes, a constraint on the chip rate corresponds to a constraint on the transmission bit rate  $R_k$ . Thus, we can equivalently impose a constraint on the bit rate instead of the chip rate. We wish to determine for each node k the source coding rate  $R_{s,k}$ , channel coding rate  $R_{c,k}$ , and power level  $S_k$ ,  $k = 1, \ldots, K$ . The objective is to optimize a function of the expected video distortions for the nodes. In this paper, the function to be optimized is derived using the Nash Bargaining Solution.

In order to reduce the computational complexity of the solution, we assume that there are two possible motion levels viewed by the sensor nodes, high motion and low motion. Thus, we assume that the K nodes are grouped into two motion classes, namely high-motion nodes,  $K_h$ , and low-motion nodes,  $K_l$  ( $K_h + K_l = K$ ). Therefore, the goal of our optimization problem is to determine the vectors  $S = (S_h, S_l)^{\top}$ ,  $R_s = (R_{s,h}, R_{s,l})^{\top}$ , and  $R_c = (R_{c,h}, R_{c,l})^{\top}$ , where  $S_h$ ,  $R_{s,h}$  and  $R_{c,h}$  are the transmission parameters for the high motion nodes, and  $S_l$ ,  $R_{s,l}$  and  $R_{c,l}$  are the corresponding values for the low-motion nodes.

In this paper, we assume that the transmission powers can take values from a continuous set of predetermined range. Specifically,  $S_h, S_l \in \mathbf{S} = [s_{\min}, s_{\max}] \subset \mathbb{R}$ , unlike [5] where power levels could only take discrete values. The source coding rates can take discrete values from a set  $\mathbf{R}_s$ , that is,  $R_{s,h}, R_{s,l} \in \mathbf{R}_s$  and the channel coding rates can also take discrete values from a set  $\mathbf{R}_c$ , that is,  $R_{c,h}, R_{c,l} \in \mathbf{R}_c$ .

Assuming that all nodes utilize the same spreading code length L, the transmission bit rate of each user is  $R_k = \frac{R_{chip}}{L}$ , where  $R_{chip}$  is the chip rate. The constraint in our optimization problem is that chip rate be the same for all nodes. Thus, this constraint corresponds to a constraint on the transmission bit rate  $R_k$ . Given this assumption, and since  $R_k = \frac{R_{s,k}}{R_{c,k}}$  for a node k, source coding rates and channel coding rates share the same transmission bit rate. Since  $R_{c,k}$  can only take values from a discrete set [8], if follows that the pairs  $(R_{s,k}, R_{c,k})^{\top}$  can take values from a finite discrete set  $\mathbf{R}_{s+c}$ , namely  $(R_{s,h}, R_{c,h})^{\top}, (R_{s,l}, R_{c,l})^{\top} \in \mathbf{R}_{s+c}$ . It should be noted that the sets  $\mathbf{R}_s, \mathbf{R}_c, \mathbf{R}_{s+c}$  shall be of the same cardinality.

## 3.1. The Nash Bargaining Solution

We next describe the Nash Bargaining Solution. Let us first define the utility function. The *utility function*  $U_k$  is given by

$$U_k = 10 \log_{10} \frac{255^2}{E\{D_{s+c,k}\}},\tag{2}$$

where  $E\{D_{s+c,k}\}$  is the expected video distortion for a node k. In this case the utility of a node is the PSNR of the received video.

The expected video distortion for a node is due to both the lossy compression and the channel errors. In this paper, we utilize Universal Rate–Distortion Characteristics (URDC), which present the expected distortion as a function of the bit error rate, after channel decoding. So, we assume the following model for the URDC for each node k:  $E\{D_{s+c,k}\} = a \left[ \log_{10} \left( \frac{1}{P_b} \right) \right]^{-b}$ , where a > 0 and b > 0 are determined using mean square optimization from a few  $(E\{D_{s+c,k}\}, P_b)$  pairs that are obtained experimentally. The parameters a and b depend on the amount of motion of the video sequence and the source coding rate. It can be shown that, using Eq. (1), Viterbi's upper bound for  $P_b$ , and the URDC model, the expected video distortion of node k can be expressed as a function of the source and channel coding rates of node k and the power levels of all nodes [2]. Thus, the expected video distortion can be written as  $E\{D_{s+c,k}\}(R_{s,k}, R_{c,k}, S)$ .

The *feasible set*, **U**, is the set of all possible vectors  $(U_1, U_2, \ldots, U_K)^{\top}$ . Each vector in **U** results from a different combination of the source coding rates, channel coding rates and transmission powers of all K nodes. The requirement for the feasible set **U** is that it should be convex, closed and bounded above. We conducted a number of experiments in order to verify the convexity of the feasible set for the NBS, considering that free disposal is allowed [3]. In all cases, we assumed two motion classes for the nodes. Pictorially, it follows that the feasible set is convex for all node distributions we considered. The feasible set with the NBS shown on it for 70 high-motion nodes and 30 low-motion nodes is depicted in Fig. 1.



Fig. 1: Illustration of the convexity of the feasible set for 70 highmotion and 30 low-motion nodes,  $d = (28, 28)^{\top} dB$ .

The centralized control unit should help nodes negotiate in order to reach a mutually acceptable agreement on a combination of set U. Each node joins the bargaining game with the goal of gaining at least as high a utility as what it would get without negotiating. The minimum utility each node can still get if negotiations fail is called the *disagreement point*. The disagreement point,  $d = (d_1, d_2, ..., d_K)^\top$ , is the vector of minimum utilities each user expects by joining the game, without cooperation. In the present work, we assume that d is the minimum acceptable utility (in our case, PSNR) for each node and is imposed by the designer of the system. Clearly, the imposed d should belong to the feasible set U.

The *bargaining set* consists of all Pareto–optimal points (elements of **U**) that assign to all nodes a utility that is at least as high as the disagreement point. An outcome of a utility allocation is Pareto–optimal if there is no other outcome that all nodes prefer. Thus, a utility allocation  $U = (U_1, U_2, ..., U_K)^{\top}$  is not Pareto–optimal if there is another allocation where each node increases its utility.

The bargaining solution that fulfills the following axioms [3] is known as the Nash Bargaining Solution.

- 1. i)  $F(\mathbf{U}, d) \ge d$ . ii)  $y > F(\mathbf{U}, d) \Rightarrow y \notin \mathbf{U}$ .
- 2. Given any strictly increasing affine transformation  $\tau(.)$  $F(\tau(\mathbf{U}), \tau(d)) = \tau(F(\mathbf{U}, d)).$

3. If 
$$d \in \mathbf{Y} \subseteq \mathbf{U}$$
, then  
 $F(\mathbf{U}, d) \in \mathbf{Y} \Rightarrow F(\mathbf{Y}, d) = F(\mathbf{U}, d)$ .

The first axiom stipulates that the solution should lie in the bargaining set. The second axiom states that if the utility function or the disagreement point are scaled by an affine transformation, it should not make a difference in the solution. The third axiom formalizes the *Independence of Irrelevant Alternatives* [3].

In order to find the Nash Bargaining Solution, we need to determine the source coding rate, channel coding rate and power for each node that result in the utility vector  $U = (U_1, U_2, \ldots, U_K)^{\top}$  that maximizes the Nash product:

$$F(\mathbf{U}, d) = \arg \max_{U} (U_1 - d_1)^{\alpha_1} (U_2 - d_2)^{\alpha_2} \cdots (U_K - d_K)^{\alpha_K},$$
(3)

subject to the requirement that  $U \ge d$ .  $a_k$  is the *bargaining power* of each node. Specifically, a node with a higher bargaining power is more advantaged by its role in the bargaining game than a node with a lower bargaining power. Here, there is no reason to assume that some nodes are more advantaged than others, thus we set  $\alpha_k = 1/K$  for all k.

After node clustering into high-motion nodes and low-motion nodes, the vectors of utilities and disagreement point become  $U = (U_h, U_l)^{\top}$  and  $d = (d_h, d_l)^{\top}$ , respectively. Therefore, we need to maximize

$$F(\mathbf{U},d) = \arg\max_{U} (U_h - d_h)^{\frac{K_h}{K}} (U_l - d_l)^{\frac{K_l}{K}}, \qquad (4)$$

Since the transmission powers can take values from a continuous set, while the source and channel coding rates can take values from discrete sets, the optimization problem of Eq. (4) is a mixed–integer problem, which we solve using the particle swarm optimization algorithm.

#### 3.2. The Particle Swarm Optimization algorithm

Particle Swarm Optimization (PSO) is a population–based, stochastic optimization algorithm [6] and it is categorized as a swarm intelligence algorithm. PSO exploits a population (called a *swarm*) of search points (called *particles*) to probe the search space. Each particle moves in the search space with an adaptable velocity, recording the best position it has ever visited. In minimization problems, such positions have the lowest function values. The velocity is adapted based on information coming from the particle itself as well as from the rest of the swarm. More specifically, each particle assumes a "neighborhood" that consists of some other particles. The best position ever attained by any member of the neighborhood is then communicated to the particle and influences its velocity's update. To put it formally, let f(x),  $x \in V \subset \mathbb{R}^n$ , be the objective function under minimization. Then, a swarm to tackle this problem consists of N particles,  $\mathbb{S} = \{x_1, x_2, \ldots, x_N\}$ , which are *n*-dimensional vectors,  $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^\top \in V$ ,  $i = 1, 2, \ldots, N$ . The velocity,  $v_i = (v_{i1}, v_{i2}, \ldots, v_{in})^\top$ , of the *i*th particle, as well as its *best position*,  $p_i = (p_{i1}, p_{i2}, \ldots, p_{in})^\top \in V$ , are also *n*-dimensional vectors. The neighborhoods are usually defined based on the particles' indices. The most common neighborhood topology is the "ring" topology, where the neighborhood of a particle consists of particles with neighboring indices. Thus, a neighborhood of radius m of  $x_i$  is the set of indices,  $NB_i = \{i - m, \ldots, i, \ldots, i + m\}$ , where index 1 is assumed to follow immediately after N.

Let  $g_i \in NB_i$  denote the index of the particle that attained the best previous position among all the particles in the neighborhood of  $x_i$ , i.e.,  $f(p_{g_i}) \leq f(p_j), \forall j \in NB_i$ , and let t be the iteration counter. Then, the velocity and position of  $x_i$  are updated according to the equations [6]:

$$v_i(t+1) = \chi \left[ v_i(t) + c_1 R_1 \left( p_i(t) - x_i(t) \right) + c_2 R_2 \left( p_{g_i}(t) - x_i(t) \right) \right],$$
  
$$x_i(t+1) = x_i(t) + v_i(t+1),$$
 (5)

where  $\chi$  is a parameter called the *constriction coefficient*;  $c_1$ ,  $c_2$  are positive acceleration parameters called *cognitive* and *social* parameter, respectively; and  $R_1$ ,  $R_2$  are vectors with components uniformly distributed in the range [0, 1]. All vector operations in Eqs. (5) are performed componentwise. The best position of a particle is updated as soon as a better position (i.e., one with lower function value) is discovered by the particle. Clerc and Kennedy [9] studied the stability of PSO, proposing values of its parameters that promote convergence of the algorithm towards the most promising solutions in the search space. Its efficiency and the minor required implementation effort, rendered PSO one of the most popular intelligent optimization approaches.

## 4. EXPERIMENTAL RESULTS

In order to evaluate the proposed game—theoretic multinode resource allocation method, we conducted a number of experiments, some of which are presented here. Since we assumed two motion classes, the "Foreman" video sequence was used to represent the scene viewed by a high—motion node, while "Akiyo" was used to represent the scene viewed by a low—motion node, both at QCIF resolution. Thus, it is necessary to have two sets of URDC curves, one for each level of motion. The characteristics were obtained for both video sequences at a frame rate of 15 frames/s and the H.264/AVC High profile for 4:2:0 color format video was selected. The data points used to obtain the parameters a and b in the URDC model were obtained by corrupting the video stream with packet errors based on a bit error rate  $P_b$ , decoding the corrupted video bit stream with the H.264/AVC codec, calculating the distortion, repeating this experiment 300 times and then taking the average distortion.

We assumed Binary Phase Shift Keying (BPSK) modulation and RCPC codes with mother rate 1/4 from [8]. The total bandwidth  $W_t$ was set to 20 MHz. Moreover, we assumed a target bit rate of  $R_k =$ 96000bps. The set of admissible source and channel coding rate combinations is  $\mathbf{C} \in \{1 : (32kbps, 1/3), 2 : (48kbps, 1/2), 3 : (64kbps, 2/3)\}$ , for a node k. The power levels assumed continuous values from the set  $\mathbf{S} = [5.0, 15.0]$ , (representing Watts). Let  $C_h$  and  $C_l$  denote the index of the source and channel coding rate combination selected for the high-motion and the low-motion users, respectively ( $C_h, C_l \in \{1, 2, 3\}$ ).

Regarding PSO, a swarm of 20 particles was used under the ring topology of radius 1. Each particle consisted of four unknowns, namely  $S_h$ ,  $S_l$  (continuous values) and  $C_h$ ,  $C_l$  (discrete values). In our implementation, the discrete parameters were allowed to take continuous values for the position and velocity update, although they were rounded to the nearest integer for the evaluation of the particle. The default PSO parameter values,  $\chi = 0.729$ ,  $c_1 = c_2 = 2.05$  [9], were used. Since PSO is a stochastic algorithm, its performance was assessed on average over a number of 30 independent experiments and it was allowed to execute 500 iterations for each problem. At each experiment, the best detected solution was recorded. We can see from Eq. (1) that multiplying all powers with the same constant does not change  $E_k/N_0$ . This is because we assumed that thermal noise is negligible and the AWGN is entirely due to interference. Thus, the optimization essentially determines the optimal  $S_h/S_l$  ratio rather than specific values for the powers.

We compared the Nash Bargaining Solution for  $d = (28, 28)^{\top}$  dB to a method that minimizes the average distortion of the nodes (MAD) and a method that minimizes the maximum distortion among the nodes (MMD). Tables 1, 2 and 3 show the results of the three methods for 70 high-motion and 30 low-motion nodes, 50 high-motion and 50 low-motion nodes, and 30 high-motion and 70 low-motion nodes, respectively.  $C_h$ ,  $S_h$  and  $PSNR_h$  are the source-channel coding rate combination, transmission power and PSNR for the high-motion nodes, while  $C_l$ ,  $S_l$  and  $PSNR_l$  are the corresponding parameters for the low-motion nodes.

	$C_h$	$C_l$	$S_h$	$S_l$	$PSNR_h$	$PSNR_l$
MAD	2	1	15.0000	7.6069	29.2296	32.2581
MMD	3	1	13.4344	5.0000	29.7737	29.7737
NBS	3	1	15.0000	6.3195	29.5508	30.8520

 Table 1: Optimal resource allocation for the three criteria (MAD, MMD, NBS) for 70 high-motion and 30 low-motion nodes.

	$C_h$	$C_l$	$S_h$	$S_l$	$PSNR_h$	$PSNR_l$
MAD	3	1	15.0000	7.0428	30.9419	32.8537
MMD	3	1	13.0847	5.0000	31.6114	31.6114
NBS	3	1	10.8085	5.0000	30.9947	32.7705

**Table 2**: Optimal resource allocation for the three criteria (MAD, MMD, NBS) for 50 high-motion and 50 low-motion nodes.

	$C_h$	$C_l$	$S_h$	$S_l$	$PSNR_h$	$PSNR_l$
MAD	3	3	8.6240	5.0000	31.3844	35.1131
MMD	3	1	12.6814	5.0000	33.4049	33.4049
NBS	3	3	8.4217	5.0000	31.2350	35.2603

**Table 3**: Optimal resource allocation for the three criteria (MAD, MMD, NBS) for 30 high–motion and 70 low–motion nodes.

We can see that, in order to minimize the average distortion of the nodes, the MAD always results in a greater PSNR for the lowmotion nodes. We observed a difference between the PSNR of the low-motion and the high-motion nodes of up to 3.73 dB. The MMD minimizes the maximum distortion among the nodes. This always results in equal PSNR for the high-motion and low-motion nodes. However, the PSNR increase compared to the MAD for the highmotion nodes is lower than the PSNR decrease of the low-motion nodes. Thus, on one hand, we can say that the MMD is fair because it offers equal PSNR between the two node classes. On the other hand, the PSNR of the low-motion nodes drops significantly. The NBS can be seen as a compromise between MAD and MMD. With NBS, the low-motion nodes always get a significantly higher PSNR than with MMD. The high-motion nodes also get a higher PSNR than with MAD, except when the number of low-motion nodes is significantly larger than the number of high-motion nodes, in which case the PSNR of the high-motion nodes is slightly lower compared with MAD, and the PSNR of the low-motion nodes is slightly higher compared with MAD and significantly higher compared with MMD.

## 5. CONCLUSIONS

We have presented a resource allocation scheme for DS-CDMA visual sensor networks, which is based on the Nash Bargaining Solution from Game Theory. Since the power levels assume continuous values while the source and channel coding rate combinations can only take discrete values, the resulting optimization problem is a mixed-integer problem, which is solved using Particle Swarm Optimization. We experimentally compared the NBS with two other optimization criteria, the MAD and the MMD. The NBS can be seen as a compromise between MAD and MMD, in which high-motion nodes receive a higher PSNR than MAD in most cases, while lowmotion nodes receive a higher PSNR than MMD. Since it is important to improve the video quality of high-motion nodes, which are usually more important in surveillance applications, without reducing the video quality of the low-motion nodes too much, we believe that the NBS will be the criterion of choice in many practical applications.

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