Strongly chordal and chordal bipartite graphs are sandwich monotone

Pinar Heggernes[1] Federico Mancini[1]

Charis Papadopoulos[2] R. Sritharan[3]

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The 15th Int'l Computing and Combinatorics Conference COCOON 2009

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Overview

- *Sandwich monotone* graphs

implies that minimal completion problems are poly-time solvable

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subclass of chordal graphs

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constructive proof

- Minimal Strongly Chordal completions (characterizations)

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close related to Strongly Chordal graphs

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Sandwich monotone

- A graph class $\mathcal{C}$ is sandwich monotone:

Given two graphs $G_0 = (V, E) \in \mathcal{C}$ and $G_{|F|} = (V, E \cup F) \in \mathcal{C}$, there is an edge $f \in F$ such that $\boxed{G_{|F|} - f \in \mathcal{C}}$.

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Given $G_0 = (V, E)$ and $G_F = (V, E \cup F)$,

$$G_1 = (V, E \cup \{f_1\}) \in \mathcal{C}$$

$$\uparrow$$

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- Sandwich monotonicity $\mathcal{C}$:

Given $G_0 = (V, E)$ and $G_F = (V, E \cup F)$,

$$G_2 = (V, E \cup \{f_1, f_2\}) \in \mathcal{C}$$

$$G_1 = (V, E \cup \{f_1\}) \in \mathcal{C}$$

$$\boxed{G_0 = (V, E) \in \mathcal{C}}$$

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$$\boxed{G_F = (V, E \cup \{f_1, f_2, \ldots, f_{|F|-1}, f_{|F|}\}) \in \mathcal{C}}$$

$$G_{F-1} = (V, E \cup \{f_1, f_2, \ldots, f_{|F|-1}\}) \in \mathcal{C}$$

$$\vdots$$

$$G_2 = (V, E \cup \{f_1, f_2\}) \in \mathcal{C}$$

$$G_1 = (V, E \cup \{f_1\}) \in \mathcal{C}$$

$$\boxed{G_0 = (V, E) \in \mathcal{C}}$$

- Close related to *minimal completion problems*:

Given any graph $G = (V, E)$ and a graph class $\mathcal{C}$, we are looking for a supergraph $H = (V, E \cup F)$ such that $H \in \mathcal{C}$ and

$\boxed{\text{for every } F' \subset F\text{: } H' = (V, E \cup F') \notin \mathcal{C}}$

$\triangleright$ computing the minimum $|F|$ is usually difficult (NP-complete).

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- if $\mathcal{C}$ is sandwich monotone:

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if $\mathcal{C}$ is sandwich monotone and $\mathcal{C}$ is recognized in poly-time:

There is a polynomial time algorithm for computing a minimal $\mathcal{C}$ completion.

Why research on sandwich monotonicity? 2/2

- Listing all graphs in $\mathcal{C}$ with edge constraints

COCOON 2008

Input: $G = (V, E)$ and $H = (V, E \cup F)$ such that $H \in \mathcal{C}$

Output: list all graphs in $\mathcal{C}$ between G and H

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- Kijima, Kiyomi, Okamoto, and Uno (COCOON 2008)

Efficient solution for chordal graphs

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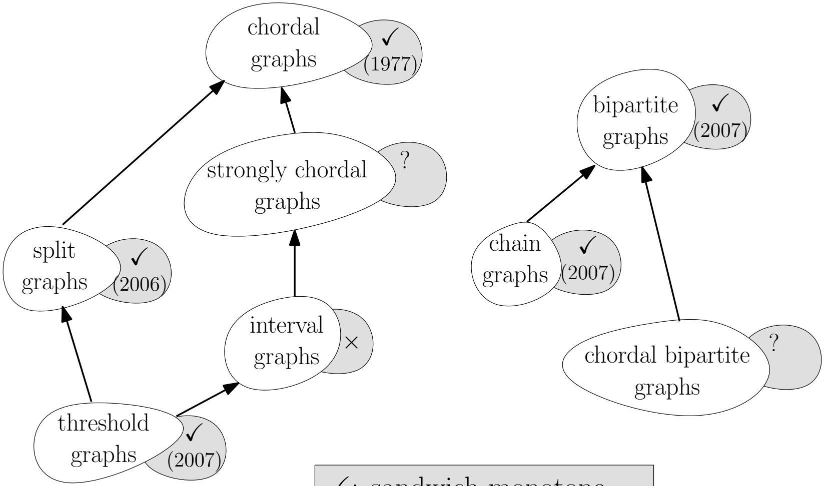
Efficient solution for chordal graphs

▷ Observing the previous algorithm...

the proposed solution can be extended to

every sandwich monotone class

Which graph classes are sandwich monotone?



✓: sandwich monotone

×: not sandwich monotone

?: unknown

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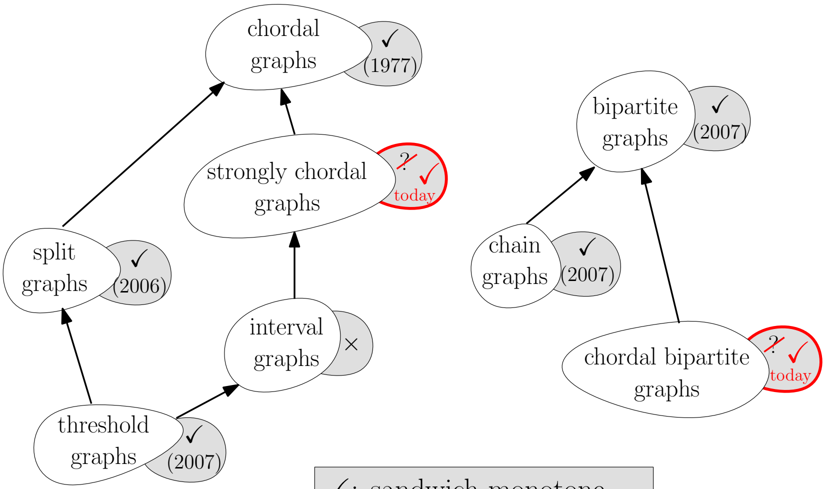

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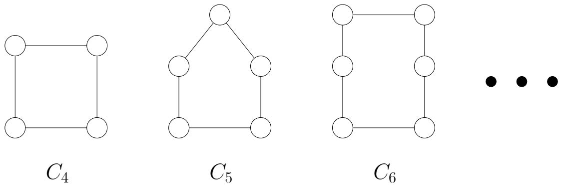
- Chordal graphs: do not contain C_k, $k \geq 4$



Strongly Chordal graphs 1/3

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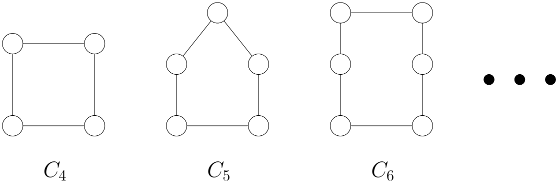
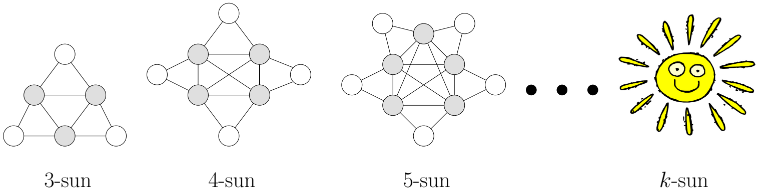
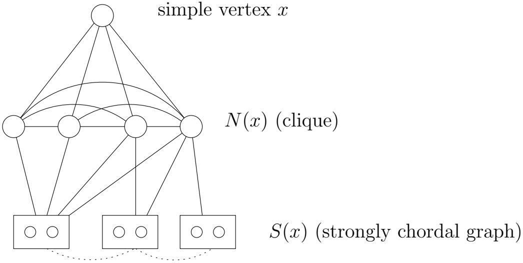


- Strongly Chordal graphs: chordal graphs having no k-sun



- Strongly Chordal graphs:

characterized through simple vertices



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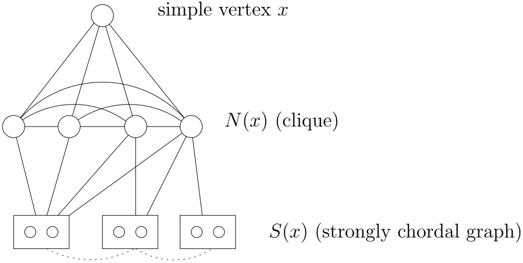

- Simple elimination ordering: given an ordering $v_1, \ldots, v_n$

For every i, v_i is simple in $G_i \equiv G[\{v_i, \ldots, v_n\}]$, $1 \leq i \leq n$.

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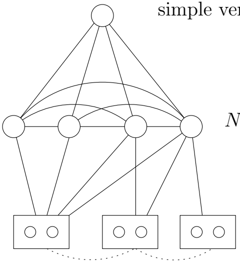

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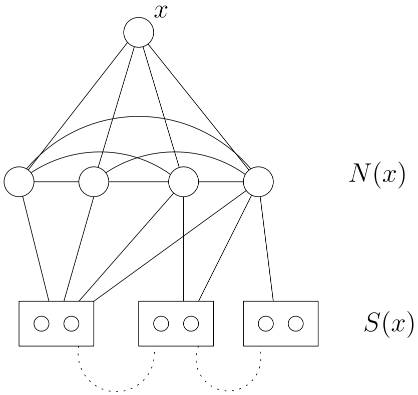
For every i, v_i is simple in $G_i \equiv G[\{v_i, \ldots, v_n\}]$, $1 \leq i \leq n$.

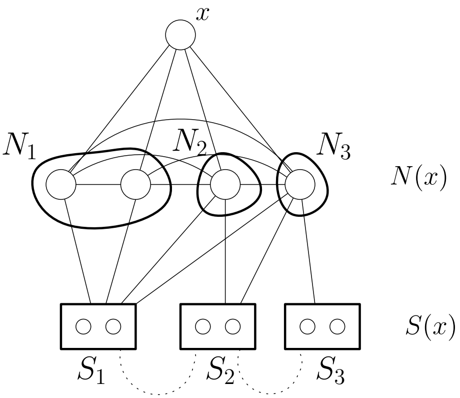
Farber, 1983

G is strongly chordal $\Leftrightarrow$ G admits a simple elimination ordering.

- Simple vertices ⇒ simple partition



- Simple vertices $\Rightarrow$ simple partition



- $N(N_1) \subset N(N_2) \subset N(N_3) \Rightarrow S_1 \prec S_2 \prec S_3$

Strongly Chordal graphs are Sandwich Monotone 1/3

- Our goal is to prove the following:
  - Given: $G = (V, E)$ and $G' = (V, E \cup F)$
    two strongly chordal graphs,
  - there is an edge $f \in F$ such that $G' - f$ is strongly chordal.

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  - Given: $G = (V, E)$ and $G' = (V, E \cup F)$
    two strongly chordal graphs,
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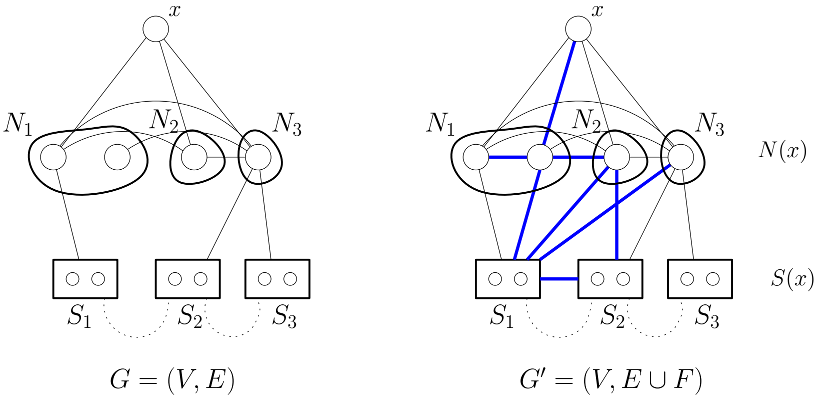
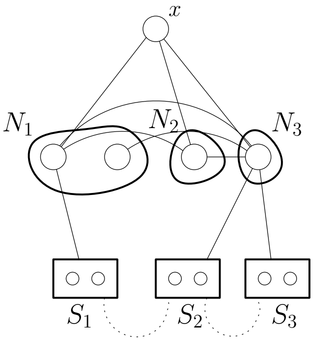
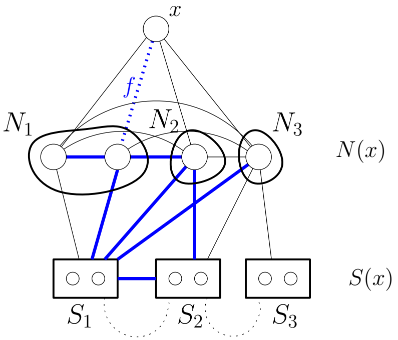
- For the proof we apply induction on $|V|$:
  - For $|V| \leq 3$: all graphs are strongly chordal
    (the statement holds)
  - Assume that the statement is true for $|V| - 1$.
  - Prove that the statement holds for $|V|$.



- G': Pick a simple vertex



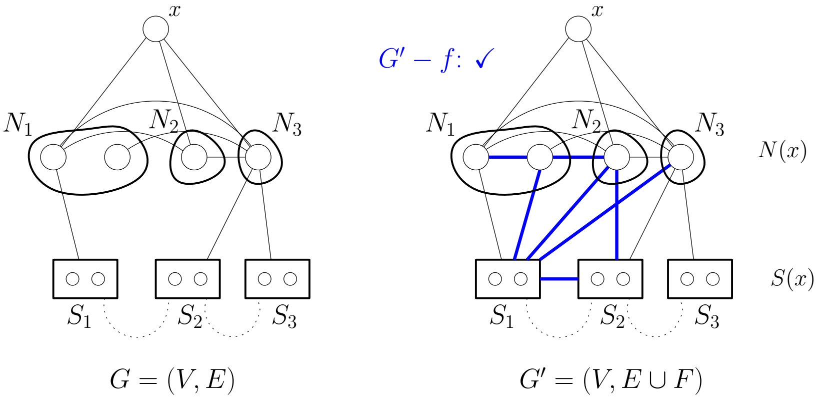
$G = (V, E)$



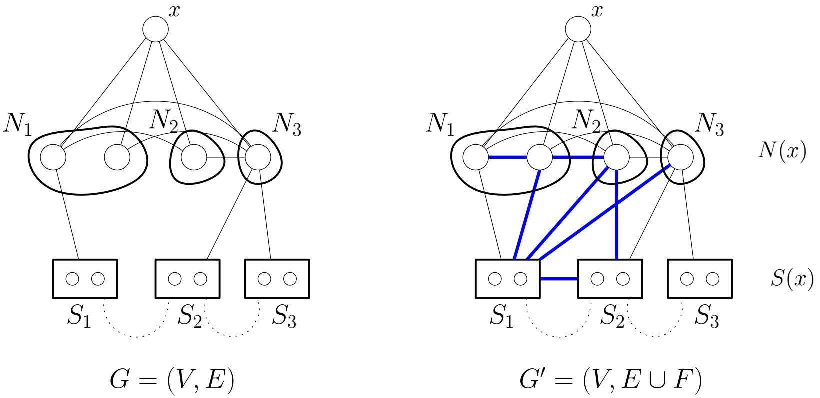
$G' = (V, E \cup F)$

- if x is incident to an added edge $f \in F$

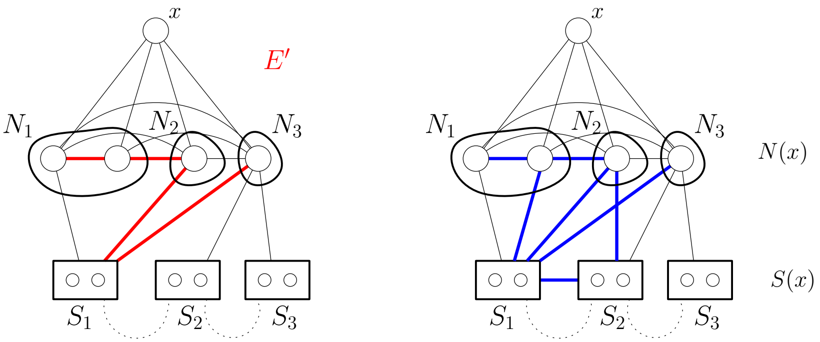
Strongly Chordal graphs are Sandwich Monotone 2/3



- then $G' - f$ has no C_4 or 3-sun $\Rightarrow$ $\boxed{G' - f \ \checkmark}$



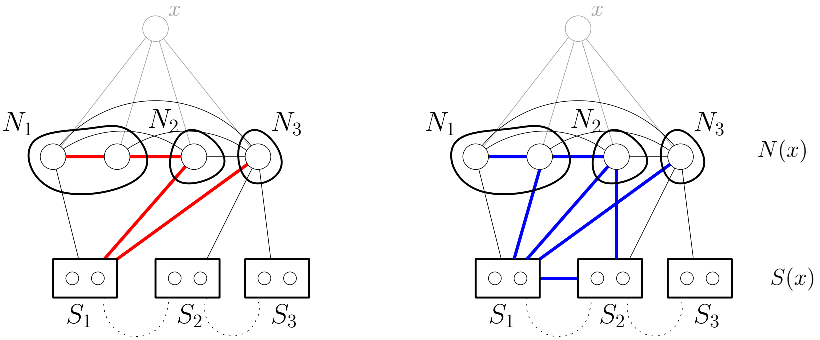
- otherwise, $F \subset N(x) \cup S(x)$



$\cancel{G = (V, E)} \Rightarrow H = (V, E \cup E')$ $\quad G' = (V, E \cup F)$

- Add certain type of edges E' to $G = (V, E) \Rightarrow H = (V, E \cup E')$
  E' : $N_G(x)$ clique and between $N(x) - S(x)$

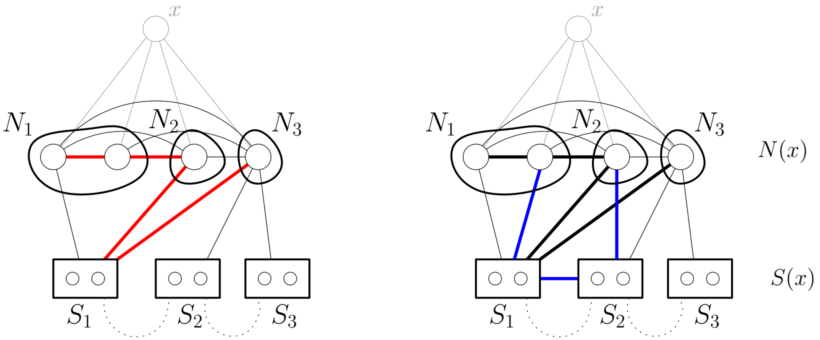
Strongly Chordal graphs are Sandwich Monotone 2/3



$\cancel{G = (V, E)} \Rightarrow H = (V, E \cup E')$ $\qquad G' = (V, E \cup F)$

- Remove x from both graphs H and G'

Strongly Chordal graphs are Sandwich Monotone 2/3

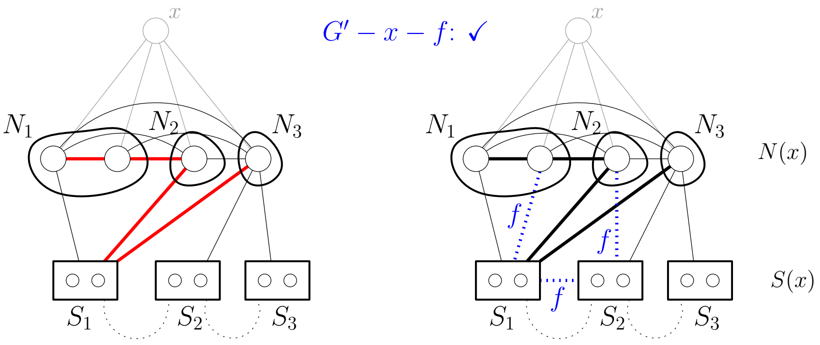


$\cancel{G = (V, E)} \Rightarrow H = (V, E \cup E')$ $\quad G' = (V, E \cup F)$

- Both graphs are on $|V| - 1$ vertices
- Both graphs are strongly chordal

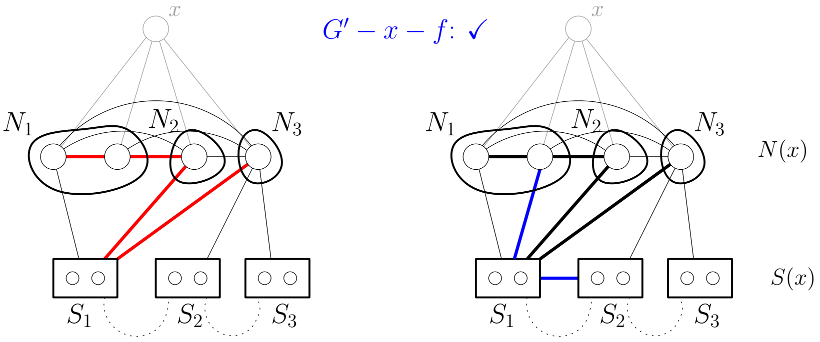
$\Rightarrow$ apply induction

Strongly Chordal graphs are Sandwich Monotone 2/3



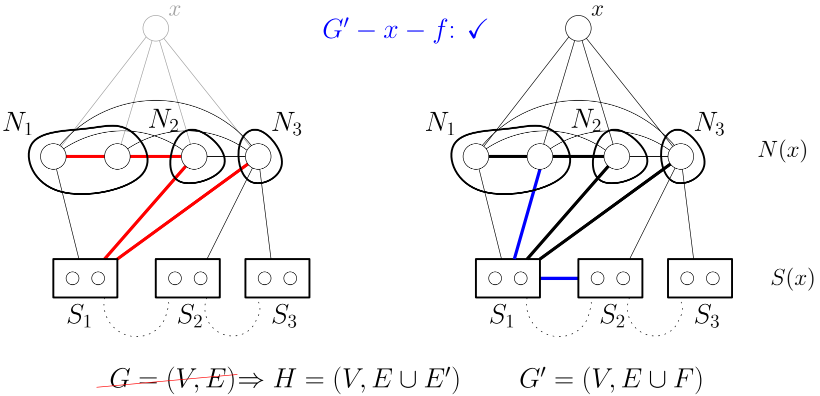
$\cancel{G = (V, E)} \Rightarrow H = (V, E \cup E')$ $\quad$ $G' = (V, E \cup F)$

- ▷ By induction $\Rightarrow$ (i) $G' - x - f$ ✓
- ▷ By the edges E' $\Rightarrow$ (ii) either $f \in S(x)$ or $f \in N_i S_i$

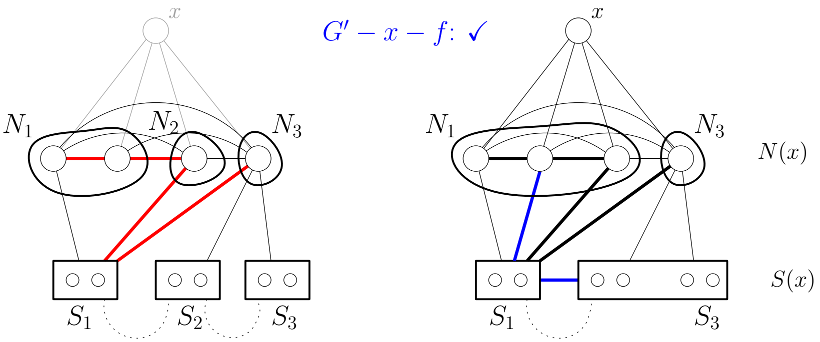


$\cancel{G = (V, E)} \Rightarrow H = (V, E \cup E')$ $\qquad G' = (V, E \cup F)$

- Construct $G' - x - f$ from $H - x$ and $G' - x$

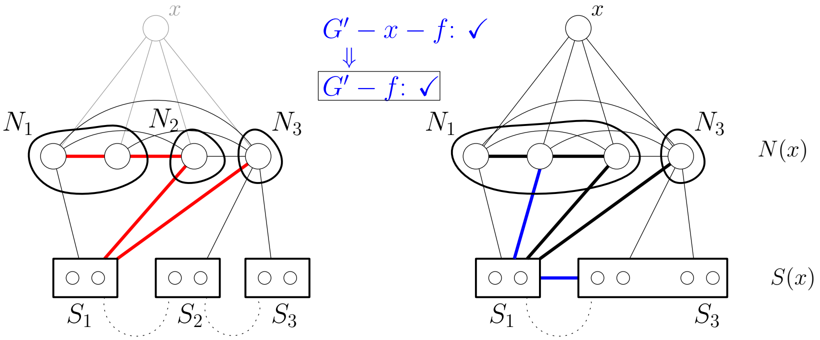

x
$G' - x - f$: ✓
N_1
N_2
N_3
$N(x)$
$S(x)$
S_1
S_2
S_3
$\cancel{G = (V, E)} \Rightarrow H = (V, E \cup E')$
$G' = (V, E \cup F)$

- Add x to $G' - x - f$



$\cancel{G = (V, E)} \Rightarrow H = (V, E \cup E')$ $\qquad G' = (V, E \cup F)$

- Add x to $G' - x - f \;\Rightarrow\; x$ is simple in $G' - f$
  (either $f \in S(x)$ or $f \in N_i S_i$)



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  (either $f \in S(x)$ or $f \in N_i S_i$)

Sketch of the proof (algorithm):

1. Pick a simple vertex x in G'
2. if x is incident to an added edge $f \in F$ then
   - 2.1 $\boxed{G' - f \checkmark}$

   else
   - 2.2 Add E' to $G = (V, E) \Rightarrow H = (V, E \cup E')$
   - 2.3 $G' - x - f$: by induction on $H - x$ and $G' - x$
   - 2.4 Add x in $G' - x - f$ as simple $\Rightarrow \boxed{G' - f \checkmark}$

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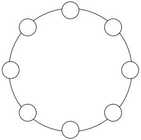
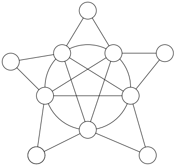
   2.4 Add x in $G' - x - f$ as simple $\Rightarrow$ $\boxed{G' - f \checkmark}$
   $N(x)$ retains the inclusion set property

- chords:

chord of a cycle: an edge between two nonconsecutive vertices
chord of a k-sun: an edge between the indep. set and the clique





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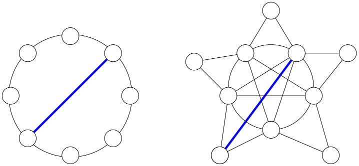

Characterizations of minimal strongly chordal completions

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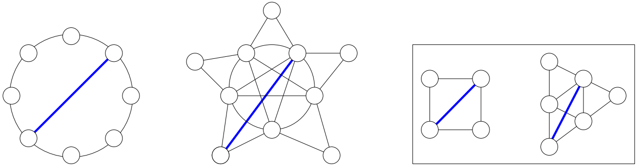

Let G be strongly chordal and e be an edge

$G - e$ is strongly chordal iff e is not the unique chord of a C_4 or the unique chord of a 3-sun.

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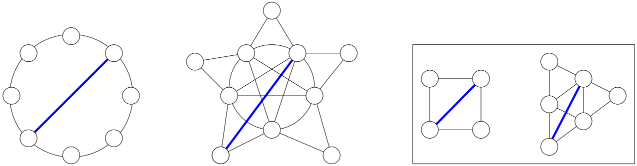
chord of a cycle: an edge between two nonconsecutive vertices
chord of a k-sun: an edge between the indep. set and the clique



Let G be strongly chordal and e be an edge

$G - e$ is strongly chordal iff e is not the unique chord of a C_4 or the unique chord of a 3-sun.

G' is a minimal strongly completion of an arbitrary graph G iff

every added edge is the unique chord of a C_4 or a 3-sun.

Overview

- *Sandwich monotone* graphs

implies that minimal completion problems are poly-time solvable

- Strongly Chordal graphs

subclass of chordal graphs

- Strongly Chordal graphs are sandwich monotone

constructive proof

- Minimal Strongly Chordal completions (characterizations)

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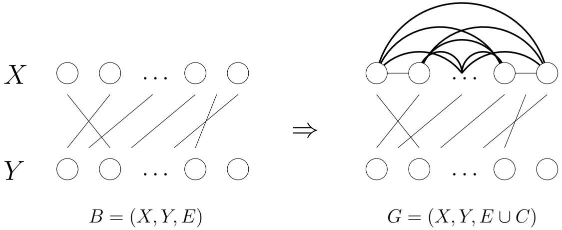
  - Let $B = (X, Y, E)$ a bipartite graph
  - Make X a clique by adding edges in $X \Rightarrow G$



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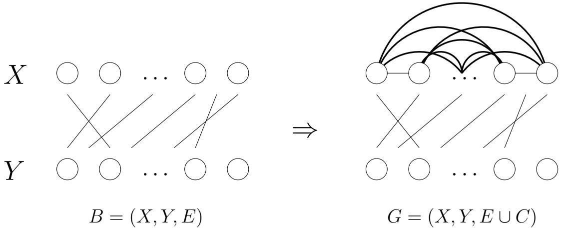

Dahlhaus, 1991

B is chordal bipartite $\Leftrightarrow$ G is strongly chordal

Chordal bipartite graphs are sandwich monotone

Sandwich monotone

$B = (X, Y, E)$ and $B' = (X, Y, E \cup F)$ two chordal bipartite graphs; there is an $f \in F$ such that $B' - f$ is chordal bipartite.

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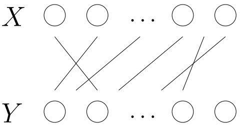

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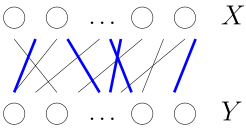

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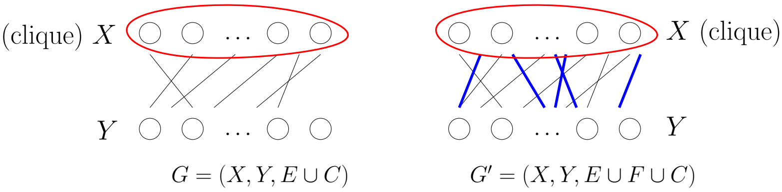

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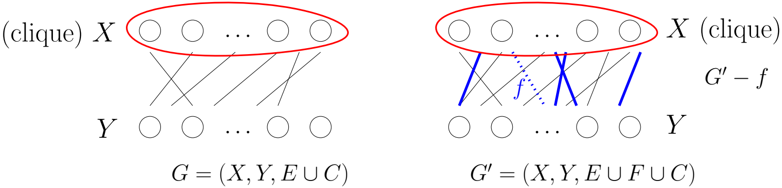

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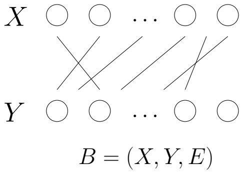

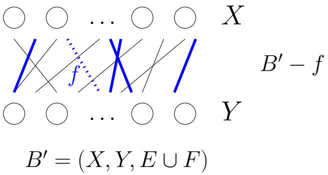

- G and G' are both strongly chordal
- By sandwich monotonicity of strongly chordal $\Rightarrow$ there is an $f \in F$ such that $G' - f$ strongly chordal
- Remove all edges in $X \Rightarrow B' - f$ is chordal bipartite

Characterizations of minimal chordal bipartite completions

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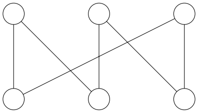

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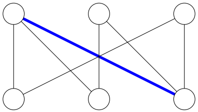

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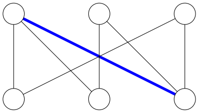

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Conclusions and Open problems

- Strongly chordal graphs and chordal bipartite graphs are sandwich monotone.
  - recognition problem $\Rightarrow \mathcal{O}(\min\{m \log n, n^2\})$
    Paige and Tarjan 1987, Spinrad 1993
  - computing a minimal completion $\Rightarrow \mathcal{O}(n^4(\min\{m \log n, n^2\}))$
    by applying a straightforward algorithm; any improvement?

- Other graph classes
  - weakly chordal: neither G nor $\bar{G}$ contain $C_5, C_6, \ldots$
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