

Encoding Watermark Integers as Self-inverting Permutations

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Abstract: *In a software watermarking environment, several graph theoretic watermark methods use integers as watermark values, where some of these methods encode the watermark integers as reducible permutation graphs (RPG; these are reducible control-flow graphs with a maximum out-degree of two). Since there is a one-to-one correspondence between self-inverting permutations and isomorphic classes of RPGs, for encoding watermark integers most of the watermarking methods use only those permutations that are self-inverting. In this paper we present an efficient algorithm for encoding integers as self-inverting permutations. More precisely, our algorithm takes as input an integer w , computes its binary representation $b_1b_2\dots b_n$, and then produces a self-inverting permutation π^* in $O(n)$ time. Moreover, we also present an algorithm for decoding a self-inverting permutation; our algorithm takes as input a self-inverting permutation π^* produced by the encoding algorithm and returns its corresponding integer w in $O(n)$ time, where n is the length of the input permutation.*

Key words: *Watermark, permutations, self-inverting permutations, encoding, decoding, graphs, algorithms.*

INTRODUCTION

Software watermarking is a technique that is currently being studied to prevent or discourage software piracy and copyright infringement. The idea is similar to digital (or, media) watermarking where a unique identifier is embedded in image, audio, or video data through the introduction of errors not detectable by human perception [6].

The Software Watermarking problem can be described as follows: Embed a structure W into a program P such that W can be reliably located and extracted from P even after P has been subjected to code transformations such as translation, optimization and obfuscation. More precisely, a Software Watermarking System can be defined as follows [10]: Given a program P , a watermark w , and a key k , a software watermarking system consists of functions:

- $\text{embed}(P, w, k) \rightarrow P'$
- $\text{recognize}(P', k) \rightarrow w$

Although digital watermarking has made considerable progress and become a popular technique for copyright protection of multimedia information [6, 12], research on software watermarking has recently received sufficient attention. The patent by Davidson and Myhrvold [7] presented the first published software watermarking algorithm. The preliminary concepts of software watermarking also appeared in paper [8] and patents [9, 11]. Collberg et al. [4, 5] presented detailed definitions for software watermarking. Authors of papers [14, 15] have given brief surveys of software watermarking research.

There are two general categories of watermarking algorithms: the static and the dynamic algorithms [4]. A static watermark is stored inside program code in a certain format, and it does not change during the program execution. According to the representation of watermark information, there are two types of static watermarks: data watermarks and code watermarks. A data watermark stores watermark information as program data, and can be stored anywhere inside a program, such as in comments or in variables.

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A code watermark is represented by choosing a particular sequence of instructions in cases (and these are common), where more than one sequence of instructions has an equivalent effect. A static code watermark may also be stored in “dead code” (which is never executed); any sequence of instructions may be used with equivalent effect in a dead-code area. For example, in a Java program, a particular order of cases in a switch statement can be used to represent a watermark number. Further discussion of static watermarking issues can be found in [7, 9, 13].

A dynamic watermark is built during program execution, perhaps only after a particular sequence of input. It might be retrieved by analyzing the data structures built when watermarked program is running. In other cases, tracing the program execution may be required. There are three kinds of dynamic watermarks: *Easter Eggs*, *Execution Trace Watermarks*, and *Dynamic Data Structure Watermarks* [4].

In 1990, Davidson and Myhrvold [7] proposed the first software watermarking algorithm which is static and embeds the watermark by reordering the basic blocks of a control flow graph. Based on this idea, Venkatesan et al. [13] proposed an algorithm which embeds the watermark by extending a method's control flow graph (CFG) through the insertion of a subgraph. The first dynamic watermarking algorithm (CT) was proposed by Collberg et al. [4]; it embeds the watermark through a graph structure which is built on a heap at runtime.

Venkatesan et al. [13] propose a software watermarking scheme which is called GTW; in such a scheme an executable program is marked by the addition of code for which the topology of the control flow graph (CFG) encodes a watermark. More precisely, the GTW process is as follows: The watermark value W is encoded as a directed graph G which, in turn, is converted into control flow graph (CFG). In [13] the construction of a directed graph G (or, watermark graph G) is not discussed. Collberg et al. [2] proposed an implementation of GTW, which they call GTWsm, and it is the first publicly available implementation of the algorithm GTW. In GTWsm the watermark is encoded as a reducible permutation graph (RPG) [3], which is a reducible control-flow graph with maximum out-degree of two, mimicking real code.

In GTWsm implementation a watermark value (integer) is encoded as a RPG; in particular, in the enumeration of Collberg et al. [3], an integer n is encoded as the RPG corresponding to the n th self-inverting permutation. Note that there is a one-to-one correspondence between self-inverting permutations and isomorphic classes of RPGs. Thus, for encoding integers the GTWsm methods use only those permutations that are self-inverting.

In this paper we propose an efficient algorithm for encoding integers as self-inverting permutations. More precisely, our algorithm takes as input an integer w , computes its binary representation $b_1b_2\dots b_n$, and then produces a self-inverting permutation π^* in $O(n)$ time. Moreover, we also propose an algorithm for decoding a self-inverting permutation; our algorithm takes as input a self-inverting permutation π^* produced by the encoding algorithm and returns its corresponding integer w in $O(n)$ time, where n is the length of the input permutation.

The paper is organized as follows: In Section 2 we define the reducible permutation graphs and the self-inverting permutations. In Section 3 we introduce the notion of the bitonic permutations which is the key-object in our algorithm for encoding integers as self-inverting permutations. In Section 4 we present our algorithm Encode-Integers-as-SIP which takes as input an integer w and produces a self-inverting permutation π^* , while in the same section we also present a recognition algorithm, that is, an algorithm which takes a self-inverting permutation π^* produced by algorithm Encode-Integers-as-SIP and returns the integer w . Finally, Section 5 concludes the paper and discusses future research directions.

REDUCIBLE PERMUTATION GRAPHS

Several graph theoretic watermarking methods encodes a watermark value in the topology of a control-flow graph (CFG) [1] and embed it in an application program P [2]. Note that, each node of a CFG represents a basic block which consists of instructions with a single entry and a single exit; two basic blocks are connected by a directed edge if, during the execution, control can pass from one basic block to the other. Moreover, note that a CFG itself also has a single entry and a single exit.

In a graph theoretic watermarking environment, the GTW method [2] forms a typical watermark embedding process. We next give an overview of the GTW method consisting of the following steps:

1. The watermark value w is split into several values w_1, w_2, \dots, w_k ;
2. The values w_1, w_2, \dots, w_k are encoded as directed graphs G_1, G_2, \dots, G_k ;
3. The generated graphs G_1, G_2, \dots, G_k are converted into CFGs W_1, W_2, \dots, W_k by generating executable code for each CFG;
4. Each W_1, W_2, \dots, W_k is marked to indicate whether it is part of the watermark, and is embedded in a specific location in the application code P;

Having embedded a watermark value w in an application program P, we are interested in designing an efficient reverse method, that is, a method which takes as input the watermarked code P and produces the watermark value w ; such a reverse method is usually called *recognition method*. The recognition method for the above described embedding method is as follows:

1. Identify the marked CFGs W_1, W_2, \dots, W_k in the application program P;
2. Each CFG W_1, W_2, \dots, W_k is decoded to compute a value;
3. The individual values are combined to yield the watermark value w ;

Some graph theoretic watermarking methods, like the GTW method, use integers as watermark values and encode them as reducible permutation graphs (RPG) [2]; these are reducible control-flow graphs with a maximum out-degree of two.

More precisely, an RPG is a reducible control-flow graph with a Hamiltonian path consisting of four pieces: (a) a *header node*, (b) the *preamble*, (c) the *body*, and (d) a *footer* [2].

There is a one-to-one correspondence between self-inverting permutations and isomorphic classes of RPGs. Thus, for encoding integers some watermarking methods use only those permutations that are self-inverting.

Let π be a permutation over the set $N_n = \{1, 2, \dots, n\}$. We think of permutation π as a sequence $(\pi_1, \pi_2, \dots, \pi_n)$, so, for example, the permutation $\pi = (1, 4, 2, 7, 5, 3, 6)$ has $\pi_1 = 1, \pi_2 = 4$, ect. Notice that $(\pi^{-1})_i$, denoted here as p_i , is the position in the sequence of the number i ; in our example, $p_4 = 2, p_7 = 4, p_3 = 6$, ect.

Definition 1: The inverse of a permutation $(\pi_1, \pi_2, \dots, \pi_n)$ is the permutation (q_1, q_2, \dots, q_n) with $q_{\pi_i} = \pi_{q_i} = i$. A *self-inverting permutation* (or, *involution*) is a permutation that is its own inverse: $\pi_{\pi_i} = i$.

By definition, every permutation has a unique inverse, and the inverse of the inverse is the original permutation. Clearly, a permutation is a self-inverting permutation if and only if all its cycles are of length 1 or 2.

BITONIC PERMUTATIONS

The key-object in our algorithm for encoding integers as self-inverting permutations is the *bitonic permutation*: a permutation $\pi = (\pi_1, \pi_2, \dots, \pi_n)$ over the set N_n is called bitonic if either monotonically increases and then monotonically decreases, or else monotonically decreases and then monotonically increases. For example, the permutations $\pi_1 = (1, 4, 6, 7, 5, 3, 2)$ and $\pi_2 = (6, 4, 3, 1, 2, 5, 7)$ are both bitonic.

In this paper, we consider only bitonic permutations that monotonically increases and then monotonically decreases. Let $\pi = (\pi_1, \pi_2, \dots, \pi_i, \pi_{i+1}, \dots, \pi_n)$ be such a bitonic permutation over the set N_n and let π_i, π_{i+1} be the two consecutive elements of π such that $\pi_i > \pi_{i+1}$. Then, the sequence $X = (\pi_1, \pi_2, \dots, \pi_i)$ is called *first increasing subsequence* of π and the sequence $Y = (\pi_{i+1}, \pi_{i+2}, \dots, \pi_n)$ is called *first decreasing subsequence* of π .

We next give some notations and terminology we shall use throughout the paper. Let w be an integer number. We denote by $B = b_1b_2\dots b_n$ the binary representation of w , where b_i is either 1 or 0 ($1 \leq i \leq n$). If $B_1 = b_1b_2\dots b_n$ and $B_2 = d_1d_2\dots d_m$ be two binary numbers, then the number $B_1||B_2$ is the binary number $b_1b_2\dots b_nd_1d_2\dots d_m$; for example, if $B_1 = 10101$ and $B_2 = 110$ are the integers 21 and 6, respectively, then the binary number $B_1||B_2 = 10101110$ is the integer 174. The binary sequence of the number $B = b_1b_2\dots b_n$ is the sequence $B^* = (b_1, b_2, \dots, b_n)$ of length n .

Let $B = b_1b_2\dots b_n$ be a binary number. Then, $\text{flip}(B) = b'_1b'_2\dots b'_n$ is the binary number such that $b'_i = 0$ (1) if and only if $b_i = 1$ (0), $1 \leq i \leq n$.

ENCODING INTEGERS

In this section, we present an algorithm for encoding an integer as self-inverting permutation. In particular, our algorithm takes as input an integer w , computes the binary representation $b_1b_2\dots b_n$ of w , and then produces a self-inverting permutation π^* in $O(n)$ time. We next describe the propose algorithm:

Algorithm Encode-Integers-as-SIP

Input: a watermark integer w ;

Output: the self-inverting permutation π^* ;

1. Compute the binary representation $B = b_1b_2\dots b_n$ of w ;
2. Construct the binary number $B' = 00\dots 0||B||1$ of length $2n+1$, and then the binary sequence $B^* = (b_1, b_2, \dots, b_n)$ of $\text{flip}(B')$;
3. Find the sequence $X = (x_1, x_2, \dots, x_k)$ of the positions of 0's and the sequence $Y = (y_1, y_2, \dots, y_m)$ of the positions of 1's in B^* from left-to-right;
4. Construct the bitonic permutation $\pi = (x_1, x_2, \dots, x_k, y_m, y_{m-1}, \dots, y_1)$ on $n' = 2n+1$ numbers;
5. Let $(z_1, z_2, \dots, z_k, z_{k+1}, z_{k+2}, \dots, z_{n'}) = (x_1, x_2, \dots, x_k, y_m, y_{m-1}, \dots, y_1)$;
 Case 1: n' even: select $n'/2$ pairs $(z_1, z_{n'})$, $(z_2, z_{n'-1})$, \dots , $(z_{n'/2}, z_{(n'+3)/2})$;
 for each selected pair (z_i, z_j) , do the following:
 $\pi_{z_j} = z_i$ and $\pi_{z_i} = z_j$;
 Case 2: n' odd: select $\lfloor n'/2 \rfloor$ pairs $(z_1, z_{n'})$, $(z_2, z_{n'-1})$, \dots , $(z_{\lfloor n'/2 \rfloor}, z_{\lfloor n'/2 \rfloor + 2})$ and the number $z_{\lfloor n'/2 \rfloor + 1}$;
 for each selected pair (z_i, z_j) , do the following:
 $\pi_{z_j} = z_i$ and $\pi_{z_i} = z_j$;
 $\pi_{z_{\lfloor n'/2 \rfloor + 1}} = z_{\lfloor n'/2 \rfloor + 1}$;
6. Return the self-inverting permutation $\pi^* = (\pi_1, \pi_2, \dots, \pi_{n'})$ on $n' = 2n+1$ numbers;

Example 1: Let $w = 12$ be the input watermark integer in the Algorithm Encode-Integers-as-SIP. We first compute the binary representation $B = 1100$ of the number 12; then we

construct the binary number $B' = 000011001$ and the binary sequence $B^* = (1, 1, 1, 1, 0, 0, 1, 1, 0)$ of $\text{flip}(B')$; we compute the sequences $X = (5, 6, 9)$ and $Y = (1, 2, 3, 4, 7, 8)$, and then construct the bitonic permutation $\pi = (5, 6, 9, 8, 7, 4, 3, 2, 1)$ on $n' = 9$ numbers; since $n'=9$ odd, we select 4 pairs $(5, 1), (6, 2), (9, 3), (8, 4)$ and the number 7 and then construct the self-inverting permutation $\pi^* = (5, 6, 9, 8, 1, 2, 7, 4, 3)$.

Next, we present a recognition algorithm, that is, an algorithm for decoding a self-inverting permutation. More precisely, our recognition algorithm, which we call Decode-SIP, takes as input a self-inverting permutation π^* produced by Algorithm Encode-Integers-as-SIP and returns its corresponding integer w . The time complexity of the decode algorithm is also $O(n)$, where n is the length of the permutation π^* . We next describe the propose algorithm:

Algorithm Decode-SIP

Input: a self-inverting permutation π^* produced from Algorithm Encode-Integers-as-SIP;

Output: an integer w ;

1. Compute the cycle representation $C = c_1c_2 \dots c_k$ of the self-inverting permutation $\pi^* = (\pi_1, \pi_2, \dots, \pi_{n'})$, where $n' = 2n+1$;
2. Initially, $i = 1, j = n'$ and all the cycles of C are unmarked;
3. While there exists an unmarked cycle c in C , do the following:
 - Select the first unmarked cycle c of C from left-to-right;
 - Case 1: the selected cycle c has length 2 and let $c = (a, b)$:
 - $\pi_i = a$ and $\pi_j = b$;
 - mark cycle c ; $i = i+1$ and $j = j -1$;
 - Case 2: the selected cycle c has length 1 and let $c = (a)$:
 - $\pi_i = a$; mark cycle c ; $i = i+1$;
4. Find the first increasing subsequence $X = (x_1, x_2, \dots, x_k)$ and then the first decreasing subsequence $Y = (y_1, y_2, \dots, y_m)$ of π ;
5. Construct the binary sequence $B^* = (b_1, b_2, \dots, b_{n'})$ as follows:
 - set 0's in positions x_1, x_2, \dots, x_k and 1's in positions y_1, y_2, \dots, y_m ;
6. Compute $B' = \text{flip}(B^*) = (b_1, b_2, \dots, b_n, b_{n+1}, \dots, b_{2n}, b_{2n+1})$;
7. Return the integer w of the binary number $B = b_{n+1}, b_{n+2}, \dots, b_{2n}$;

Example 2: Let $\pi^* = (5, 6, 9, 8, 1, 2, 7, 4, 3)$ be a self-inverting permutation produced from Algorithm Encode-Integers-as-SIP. The cycle representation of π^* is the following: $(1, 5), (2, 6), (3, 9), (4, 8), (7)$; from the cycles we construct the permutation $\pi = (5, 6, 9, 8, 7, 4, 3, 2, 1)$; then, we compute first increasing subsequence $X = (5, 6, 9)$ and the first decreasing subsequence $Y = (8, 7, 4, 3, 2, 1)$; we then construct the binary sequence $B^* = (1, 1, 1, 1, 0, 0, 1, 1, 0)$ of length 9; we flip the elements of B^* and construct the sequence $B' = (0, 0, 0, 0, 1, 1, 0, 0, 1)$; the binary number 1100 is the integer $w = 12$;

CONCLUSIONS AND FUTURE WORK

In this paper we presented an efficient algorithm for encoding watermark integers as self-inverting permutations. Our algorithm takes as input an integer w and produces a self-inverting permutation π^* in $O(n)$ time, where n is the number of bits in the binary representation $b_1b_2\dots b_n$ of w . We also presented a decoding algorithm; it takes as input a self-inverting permutation π^* produced by the encoding algorithm and returns its corresponding integer w in $O(n)$ time, where n is the length of the input permutation. Both algorithms are simple, easy implemented and very fast.

It is worth noting that our approach enable us to encode the integer $w = b_1b_2\dots b_n$ as self-inverting permutation π^* of any length; indeed, π^* can be constructed over the set $N_{n'} = \{1, 2, \dots, n'\}$, where the smallest value of n' is $O(\log n)$.

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