

# A descriptive view of combinatorial group theory

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## Abstract

It is well-known that descriptive set theory provides a framework for measuring the relative complexity of many naturally occurring classification problems. But it is less well-known that descriptive set theory also provides a framework for explaining the inevitable non-uniformity of many classical constructions in mathematics. In this tutorial, I will illustrate this point by considering some constructions from combinatorial group theory.

For example, the Higman-Neumann-Neumann Embedding Theorem states that any countable group  $G$  can be embedded into a 2-generator group  $K$ . In the standard proof of this classical theorem, the construction of the group  $K$  involves an enumeration of a set of generators of the group  $G$ ; and it is clear that the isomorphism type of  $K$  usually depends upon both the generating set and the particular enumeration that is used. One of the main results of this tutorial will be that there does not exist a more uniform construction with the property that the isomorphism type of  $K$  only depends upon the isomorphism type of  $G$ .

In the first lecture, I will explain how to formulate various uniformity problems using the notions of descriptive set theory and I will discuss the statements of the main results; and in the remaining two lectures, I will present some proofs. In contrast to much of the recent work in descriptive set theory, the proofs in this tutorial will only involve purely set-theoretical notions such as forcing, large cardinals and Borel determinacy.